Textbook OF Electricity & Magnetism

(For three years' degree course)

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PREFACE TO THE FIRST EDITION

Although there is a large number of books on Electricity and Magnetism written by distinguished foreign authors, there is hardly any book exactly suitable for a student preparing for the degree course in an Indian University. The present book is an attempt to remove this long felt want.

In writing this book I have deliberately cut down all unimportant elementary portions (which are usually done in the Intermediate stage) so much so that during the first few chapters, the book may appear to be almost a note rather than a Text book. It is however far from my intention to write a note on this subject, as will be amply borne out in later chapters where no pains have been spared to discuss all relevant subjects very thoroughly. I wish I could introduce many more important topics into this book. It is only due to shortness of time that this has not been possible in this edition. If I get an opportunity I have a mind to do this in a later edition.

I have great pleasure in acknowledging help from many of my colleagues, particularly from Sj. Kamada Majumdar of Sibpur Engineering College and from Sj. Bhabesh Kumar Som of Hoogly Mohsin College. I have also freely taken help from the well known authors.

Inspite of my best care the book has not been free from printing mistakes. This is due to the book being rushed through the press. An errata is given at the end of the book pointing out the important mistakes.

Suggestions for improvement of the book will be gratefully appreciated.

PRESIDENCY COLLEGE
CALCUTTA

D. P. Acharya

The 18th September, 1948

PREFACE TO THE FIFTH EDITION

Through the kind patronage of students and teachers the book has gone into the fifth edition. I am grateful to all.

In this edition considerable changes have been introduced. The theory of ballistic galvanometers has now been included. In the chapter on Wireless mathematical treatment has been partly introduced. Besides, several changes have been made here and there.

Due to printing mistakes several errors were introduced in the "Answers" in the preceding editions of the book. All the problems have therefore been worked out again and the answers have been checked. Many new problems have also been introduced. I shall be thankful if any mistake still found to be present in the "Answers" be pointed out to me.

I am grateful to Sj. Kamadakanta Majumdar of Sibpur B. E. College who has helped me considerably in re-writing the chapter on Wireless.

CALCUTTA

12 September, 1958 **D. P. Acharya**

PREFACE TO THE TWELFTH EDITION

This, the twelfth edition, has been thoroughly revised and brought uptodate incorporating the latest useful material for the benefit of students. Many more excercises and latest questions have been added at the end of each chapter. It is hoped that this revised edition will also prove useful to the students as the previous editions.

December, 1977 **D.P. Acharya**

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Statical Electricity

Current Electricity

MAGNETISM

CHAPTER I

GENERAL THEORY

Art 1 Introduction

Magnets may be either natural or artificial. A natural magnet known as lodestone* is found as a mineral in Asia Minor, Norway, Sweden and several other places. Artificial magnets may be prepared from soft iron or steel. In recent times an alloy known as Heuslar's alloy has been prepared. This does not contain iron or steel but can be magnetised fairly strongly. All magnets possess two remarkable properties, *viz.* (1) they attract iron filings and (2) when suspended freely from the centre of gravity by a light string, they always rest in the North-South direction. It is found that the power of attracting iron filings is concentrated at two points near the two ends. These points are known as poles. The line joining the two poles is known as the magnetic axis of the magnet. When the magnet is freely suspended the pole which points towards the North is known as North *seeking* pole or simply North pole; similarly the pole directed towards the South is called South *seeking* pole or South pole.

Art 2 Fundamental Formula

If two poles of strengths m_1 and m_2 be separated by a distance r each is acted on by a force $F=\frac{m_1 m_2}{\mu r^2}$ where μ is a constant depending upon the nature of the medium in which the poles are placed and is called the permeability of the medium.

If the two poles are like the force is of repulsion; if they are unlike the force is of attraction. In either case the magnitude of the force between two poles is given by the above expression.

For air the permeability is one; hence if the poles be placed in air the force between them is equal to $\frac{m_1 m_2}{r^2}$. In

* Magnetite is the modern name of this mineral

this equation we notice that if $r=1$ cm, $F=1$ dyne and $m_1=m_2$, then $m_1=m_2=1$ C. G. S. unit. Thus we have the following definition of a unit pole :—

Unit Pole

If we have in air two poles of equal strength at a distance of one cm apart and if the force between them be one dyne then each of the two poles is said to be a unit pole.

In practice we never come across single poles. We deal with magnets which always contain a pair of poles,—a North pole and a South pole—both of the same strength.

It follows that we can never have bodies charged with one kind of magnetism in the way in which bodies are charged with positive or negative electricity.

Magnetic moment

The magnetic moment of a magnet is defined to be the couple* acting on the magnet when it is placed at right angles to a uniform field of unit strength and is measured by the product of the strength of any one pole and the distance between the poles. Thus $M=m\times 2l$ where $M=$ Magnetic moment, $m=$ pole strength, $2l=$ distance between the poles.

We shall see that in all problems on magnetism our ultimate formula always contains the magnetic moment and not the pole-strength. The magnetic moment of a magnet may therefore be regarded as more fundamental than the pole-strength.

Art 3
Magnetic intensity

The magnetic intensity at any point is defined to be the force experienced by a unit North pole placed at that point.

Thus if we consider a point at a distance r from a pole of strength m the magnetic intensity at the point is given by $F=\frac{m.1}{\mu r^2}=\frac{m}{\mu r^2}$. If the medium be air the intensity is $\frac{m}{r^2}$.

It follows from the definition that if we have a pole of strength m' placed at a point where the intensity is F, the force on the pole is m' F.

It is to be noted that the intensity is a vector quantity, *i. e.* it has both magnitude and direction. The magnitude

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* Vide Art 7

is given by $\frac{m}{\mu r^2}$ and the direction is indicated by the fact that the intensity is the force experienced by a unit *north* pole. It is obvious that the force on a unit *south* pole is equal in magnitude but is directed in the opposite direction.

The magnetic intensity is also known as the strength of the field or field strength. The word 'field' alone often conveys the same idea.

Magnetic potential

The magnetic potential at any point is the work done in bringing a unit North pole from infinity up to the point. Hence the potential difference between two points is the work done in carrying a unit North pole from one point to the other.

Consider two points A and B on the X axis at distances x and $x+dx$ from an arbitrary origin O. Let the potential at A be V and that at B V + dV. Then the potential difference $V_A - V_B = V - (V + dV) = -dV$.

Fig 1

This must be equal to the work done in carrying a unit north pole from B to A. If F be the magnetic intensity along X axis at the point A*, the force on a unit north pole between A and B is F; and since the distance traversed is $AB = dx$, we have

$$-dV = F dx \quad \text{or} \quad F = -\frac{dV}{dx} \quad \ldots \quad (1)$$

In this equation F measures the intensity along X axis. If the actual intensity be in any oblique direction the component along X axis is $-\frac{dV}{dx}$; similarly the component along Y axis is $-\frac{dV}{dy}$ and that along Z axis is $-\frac{dV}{dz}$.

Equation (1) gives us the relation between the magnetic intensity and the magnetic potential at any point. It is to be noted that unlike magnetic intensity magnetic potential

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* A and B are supposed to be so close that the magnetic intensity at A is the same as that at B or at any point between A and B.

is a scalar quantity, *i. e.* it has magnitude but no direction.

Consider a point A at a distance x from a pole of strength m. Then the intensity at A is $\frac{m}{\mu x^2}$

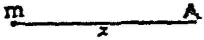


Fig. 2

i. e. $-\frac{dV}{dx}=\frac{m}{\mu x^2}$ or $dV=-\frac{m dx}{\mu x^2}$

$$\therefore \quad V=-\int\frac{m dx}{\mu x^2}+C=\frac{m}{\mu x}+C,$$

where C is the constant of integration.

The potential at infinity is supposed to be zero, *i. e.* $V=0$ when $x=\infty$: we thus have $C=0$.

Hence $$V=\frac{m}{\mu x} \quad \ldots \quad \ldots \quad (2)$$

If the medium be air $$V=\frac{m}{x} \quad \ldots \quad \ldots \quad (2a)$$

Throughout the remaining portion of this book we shall always assume the medium to be air unless otherwise stated.

Potential and intensity due to a bar magnet

First Case

Art 4
End on position

Fig. 3

Consider a point P on the axial line of the magnet NS whose centre is O. The point P is said to be in 'Tan A' position of Gauss or in 'End on' position with respect to the magnet NS.

Let $OP=r$, $NS=2l$, and m be the pole strength. Then the magnetic moment $M=m\times 2l$.

Potential. $$V_P=\frac{m}{NP}-\frac{m}{SP}=\frac{m}{r-l}-\frac{m}{r+l}$$

$$=\frac{2ml}{r^2-l^2}=\frac{M}{r^2-l^2} \quad \ldots \quad \ldots \quad (3)$$

If the magnet be small $$V_P=\frac{M}{r^2} \quad \ldots \quad \ldots \quad (3a)$$

Intensity. $F_P = \frac{m}{NP^2} - \frac{m}{SP^2} = \frac{m}{(r-l)^2} - \frac{m}{(r+l)^2}$

$$= \frac{4mrl}{(r^2-l^2)^2} = \frac{2Mr}{(r^2-l^2)^2} \quad \ldots \quad \ldots \quad (4)$$

If the magnet be small $F_P = \frac{2M}{r^3}$ ... ... (4a)

This intensity acts in the direction NP produced, *i. e.* *along* the axis of the magnet, *away* from the North pole.

If a magnet be placed in the N-S direction with its *North pole pointing south,* then there are two points on the axis produced at equal distances on either side of the magnet such that the intensity due to the magnet at any of these points, is equal and opposite to the horizontal component* of Earth's magnetism.

Thus, at these points

$$\frac{2Mr}{(r^2-l^2)^2} = H \text{ or } \frac{M}{H} = \frac{(r^2-l^2)^2}{2r}.$$

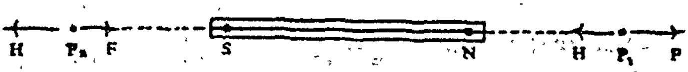


Fig. 4

Neutral points

Such points where the total intensity is nil are called neutral points. Evidently from a knowledge of the positions of these neutral points the value of $\frac{M}{H}$ can be found out.

Second case

Broadside on position

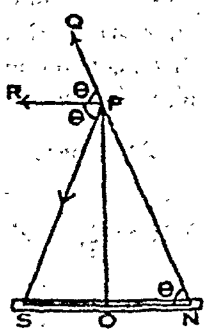


Fig 5

Consider a point P on the equatorial line of the magnet NS. The point is said to be in 'Tan B' position of Gauss or in 'Broad-side on' position with respect to the magnet NS. As before let OP=r, NS=$2l$.

* Vide Art 12

Potential. $V_P = \frac{m}{NP} - \frac{m}{SP} = 0$ ... ... (5)

Intensity. Intensity at P due to North pole $= \frac{m}{NP^2}$ along NP produced, *i. e.* along PQ. Intensity at P due to South pole $= \frac{m}{SP^2}$ along PS.

These two components are numerically equal. Hence their resultant bisects the angle between them, *i. e.* acts along PR in a direction parallel to NS.

Let $\angle PNO = \theta$, then $\angle QPR = \angle RPS = \theta$. Hence the resultant intensity at P in the direction PR is

$$F_P = \frac{m}{NP^2}\cos\theta + \frac{m}{SP^2}\cos\theta = \frac{2m}{NP^2}\cos\theta$$

$$= \frac{2m}{r^2+l^2}\,\frac{l}{\sqrt{(r^2+l^2)}} = \frac{M}{(r^2+l^2)^{3/2}} \quad \ldots \quad \ldots \quad (6)$$

If the magnet be small $F_P = \frac{M}{r^2}$ ... ... (6a)

This intensity acts in a direction *parallel* to the magnet and *away* from the North pole.

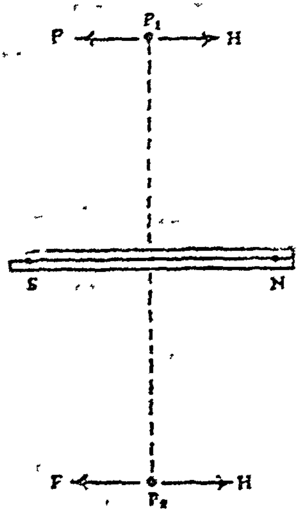


Fig 6

If a magnet be placed in the N-S direction with its *North pole pointing north* then the neutral points are on the equatorial line at equal distances on either side of the magnet.

At these points $\frac{M}{(r^2+l^2)^{3/2}} = H$ or $\frac{M}{H} = (r^2+l^2)^{3/2}$

Clearly $\frac{M}{H}$ can be determined from these neutral points also.

Third case

General case

Consider a point P in the neighbourhood of a small* magnet NS whose centre is O. Join OP, NP, SP. Let $OP = r$, $NS = 2l$, $\angle PON = \theta$.

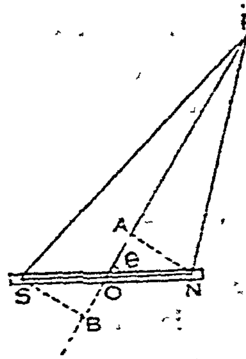


Fig. 7

Potential

1st Method. From N and S drop NA and SB perpendiculars to OP produced if necessary. Since the magnet is small NP may be regarded as equal to AP and SP to BP.

$$\therefore \quad V_P = \frac{m}{NP} - \frac{m}{SP} = m\,\frac{SP - NP}{NP.SP} = m\,\frac{BP - AP}{AP.BP}$$

$$= m\,\frac{AB}{(OP - OA)(OP + OB)} = m.\,\frac{2OA}{(r - l\cos\theta)(r + l\cos\theta)}$$

$$= m\,\frac{2l\cos\theta}{r^2} \quad [\text{ neglecting } l^2]$$

$$= \frac{M\cos\theta}{r^2} \quad \ldots \quad \ldots \quad \ldots \quad (7)$$

As the length of the magnet increases the value of $M(=2l)$ increases and hence the potential increases. If, however, the length becomes very large $SP \neq BP$ and $NP \neq AP$ and hence the above *proof fails. In this case no simple expression can be arrived at for the potential or for the intensity at the point.*

2nd Method. Resolve the magnetic moment M along OP and perpendicular to OP ; these components are $M\cos\theta$ and $M\sin\theta$. With respect to the first component the point P is in 'tan A' position and the potential at P is therefore $\frac{M\cos\theta}{r^2}$ [from (3a)]. With respect to the second component the point P is in 'Tan B' position and the potential at P is

zero [from (5)]. Hence the total potential at $P = \frac{M \cos\theta}{r^2}$.

The agreement of this result with that obtained by the first method justifies us in assuming that the magnetic moment of a magnet may be resolved like any other vector quantity.

Intensity

First Method

Resolve the magnetic moment along OP and perpendicular to OP. These components are $M \cos\theta$ and $M \sin\theta$. Due to first of these the intensity at P is $\frac{2M\cos\theta}{r^3}$ along OP produced *i.e.* along PA [from (4a)] and due to the second the intensity is $\frac{M\sin\theta}{r^3}$ perpendicular to OP and away from the north pole, *i. e.* along PB [from (6a)]

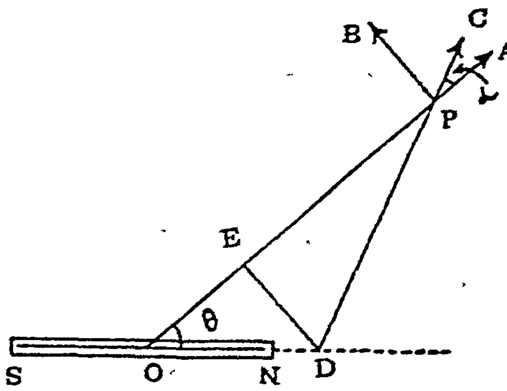


Fig. 8

Hence the resultant intensity at P is

$$F_P = \sqrt{\left(\frac{2M\cos\theta}{r^3}\right)^2 + \left(\frac{M\sin\theta}{r^3}\right)^2} = \frac{M}{r^3}\sqrt{4\cos^2\theta + \sin^2\theta}$$

$$= \frac{M}{r^3}\sqrt{1+3\cos^2\theta} \quad \ldots \quad \ldots \quad (8)$$

The direction of the resultant is along PC making an angle $\angle APC = \alpha$ with PA where

$$\tan\alpha = \frac{M\sin\theta}{r^3} \Big/ \frac{2M\cos\theta}{r^3} = \tfrac{1}{2}\tan\theta \quad \ldots \quad (8a)$$

A simple geometrical construction for finding this direction PC can be deduced readily. For, producing CP to meet SN produced at D and dropping DE perpendicular to OP, we notice that

$$\tan\theta = \frac{DE}{OE} \text{ and } \tan\alpha = \tan\angle DPE = \frac{DE}{EP}$$

$$\therefore \quad \text{from (8a)} \quad \frac{DE}{EP} = \tfrac{1}{2}\frac{DE}{OE} \quad \therefore \quad EP = 2OE.$$

Thus E is a point of trisection of OP nearer to O.

Hence we obtain the following geometrical method for determining the direction of the resultant intensity at P :—

Trisect OP and take the point E such that PE = 2OE. At E draw ED perp. to OP meeting SN produced at D. Join DP. Then DP produced gives us the direction of the intensity at P, *i. e.* of the line of force through P.

Problem. Find by this geometrical method the direction of the intensity at the point P when the point is on the other side of the equatorial line, *i. e.* in the south polar region of the magnet.

Second method

From (7) the potential at P $\quad V = \dfrac{M\cos\theta}{r^2}$

$\therefore$ Intensity at P along OP $(=r) = -\dfrac{dV}{dr}$

$$= -\frac{d}{dr}\left(\frac{M\cos\theta}{r^2}\right) = \frac{2M\cos\theta}{r^3}$$

And intensity at P perp. to OP

$$= -\frac{dV}{rd\theta} \qquad [\because \text{ a small distance measured perp. to OP is } rd\theta\,]$$

$$= -\frac{1}{r}\cdot\frac{d}{d\theta}\left(\frac{M\cos\theta}{r^2}\right) = \frac{M\sin\theta}{r^3}$$

We thus find the components along OP and perp to OP ; we can then find the resultant intensity at P as in the first method.

We now proceed to determine the position of the neutral point in the special case when a small magnet NS is placed in East-West direction with its North pole pointing East. Consider a point P (in the Southern region of the magnet

Article 4 (a) NS) at a distance r from the centre O of the magnet NS, the line OP making an angle θ with the axis of the magnet. Then the two components of the

magnetic intensity at the point P due to the magnet NS are $F_1\left(=\frac{2M\cos\theta}{r^3}\right)$ along OP produced and $F_2\left(=\frac{M\sin\theta}{r^3}\right)$ in a direction perpendicular to OP where M is the magnetic moment of the magnet. We can resolve these components along E-W and also along N-S directions. If P be a neutral point the components

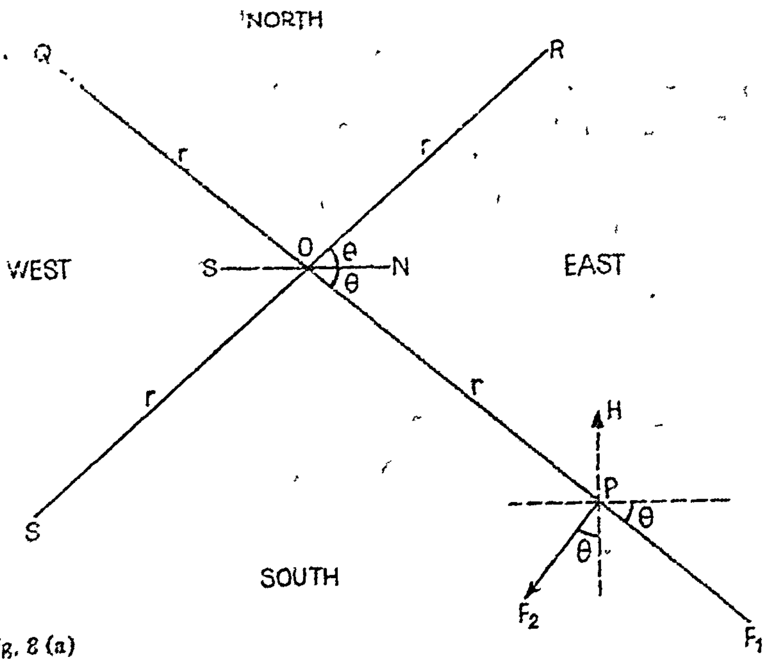


Fig. 8 (a)

$F_1\cos\theta$ and $F_2\sin\theta$ along E-W direction must cancel each other and the components $F_1\sin\theta$ and $F_2\cos\theta$ along N-S direction must together be equal and opposite to H (Earth's Horizontal Component). Thus

$$F_1\cos\theta=F_2\sin\theta \quad\text{or}\quad \frac{2M\cos^2\theta}{r^3}=\frac{M\sin^2\theta}{r^3} \quad\therefore\ \tan\theta=\sqrt{\cdot}$$

$$\text{and}\quad F_1\sin\theta+F_2\cos\theta=H \quad\text{or}\quad \frac{3M\sin\theta\cos\theta}{r^3}=H \qquad \text{(a)}$$

$$\text{Since } \tan\theta=\sqrt{2}\ \therefore\ \sin\theta=\frac{\sqrt{2}}{\sqrt{3}} \text{ and } \cos\theta=\frac{1}{\sqrt{3}}$$

$\therefore$ from (a) $\frac{3M}{r^3} \cdot \frac{\sqrt{2}}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}} = H \qquad \therefore \frac{M}{H} = \frac{r^3}{\sqrt{2}}$

Thus from a knowledge of the position of the neutral point r and θ can be determined; and hence $\frac{M}{H}$ can be found out.

N.B. (1) It may be easily seen that if PO be produced to Q beyond O on the other side of the magnet such that OP = OQ then Q is also a neutral point.

(2) If the N pole of the magnet points West the neutral points are R and S where the straight line ROS makes the same angle θ with the magnet and OR = OS = OP = OQ.

A uniformly magnetised bar magnet 10 cms long and of moment 200 is placed horizontally with its axis in the magnetic meridian and the north pole pointing north. A small compass needle placed at a distance of 10 cms east of the centre of the bar, is observed to be in neutral equilibrium. Find the horizontal intensity of the earth's field. C. U. 1939

OP = 10 cms. NS = $2l$ = 10 cms $\therefore$ l = 5 cms. Intensity at P due to the magnet

$$F = \frac{M}{(r^2+l^2)^{3/2}} = \frac{200}{(10^2+5^2)^{3/2}}$$

= 0·143 C. G. S. unit.

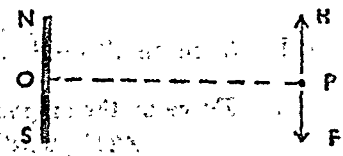


Fig. 9

At the neutral point H = F = 0·143 C. G. S. unit.

Find the value of the potential at a point situated on a line passing through the middle point of a magnet of moment 30 and making an angle of 60° with its axis, the point being 5 cms away from the mid point of the magnet.

C. U. 1943, 1951, 1955.

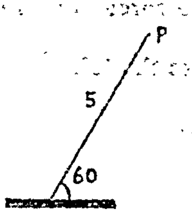


Fig. 10

$$V_P = \frac{M \cos\theta}{r^2} = \frac{30 \cos 60^\circ}{5^2} = \frac{15}{25} = 0·6$$

C. G. S. unit

Action between two small magnets.

Art 5

First Case

Consider two small magnets NS and N′S′ lying along the same straight line with their centres O and O′ separated by a distance r. Let m, $2l$ and M be the pole strength, magnetic length and moment of the magnet NS and let m', $2l'$ and M′ be the corresponding quantities of the magnet N′S′.

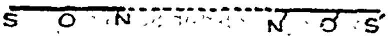


Fig. 11

Action of NS *on* N′S′.

Intensity at O′ due to NS $=\frac{2M}{r^3}$ along OO′ to the right.

$$\therefore \text{ Intensity at N}' = \frac{2M}{r^3} - \frac{d}{dr}\left(\frac{2M}{r^3}\right) l' = \frac{2M}{r^3} + \frac{6Ml'}{r^4}$$

$$\text{and intensity at S}' = \frac{2M}{r^3} + \frac{d}{dr}\left(\frac{2M}{r^3}\right) l' = \frac{2M}{r^3} - \frac{6Ml'}{r^4}$$

$$\therefore \text{ Force on N}' = m'\left(\frac{2M}{r^3} + \frac{6Ml'}{r^4}\right) \quad \text{to the right}$$

$$\text{and force on S}' = m'\left(\frac{2M}{r^3} - \frac{6Ml'}{r^4}\right) \quad \text{to the left}$$

$\therefore$ Force on the magnet N′S′

$$= m'\left(\frac{2M}{r^3} + \frac{6Ml'}{r^4}\right) - m'\left(\frac{2M}{r^3} - \frac{6Ml'}{r^4}\right)$$

$$= \frac{12Mm'l'}{r^4} = \frac{6MM'}{r^4} \quad \text{to the right}$$

In a similar way it can be proved that the force on the magnet NS by N′S′ is also equal to $\frac{6MM'}{r^4}$ but to the left.

Mutual Potential Energy.

$$\text{Potential at O}' \text{ due to NS} = \frac{M}{r^2}$$

$$\therefore \text{ Potential at N}' = \frac{M}{r^2} - \frac{d}{dr}\left(\frac{M}{r^2}\right) l' = \frac{M}{r^2} + \frac{2Ml'}{r^3}$$

$$\text{and potential at S}' = \frac{M}{r^2} + \frac{d}{dr}\left(\frac{M}{r^2}\right) l' = \frac{M}{r^2} - \frac{2Ml'}{r^3}$$

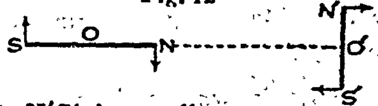
$\therefore$ Potential energy

$$=m'\left(\frac{M}{r^3}+\frac{3Ml'}{r^3}\right)-m'\left(\frac{M}{r^3}-\frac{2Ml'}{r^3}\right)$$

$$=\frac{4Mm'l'}{r^3}=\frac{2MM'}{r^3}.$$

Second Case

Fig. 12



Action of NS *on* N'S'

Intensity at $O'=\frac{2M}{r^3}$ to the right. Since the magnet N'S' is small we may assume that the intensity at N' or at S' is the same as that at O'.

Hence force on $N'=m'\frac{2M}{r^3}$ to the right

and force on $S'=m'\frac{2M}{r^3}$ to the left

Thus N'S' is acted on by a *clockwise* couple of moment

$$m'\frac{2M}{r^3}\times 2l'=\frac{2MM'}{r^3}.$$

Action of N'S' on NS.

Intensity at $O=\frac{M'}{r^3}$ downwards. Assuming this to be also the intensity at N or at S, we have

$$\text{force on } N=m\,\frac{M'}{r^3} \quad \text{downwards}$$

and $$\text{force on } S=m\,\frac{M'}{r^3} \quad \text{upwards}$$

$\therefore$ The magnet NS is also acted on by a *clockwise* couple of moment $m\,\frac{M'}{r^3}\times 2l=\frac{MM'}{r^3}$

Considering therefore the two magnets as belonging to one system the resultant couple acting on this system is *clockwise* and is of moment equal to $\frac{3MM'}{r^3}$

Magnetic paradox. We are thus led to the following paradoxical conclusion :—

If we have two small magnets placed in the above way on a large piece of cork a resultant couple of moment equal to $\frac{3MM'}{r^3}$ will act on this system. If we therefore allow the cork to float in water the cork will rotate continuously. But this is impossible as it goes against the principle of conservation of energy.

This is what is known as the magnetic paradox. The explanation lies in the fact that although the magnets are small the intensity at the centre of a magnet cannot strictly be regarded as equal to that at either of the poles. If we allow for this variation by a more rigorous calculation it is found that another couple, anticlockwise and of moment $\frac{3MM'}{r^3}$ acts on the system. This couple neutralises the previous one and the system is therefore in equilibrium.

Mutual Potential Energy.

Potential at O′ due to $NS = \frac{-M}{r^2}$. Assuming this to be the same at N′ and also at S′, we have

$$\text{Potential energy} = m'.\frac{M}{r^2} - m'.\frac{M}{r^2} = 0.$$

Exercise 1.

1. What is meant by a pole of unit strength ? Define unit potential and unit field and establish the relation between the potential and the field at any point.

2. The repulsive force between two poles is 60 dynes when they are 9 cms apart. What will it be if the distance be decreased to 6 cms ? Ans. 135 dynes.

3. A short magnet is free to rotate about an axis perpendicular to the axis of the magnet and passing through its centre. Obtain an expression for the potential at a fixed point in the

plane of rotation and at an appreciable distance from the centre of the magnet.

Discuss how the potential at the point changes with the angle of rotation of the magnet. C. U. 1933

4. Two short bar magnets are arranged so that the axis of one produced bisects the axis of the other at right angles. Find the couple on each due to the other.

5. The force of attraction between two magnetic poles at a distance of 3 cms from each other, is equal to the weight of a gramme. If one of the poles be of strength 60 find the strength of the other. Ans. 147·2

6. A magnetic pole is acted on by a force of 20 dynes when placed in the magnetic field of strength 0·25. Find the strength of the pole. Ans. 80

7. What is meant by a neutral point in a magnetic field?

The neutral point of a short magnet is 24 cms from the centre of the magnet which lies with its axis north and south and the N pole pointing to the north. If the value of H be 0·21 C. G. S. unit what is the moment of the magnet? C. U. 1934.

Ans 2903 C. G. S. units.

8. Explain what is meant by magnetic moment.

Prove that the intensity of the magnetic field due to a small bar magnet, "end on" is twice that due to the same magnet "broadside on" at the same distance.

Two short bar magnets of moments 108 and 192 units are placed along two lines drawn on the table at right angles to each other. Find the intensity of the field at the point of intersection of the lines, the centres of the magnets being respectively 30 and 40 cms from the point. C. U. 1946

Ans. 0·01 making an angle of $\tan^{-1}\frac{3}{4}$ with the line joining the point to the centre of the magnet of moment 108 units.

9. Find the magnitude and direction of the magnetic field due to a small magnet of moment 50 at a point situated on a line passing through the middle of the magnet and making an angle of 60° with its axis, the point being at a distance of 12 cms from the centre of the magnet. Explain by an accurate

drawing how the direction of the field may be determined graphically.

Ans. 0·383 C. G. S unit making an angle of 40°54′ with the line joining the point with the centre of the magnet.

10. A magnet of pole strength 1000 and length 12 cms is placed on a drawing board. Find the intensity of the field at a point in (i) 'tan A' position and (ii)'tan B' position at a distance of 20 cms in each case from the middle point of the magnet.

Ans. (s) 3·62 C. G.S. unit (ii) 1·32 C. G. S unit.

C. U. Questions.

1961. Explain what is meant by magnetic moment.

Prove that the intensity of magnetic field due to a small bar magnet in end-on' position is twice that due to the same magnet in broadside on position at the same distance.

1966. Define magnetic potential. Derive an expression for the potential at any point due to a short magnet. Hence calculate the radial and transverse field intensities.

1967. Two magnets A and B were placed with their axes in line and their north poles opposing and 10 cm. apart. Each magnet was 10 cm long. Magnet B had a known pole strength of 100 units. Magnet A was of unknown pole strength. Plotting the fields by means of iron filings in the absence of any outside field showed a neutral point, i.e. zero force to occur at P 4 cm distant from the north pole of A and 6 cm. distant from the north pole of B. Calculate (a) the strength of A, taking into account the effect of the north poles only and (b) the strength of A including the south poles as well.

Derive any formula used.

1968. Find an expression for the magnetic potential at a point near a short magnet; hence deduce the values of radial and transverse intensities at that point.

1971, 1974, 1975. Define a unit magnetic pole, magnetic intensity, moment of a magnet and magnetic potential at a point in a magnetic field.

Deduce an expression for the magnetic potential at a point due to a very short bar magnet. How does the length of the bar magnet affect the potential ?

1973, 1976. Define magnetic potential. Derive an expression for the magnetic potential at any point due to a short magnet. Hence calculate the field intensity at the point.

CHAPTER II

LINES OF FORCE, UNIFORM MAGNETIC FIELD, MAGNETIC SHELL.

Art. 6 A magnetic line of force is a line such that at any point on it the magnetic intensity is tangential to it.

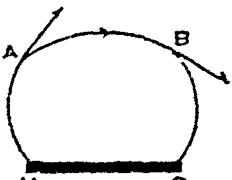


Fig. 13

Lines of force originate from north poles and end on south poles. Thus in Fig. 13 at points A, B,... on the line of force NABS, the intensity is tangential to the line and is directed away from the north pole.

It is obvious from the above definition that two lines of force can never intersect each other. For, if possible let two lines of force intersect at A.

Fig. 14

Then at A we can draw two tangents, one to each of the lines of force. Thus, on this supposition the intensity at A will have two directions. This is physically impossible; because the direction of the intensity at every point is always unique.

If the potential be the same at all points on a surface the surface is said to be equipotential.

Since the work done in carrying a unit north pole from one point to another is the difference of potential between those two points, it is obvious from the above definition (of an equipotential surface) that no work is done in moving a magnetic pole along an equipotential surface i. e. the magnetic intensity at any point has no component tangential to the equipotential surface. Or, in other words the magnetic intensity is at every point perpendicular to the equipotential surface. A line of force therefore intersects an equipotential surface at right angles.

Art 7
Tube of force

Lines of force coming out of a pole may be divided into a number of groups, each group being called a tube of force. Obviously such grouping may be done in an infinite number of ways. Thus in fig 15 there are eight groups or eight tubes of force. In magnetism the grouping is so made that $4\pi m$ tubes of force come out of a pole strength m. It is however customary not to use two separate expressions,—lines of force and tubes of force. Lines of force which are, strictly speaking, geometrical lines as defined earlier have got little importance. The tubes of force are therefore always referred to as lines of force. Thus we say that $4\pi m$ *lines of force* come out of every pole of strength m.

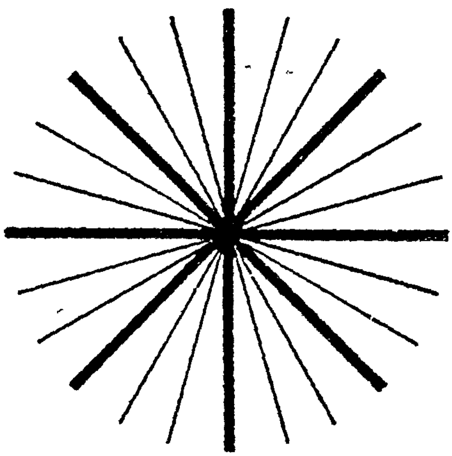


Fig. 15

If we consider an imaginary sphere of radius r round the pole all these $4\pi m$ lines of force cross this sphere. The surface area of this sphere being $4\pi r^2$ the number of lines of force crossing a unit area of this sphere $=\frac{4\pi m}{4\pi r^2}=\frac{m}{r^2}$ But intensity at any point on the surface of the sphere is $\frac{m}{r^2}$.

Hence we have the important conclusion :—

The intensity at any point is equal to the number of lines of force crossing unit area round the point, the unit area being taken perpendicular to the lines.

Uniform field

If the field be uniform the number of lines of force crossing unit area must everywhere be constant. This is possible only if the lines of forcs are parallel.

The poles of the Earth are so far away that the Earth's field at any place may be regarded as uniform. Lines of force at any place due to Earth's magnetism are therefore parallel.

Let a pivoted* magnet NS of pole strength m be inclined at an angle θ to the horizontal component H of the Earth's field. Then the force on each of the two poles is mH. Since the forces on the two poles are equal, parallel and oppositely directed, they constitute a couple. The moment of this couple $= m\mathrm{H} \times$ arm of the couple $= m\mathrm{H} \times 2l \sin\theta$ where $2l$ is the length of the magnet. But $m \times 2l = \mathrm{M}$ the magnetic moment of the magnet.

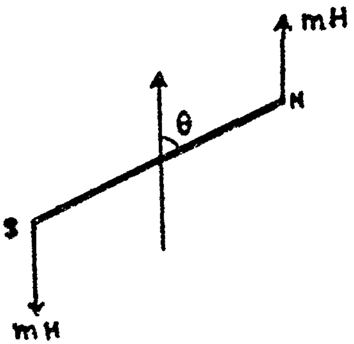


Fig. 16

Hence the couple on the magnet $= \mathrm{MH} \sin\theta$ ... (9)

In this equation if $\mathrm{H}=1$ and $\theta = 90$, M becomes equal to the couple on the magnet. Hence we get the definition of the magnetic moment. [Vide Art 2]

Period of oscillation

A pivoted magnet when undisturbed rests in the N-S direction, *i. e.* along the direction of H. If however it is rotated from its mean position of rest and then left to itself it oscillates a number of times before again coming to rest. To find the period of this oscillation we proceed thus :—

At any instant let θ be the inclination of the magnet to the direction of H. From the well-known law of Rigid Dynamics we have for an oscillating system,

Moment of intertia** $\times$ angular acceleration

---

* By a pivoted magnet we mean a magnet balanced on a pointer so that it can rotate only in the horizontal plane. *Compare* freely suspended needle [Art 11, foot-note.]

** If the magnet be rectangular in shape of length L and breadth B and if M be the mass of the magnet the moment of inertia is given by $\mathrm{I} = M \dfrac{\mathrm{L}^2 + \mathrm{B}^2}{12}$

$=$ moment of the external couple.

Hence for the oscillating magnet

$$I \times \frac{d^2\theta}{dt^2} = -MH \sin\theta$$

where I is the moment of inertia of the magnet. The minus sign is used because the couple acts in the direction of θ decreasing.

If θ is small, $\sin\theta = \theta$.

$$\therefore \quad I\frac{d^2\theta}{dt^2} = -MH\theta \quad \text{or} \quad \frac{d^2\theta}{dt^2} = -\frac{MH}{I}\theta$$

$\frac{MH}{I}$ being a constant the angular acceleration is proportional to the angular displacement; and the minus sign indicates that the acceleration is directed towards the mean position of rest. Hence the motion is simple harmonic and the time period is given by

$$T = 2\pi\sqrt{\frac{I}{MH}} \quad \ldots \quad (10)$$

N. B. (1) Here H represents the Earth's horizontal component. If there be other magnets in the neighbourhood H is the resultant field due to the Earth and the magnets.

(2) Instead of the magnet being pivoted if it be placed horizontally on a stirr-up suspended by a thin string and if the stirr-up (with the magnet) be allowed to execute angular oscillations, the time period is still given by (10). Thus if T the period of oscillation be measured by a stop-watch and if I be calculated from the dimensions of the magnet the product MH can be found out by equation (10). [Vide Vibration Magnetometer, Art 15].

Magnetic moments of two magnets may also be compared by equation (10). Thus if M_1 and M_2 be the magnetic moments of two magnets and if T_1 and T_2 be the periods of oscillation when the two magnets are successively placed on the stirr-up, we have

$$T_1 = 2\pi\sqrt{\frac{I_1}{M_1 H}} \text{ and } T_2 = 2\pi\sqrt{\frac{I_2}{M_2 H}}.$$

Hence by division $\frac{T_1}{T_2} = \sqrt{\frac{M_2}{M_1} \cdot \frac{I_1}{I_2}}$ $\therefore$ $\frac{M_1}{M_2} = \frac{T_2^2}{T_1^2} \cdot \frac{I_1}{I_2}$.

The field strengths at two points on the axis of a bar magnet at distances 25 cms and 20 cms from the centre of the magnet, are in the ratio 125 : 256. Find the distance between the poles.

Field strength at a point on the axis is $F = \frac{2Mr}{(r^2 - l^2)^2}$.

Hence if F_1 and F_2 be the field strengths at the two points,

$$F_1 = \frac{2M \times 25}{(25^2 - l^2)^2} \text{ and } F_2 = \frac{2M \times 20}{(20^2 - l^2)^2} \therefore \frac{25(20^2 - l^2)^2}{20(25^2 - l^2)^2} = \frac{F_1}{F_2} = \frac{125}{256}.$$

$$\text{or } \frac{(20^2 - l^2)^2}{(25^2 - l^2)^2} = \frac{25}{64} \quad \therefore \quad \frac{20^2 - l^2}{25^2 - l^2} = \frac{5}{8} \quad \text{or } l = 5$$

Hence the distance between the poles $= 2l = 10$ cms.

A bar magnet is placed in the magnetic meridian with its north pole pointing north. A magnetic needle suspended horizontally, vertically over the magnet makes 20 oscillations per minute. If the polarity of the bar magnet be reversed the needle makes 32 oscillations per minute. How many oscillations will it make when the magnet is removed ?

When the north pole of the magnet is directed towards north the field H' (at the centre of the needle) due to the magnet is opposite to the field H due to the Earth. When the polarity is reversed they act in the same direction. Hence

$$2\pi\sqrt{\frac{I}{M(H - H')}} = \frac{60}{20} = 3 \qquad \ldots \qquad (1)$$

$$\text{and } 2\pi\sqrt{\frac{I}{M(H + H')}} = \frac{60}{32} = \frac{15}{8} \qquad \ldots \qquad (2)$$

And if n be the number of oscillations per minute when the magnet is removed, $\frac{60}{n} = 2\pi\sqrt{\frac{I}{MH}}$ $\ldots$ (3)

Squaring (1) and (2) $\frac{4\pi^2 I}{M(H - H')} = 9$ and $\frac{4\pi^2 I}{M(H + H')} = \frac{225}{64}$

$\therefore$ by division $\frac{H + H'}{H - H'} = \frac{9 \times 64}{225} = \frac{64}{25}$ $\therefore$ $\frac{H'}{H} = \frac{39}{89}$ or $H' = \frac{39H}{89}$.

Hence from (1). $2\pi\sqrt{\dfrac{I}{M\left(H-\dfrac{39H}{89}\right)}}=3$ or $2\pi\sqrt{\dfrac{89}{50}}\sqrt{\dfrac{I}{MH}}=3$

$\therefore$ from (3) $\dfrac{60}{n}=2\pi\sqrt{\dfrac{I}{MH}}=3\sqrt{\dfrac{50}{89}}$ $\therefore n=20\sqrt{\dfrac{89}{60}}$

$=26{\cdot}68$ oscillations per minute.

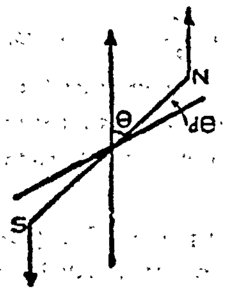


Fig. 17

Let the magnet NS make an angle θ with the magnetic field H. Then the couple on the magnet is MH $\sin\theta$. If the magnet be rotated through an additional angle $d\theta$

the work done $=$ MH $\sin\theta\, d\theta$.

[In linear motion work done = force × distance; in rotatory motion work done = couple × angular distance]

Hence when the magnet is rotated from one position to another total work done $=\int$ MH $\sin\theta\, d\theta$ ...... (11)

the limits of integration being the values of θ corresponding to the initial and final positions of the magnet.

Problem. Calculate the work done when the magnet is rotated through 90° from the magnetic meridian.

$$\text{Work done}=\int_0^{\pi/2}\text{MH}\sin\theta\, d\theta=-\text{MH}\Big[\cos\theta\Big]_0^{\pi/2}=\text{MH}$$

Potential energy of a magnet in a uniform field

Consider a system consisting of a small magnet NS whose centre is O and a pole of strength m placed at a point P. Let OP ($=r$) make an angle θ with the axis of the magnet. Due to the magnet NS the potential at P is $\dfrac{M\cos\theta}{r^2}$ *i.e.* the work done in carrying a unit North pole *from* infinity to P is $\dfrac{M\cos\theta}{r^2}$.

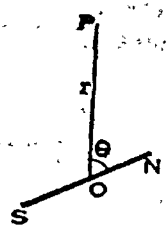


Fig. 18

Hence the work done in bringing the north pole of strength m to the point P, is $\frac{mM \cos\theta}{r^2}$ *i. e.* the potential energy of the system—the pole m at P and the small magnet NS—is $\frac{mM \cos\theta}{r^2}$. Now suppose the pole m is removed to infinity but at the same time m is made so large that $\frac{m}{r^2}$ is finite and equal to H. The field due to the pole m thus becomes uniform and of strength H. The magnet NS is placed in this uniform magnetic field of strength H and its potential energy is equal to MH $\cos\theta$.

N. B. 1. It is to be noted that the north pole at P repels the north pole N of the magnet. If, however, the magnet be placed in Earth's uniform field H of course represents Earth's horizontal component but since the north pole of the Earth *attracts* the north pole of the magnet, the potential energy in this case is $-$MH $\cos\theta$.

2. Since the pole m is ultimately removed to infinity so that the field is uniform, the initial restriction that the magnet NS is small, may be removed. Thus the formula is true for all magnets, large or small, placed in a uniform magnetic field.

Art 8 Magnetic needle under the joint action of Earth's field and another magnet.

Tan A position

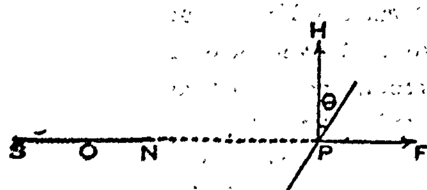


Fig. 19

Consider a magnetic needle placed at the point P in the end-on position (or tan A position) with respect to the magnet NS. The needle is therefore acted on by two intensities, (1) F due to the magnet, given by $F = \frac{2Mr}{(r^2 - l^2)^2}$ along OP produced M, r, l having their usual meanings, and (2) H the Earth's

horizontal component. If the magnet NS be placed in East-West direction, F acts along the same line; since H acts along N-S direction, F and H are at right angles to each other. The resultant intensity therefore makes an angle θ with the direction of H where $\tan\theta = \frac{F}{H}$. Thus the needle which, in the absence of the magnet NS, lies along the magnetic meridian, is deflected through an angle θ when the magnet is brought in. The angle θ is given by

$$\tan\theta = \frac{F}{H} = \frac{2Mr}{H(r^2 - l^2)^2}$$

$$\therefore \quad \frac{M}{H} = \frac{(r^2 - l^2)^2}{2r} \tan\theta \quad \cdots \quad (12)$$

If the magnet NS be small, l^2 may be neglected in comparison to r^2; in that case

$$\frac{M}{H} = \tfrac{1}{2} r^3 \tan\theta \quad \cdots \quad (12a)$$

Tan B position

If the point P be taken in broadside-on position (or Tan B position) with respect to the magnet NS, the intensity due to the magnet is $\frac{M}{(r^2 + l^2)^{3/2}}$.

If the magnet be placed in East-West position, this intensity is at right angles to H. Hence in this case a needle placed at P is deflected through an angle θ given by

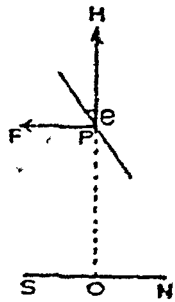


Fig. 20

$$\tan\theta = \frac{M}{H(r^2 + l^2)^{3/2}}$$

$$\therefore \quad \frac{M}{H} = (r^2 + l^2)^{\frac{3}{2}} \tan\theta \quad \cdots \quad (13)$$

If l be small, $\quad \frac{M}{H} = r^3 \tan\theta \quad \cdots \quad (13a)$

The value of $\frac{M}{H}$ can therefore be found out by either

of the two equations (12) or (13). The value of l in the equations is an uncertain factor which cannot usually be determined with sufficient accuracy. This l can however be eliminated if the needle be placed at two different distances from the magnet. (Vide appendix A)

So far we have assumed without any proof the fundamental proposition, viz, the force between two poles varies inversely as the square of the distance between the poles,—this is known as the Law of Inverse Square. That the force between two poles depends upon the distance, is obvious. That it varies inversely as the square of the distance, requires a proof which we now proceed to discuss.

Art. 9 Inverse Square Law

Let us assume that the force between two poles varies inversely as the n^{th} power of the distance, *i. e.* let the force between two poles be equal to $\frac{m_1 m_2}{r^n}$; the intensity at a distance r is therefore given by $F = \frac{m_1}{r^n}$.

Hence, considering the tan A position, the intensity at P is given by

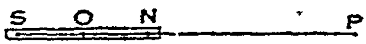


Fig. 21

$$F_1 = \frac{m}{(r-l)^n} - \frac{m}{(r+l)^n} = \frac{m}{r^n}\left(1 - \frac{l}{r}\right)^{-n} - \frac{m}{r^n}\left(1 + \frac{l}{r}\right)^{-n}$$

$$= \frac{m}{r^n}\left(1 + \frac{nl}{r}\right) - \frac{m}{r^n}\left(1 - \frac{nl}{r}\right), \text{ neglecting square and higher powers of } \frac{l}{r}.$$

$$= \frac{2mnl}{r^{n+1}} = \frac{nM}{r^{n+1}}.$$

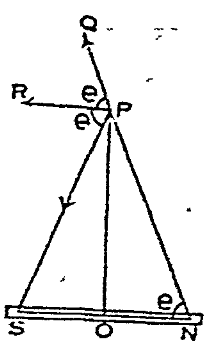


Fig. 22

Similarly for tan B position the intensity at P due to north pole

$$= \frac{m}{NP^n} = \frac{m}{(r^2 + l^2)^{n/2}} \text{ along NP produced}$$

and the intensity at P due to south pole

$$= \frac{m}{SP^n} = \frac{m}{(r^2 + l^2)^{n/2}} \text{ along PS}$$

Resolving these two in a direction

parallel to the magnet the resultant intensity at P is

$$F_2 = \frac{2m}{(r^2+l^2)^{n/2}} \cos\theta = \frac{2m}{(r^2+l^2)^{n/2}} \cdot \frac{l}{(r^2-l^2)^{\frac{1}{2}}} = \frac{M}{(r^2+l^2)^{\frac{n-1}{2}}}$$

$$= \frac{M}{r^{n+1}\left(1+\frac{l^2}{r^2}\right)^{\frac{n+1}{2}}} = \frac{M}{r^{n+1}} \text{ neglecting } \frac{l^2}{r^2}$$

Hence if we place a magnetic needle at P in the two positions (the magnet NS being placed E-W), the needle will be deflected by angles θ_1 and θ_2 in the two cases, given by

$$\tan\theta_1 = \frac{F_1}{H} = \frac{nM}{Hr^{n+1}} \text{ and } \tan\theta_2 = \frac{F_2}{H} = \frac{M}{Hr^{n+1}}.$$

$$\therefore \quad \frac{\tan\theta_1}{\tan\theta_2} = n.$$

If the values of θ_1 and θ_2 be actually obtained experimentally, it is found that $\frac{\tan\theta_1}{\tan\theta_2} = 2$. Hence we conclude that $n = 2$. This is known as Gauss' proof of the law of inverse square.

The inverse square law may also be verified by the Torsion balance. (Vide Art 28)

Art 10

Magnetic shell

If a thin sheet of a magnetic substance be magnetised at right angles to its surface, *i. e.* if north polar magnetism be distributed over one face and south polar magnetism over another, then we have what is known as a magnetic shell.

If at any point on its surface we consider an elementary area ds the amount of pole strength over this area $= m\,ds$, where m is the pole strength per unit area at this point. Then the magnetic moment of the elementary magnet whose end face is ds,

$= m\,ds \times t$ where t is the thickness of the shell at the point.

$= \phi\, ds$ where $\phi = mt$.

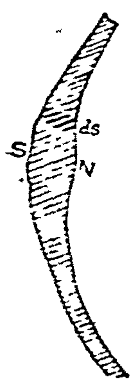


Fig. 23

This ϕ or the product $m \times t$ is called the strength of the shell a the point. The strength of a shell at any point is therefore defined a the product of the pole strength per unit area and thickness of th shell at the point. It is therefore the magnetic momen of the shell per unit area. If this strength be consta at all points of the surface the shell is said to be un form. It is to be noted that in a uniform magnetic shell neith m nor t need separately be constant; it is enough if the produ $m \times t$ remains constant.

Uniform shell

Potential due to a uniform magnetic shell.

Consider an elementary area AB ($=ds$) on the she The elementary magnet whose end face is AB is of leng t the thickness of the shell. Its moment is therefo equal to $mds \times t = \phi ds$ whe m is the pole-strength p unit area and ϕ the stren of the shell. The potent at any point P due to t elementary magnet$=\frac{\phi\, ds\, c}{r^2}$ [from (7], where r is distance of P from ds* an is the angle between r and normal to ds. Join PA, and drop AC perpendic to PB.

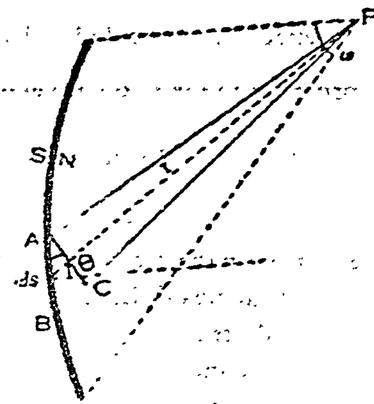


Fig. 24

Since r is perpendicular to AC the angle between and ds is equal to the angle between r and the normal to d i. e. $\angle BAC = \theta$ $\quad\therefore\quad ds \cos\theta (= AB \cos\theta) = AC$

Hence the total potential at P

$$= \int \frac{\phi\, ds \cos\theta}{r^2} = \int \frac{\phi AC}{r^2}$$

* Since ds is infinitesimally small the distance of P from A or from any point on ds, may be taken to be equal to r.

$= \int \phi d\omega,$ where $d\omega$ is the solid angle APB subtended at P by ds.

$= \phi \int d\omega$ if the shell be uniform ϕ is constant and may be taken outside the integral sign.

$= \phi\omega$ ... ... (14)

where ω is the total solid angle subtended by the entire shell at P.

N. B. (1) If P faces the south polar side of the shell the potential at P is $-\phi\omega$.

(2) If the medium, instead of being air, be a substance whose permeability is μ the potential at P is $\frac{\phi\omega}{\mu}$

Work done

Consider a uniform shell AB of any shape and consider two points P and Q, P on the north polar side and Q on the south polar side of the shell, the points being close to each other. Then $V_P = \phi \times$ solid angle APB and $V_Q = -\phi \times$ solid angle AQB. If the points P and Q be sufficiently close solid angle AQB $= 4\pi -$ solid angle APB $= 4\pi - \omega$ if $\omega =$ solid angle APB.

$$\left.\begin{aligned} \therefore\ V_P &= \phi\omega \\ \text{and } V_Q &= -\phi(4\pi - \omega) = \phi(\omega - 4\pi) \\ \therefore\ V_P - V_Q &= 4\pi\phi \end{aligned}\right\} \ldots \quad (15)$$

Hence the work done in carrying a unit North pole from a point close to the shell on one side to another point also close to the shell, but on the other side $= 4\pi\phi$. It is important to note that the path along which the unit North pole is taken from P to Q, must nowhere cross the shell.

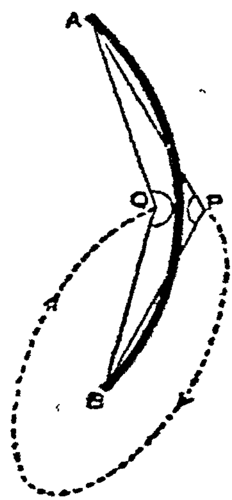


Fig. 25

Potential at any point within a uniform shell

Let P be any point within a uniform shell of strength ϕ. Let AB be a line passing through P and perpendicular

to the faces of the shell. Through P imagine a plane passing within the shell so that the shell is decomposed into two uniform shells whose thicknesses are AP and BP. Since the strength of a shell is propor-

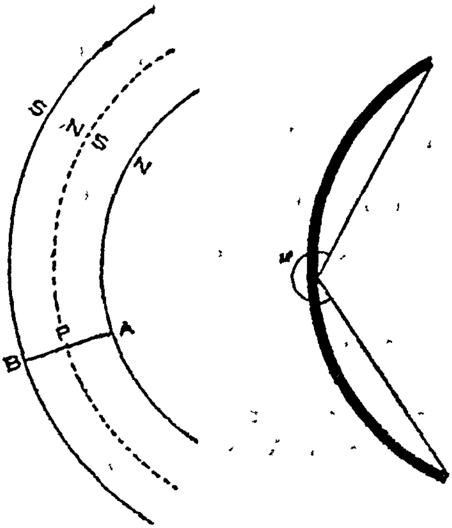


Fig. 26

tional to its thickness the strengths of the two component shells are $\Phi \dfrac{AP}{AB}$ and $\Phi \dfrac{BP}{AB}$. If A be on the north polar face of the original shell the point P is on the south polar face of the first component shell (whose thickness is AP) and on the north polar face of the other component shell (whose thickness is BP).

Hence total potential at P

$$=\phi \frac{BP}{AB} w + \phi \frac{AP}{AB} (\omega - 4\pi), \text{ [from (15)]}$$

where ω is the solid angle subtended by the shell at P.

$$=\phi\omega \frac{AP+BP}{AB} - 4\pi\phi \frac{AP}{AB}$$

$$=\phi \left\{ \omega - 4\pi \frac{AP}{AB} \right\}$$

Special Cases :—

(1) If P coincides with A, AP = 0, $V_A = \Phi\omega$

(2) If P coincides with B, AP = AB, $V_B = \Phi(\omega - 4\pi)$

Article 10 (a)

Let AB be a uniform circular shell of strength ϕ. Let P be a point on the axis of the shell at a distance b from the centre O of the shell i.e. OP=b. On the shell two concentric circles are drawn with centre O and radii equal to r and $r+dr$. The elementary portion between these two circles is of area $2\pi rdr$. The line joining P to any point C on this elementary area makes an angle θ with the normal to the shell; hence

$\angle CPO = \theta$, $PC = \sqrt{b^2+r^2}$ and $\cos\theta = \dfrac{b}{\sqrt{b^2+r^2}}$.

Since ϕ is the strength of the shell the magnetic moment of the elementary portion of the shell is $2\pi rdr\phi$. Thus the potential at P due to this elementary portion is $\dfrac{2\pi\, rdr.\phi\cos\theta}{PC^2}$ [Vide (7) Art 4].

Hence the total potential at P due to the entire shell is given by

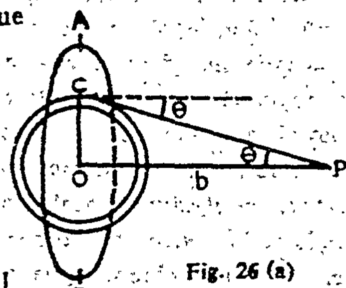


Fig. 26 (a)

$$V = \int_b^a \frac{2\pi\, rdr.\,\phi\cos\theta}{PC^2}$$

where a is the radius of the shell

$$= 2\pi\phi\int_0^a \frac{rdr}{b^2+r^2}\cdot\frac{b}{\sqrt{b^2+r^2}} = 2\pi b\phi\int_0^a \frac{rdr}{(b^2+r^2)^{\frac{3}{2}}}$$

Let $x^2 = b^2+r^2$ $\therefore$ $xdx = rdr$.

When $r=0$, $x=b$ and when $r=a$ $x=\sqrt{a^2+b^2}$

Hence changing the variable and also the limits

$$V = 2\pi b\phi \int_{b}^{\sqrt{a^2+b^2}} \frac{x\,dx}{x^3} = 2\pi b\phi \int_{b}^{\sqrt{a^2+b^2}} \frac{dx}{x^2}$$

$$= -2\pi b\phi \left[\frac{1}{x}\right]_{b}^{\sqrt{a^2+b^2}} = -2\pi b\phi \left\{\frac{1}{\sqrt{a^2+b^2}} - \frac{1}{b}\right\}$$

$$= 2\pi\phi \left\{1 - \frac{b}{\sqrt{a^2+b^2}}\right\} \qquad \ldots \text{15(a)}$$

Exercise II

1. How did Gauss prove that the force between two magnetic charges varies inversely as the square of the distance between them. C. U. 1930

2. Discuss the significance of the "inverse square law" as applied to Electricity and Magnetism. Devise an experiment to prove the validity of the law for a magnetic field. C. U. 1944

3. Two bar magnets of moments M and M' and of equal length are placed on a floating piece of cork so that they are at right angles to each other, their north poles being in contact. If $M = 2M'$ find (*a*) the angle which the magnet M makes with the magnetic meridian in the equilibrium position (*b*) through what angle the system would rotate when the poles of the magnet M' are reversed. Ans. (*a*) 26° 34′ (*b*) 53° 8′.

4. Two magnets, the moment of one being double that of the other, are rigidly connected at the centre so that they make an angle of 60° with each other, similar poles being near each other. If this combination be suspended from the centre find the angles which the magnets in the equilibrium position make with the magnetic meridian. Ans. 19° 6′ and 40° 54′.

5. A bar magnet hung horizontally by a fine wire lies in the magnetic meridian when the wire is without any twist. When the top of the wire is twisted through 150° the magnet is deflected through 60° Through what further angle must the top of the wire be rotated so as to bring the magnet perpendicular to the magnetic meridian? Ans 23·2°

6. A magnet 20 cms. long and of pole strength 20 units is suspended at a place where H=0·18. Find the couple required to deflect the magnet (i) through 30° (ii) through 90°. Find also the work done in turning the magnet from the magnetic meridan through 90° Ans. (i) 36 (ii) 72 ; 72 ergs.

7. A small magnet makes 20 oscillations per minute. When a bar magnet is placed in the magnetic meridian due north of the small magnet, the latter makes 28 oscillations per minute. What would be the frequency if the bar magnet be placed in the same position with its poles reversed ?

Ans. 4 oscillations per minute.

8. A horizontally suspended magnet vibrates 12 times a minute at a place where H=0·18. How many times per minute will it vibrate at another place where H=0·25 ? Ans $10\sqrt{2}$

9. What are the factors that affect the period of oscillation of the vibration magnetometer ?

The period of oscillation of a vibrating magnet is T when under the influence of the Earth's field. When a bar magnet of magnetic length 16 cms., is placed with its centre 20 cms. east of the vibrating magnet and with its axis parallel to the magnetic meridian the period is reduced to T/2. What is the pole strength of the bar magnet and which pole points north ? [H=0·2]. Ans 374·9 ; South pole.

10. What is a magnetic shell ? When is it said to be uniform ? Obtain an expression for the potential at a point due to a uniform magnetic shell.

A uniform magnetic shell is of the shape of a circular disc. Find the potential at any point on the axis of the disc.

C. U. Questions

1963. What is meant by a magnetic shell ? Derive an expression for the potential at a point due to a uniform magnetic shell. Apply it for the calculation of the potential at any point on the diameter of a hemispherical shell.

1964. Find an expression for the work done in deflecting a

magnet from its position of rest in a uniform magnetic field.

1965. What is a magnetic shell? Show that the potential at any point due to a uniform magnetic shell is given by the product of the shell strength and the solid angle subtended by the shell at the point.

A magnetic shell is in the form of a disc of radius 6 cm and thickness 5 mm. Its surface density of polarity is 5 C.G.S. units per cm^2. Calculate the potential on the axis of the disc at a distance of 8 cm from its centre.

1965. (New course) Show that the period of oscillation of a magnet of moment M in a field of strength H is given by $T=2\pi\sqrt{\frac{I}{MH}}$, where I is the moment of inertia of the magnet about the axis of suspension.

Describe the magnetometer working on the above relation and show how this can be used to compare the Earth's magnetic field at two different places.

1970. What do you mean by strength of a magnetic shell? Obtain an expression for the potential and intensity at a point due to a magnetic shell.

A magnetic shell is in the form of a circular disc of uniform thickness. Find the potential at a point on the axis of the disc.

1972. Calculate the work done when a bar magnet of length $2l$, originally placed along the magnetic meridian of horizontal intensity H, is rotated through 90°.

CHAPTER III

TERRESTRIAL MAGNETISM

As is well-known from the behaviour of the compass needle the Earth is a huge magnet with its magnetic north and south poles situated somewhere close to corresponding geographic poles.

Art 11

It is to be noted that since unlike poles attract each other in any magnet the pole which is attracted towards the north pole of the Earth should be called a south pole; but it is our convention to call it a north seeking pole or a north pole. Thus the north pole of a magnet is *attracted* by the north pole and *repelled* by the south pole *of the earth.*

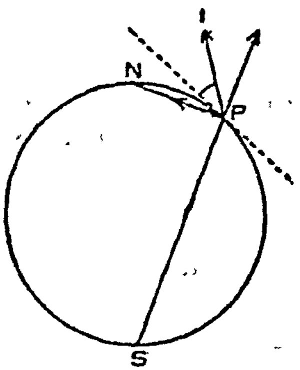


Fig. 27

At any place P on the Earth's surface the intensity due to Earth's magnetism is partly due to the north pole and partly due to the south pole of the Earth, *i. e.* a unit north pole at P is attracted towards the north pole and repelled by the south pole of the Earth, these forces varying inversely as the squares of the corresponding distances of P from the two poles. The resultant intensity or the total intensity I is, in general, inclined to the horizon of the place. This angle of inclination to the horizon is known as the inclination or dip at the place. Thus we have the following definition :—

Inclination or dip

The inclination or dip at any place is the angle which the total intensity due to Earth's magnetism makes with the horizon of the place.

A freely* suspended magnetic needle lies

* By a freely suspended needle we mean a needle suspended from its centre of gravity, so that it can rotate in all possible planes. *Compare* pivoted needle, Art 7, footnote.

along the direction of the total intensity. The inclination to the horizon of a freely suspended needle therefore measures the inclination at any place.

H and V

If we resolve the total intensity I along the horizon and along the vertical, the two components are known as Earth's horizontal component and Earth's vertical component. They are generally represented by the the letters H and V respectively. Thus if θ be the inclination

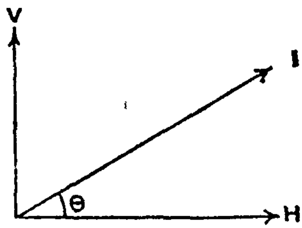


Fig. 28

$$\left.\begin{array}{ll} H = I\cos\theta & \dfrac{V}{H} = \tan\theta \\ V = I\sin\theta & H^2 + V^2 = I^2 \end{array}\right\} \quad \ldots \quad (16)$$

It is obvious that of the four quantities I, H, V and θ if two can be determined experimentally the other two can be obtained by calculation from the above four equations.

Art 12

The three elements of Earth's magnetism are (1) Declination (2) Inclination or Dip and (3) Earth's Horizontal component.

Declination

Of these the first remains to be defined. Geographical poles of the Earth are situated at the ends of the diameter about which the earth rotates. Magnetic poles though very near to the corresponding geographical poles are not exactly coincident with them. At any place on the surface of the Earth let us imagine two planes—one passing through the magnetic north, magnetic south and the zenith of the place and the other through the geographical north, geographical south and the zenith. These planes are known as magnetic meridian and geographical meridian respectively and the angle between these two planes is called the declination of the place.

We shall now discuss methods for determining these three elements.

Art 13 Declination (D)

Place a compass needle on a piece of paper stretched over a horizontal drawing board. The positions of the two ends of the needle are marked on the paper with a pencil. The line joining these two points indicates the direction of the magnetic meridian.

Magnetic meridian

Fix up a rod vertically on the same drawing board out in the open sun. In the morning when the Sun is in the East the shadow of the rod is towards the West and is fairly long. In the evening when the Sun goes towards the West the shadow is towards the East and is also long. At about mid-day when the Sun is almost* over-head the shadow is of minimum size and lies along the geographical North-South direction. Hence the proceedure :—

Geographical meridian

Watch the shadow ; when its length is minimum draw a line along the shadow. This line gives us the direction of the geographical meridian.

The direction of the magnetic and the geographical meridians being obtained in this way the angle between them gives us the declination D at the place.

Art. 14 Inclination or Dip

Inclination or Dip at any place can be best determined by an apparatus known as "Dip circle". It essentially consists of a fairly long magnetic needle supported at the centre of a vertical circle AA. The circle is graduated in each quadrant from 0° to 90°, the line joining 0° – 0°

Dip Circle

*At places on the Earth between the tropic of Cancer and Tropic of Capricorn the Sun comes exactly over-heard at midday only on two days in the year. At places outside this region the Sun is never overhead on any day in the year.

being horizontal. The needle is so supported that it can rotate only in the vertical plane, *i. e.* in the plane of the graduated circle AA. This circle can also be rotated about a vertical axis; the amount of rotation can be read off from another horizontal graduated circle BB. The whole instrument rests on three levelling screws by means of which it may be levelled. It is obvious that when the plane of the vertical circle coincides with the magnetic meridian the inclination of the needle to the horizon gives us the dip.

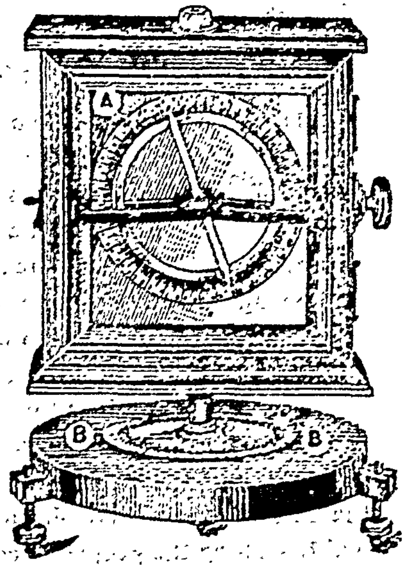


Fig. 29

There are two ways of using this instrument.

First method

In any position let the plane of the vertical circle AA make an angle δ with the magnetic meridian. Then the component of H along the plane AA, is $H \cos \delta$; the other component $H \sin \delta$ perp to the plane has no effect on the needle (because the needle can rotate only in the plane AA). Thus the needle is effectively acted on by two intensities, the vertical component V acting vertically and the component $H \cos \delta$ acting horizontally. *If in this position* θ_1 *be the inclination of the* needle to the horizon (*i. e.* apparent dip)

$$\tan \theta_1 = \frac{V}{H \cos \delta}.$$

The vertical circle is now rotated through 90° and the new apparent dip θ_2 is again noted. The plane AA now makes an angle $90^\circ + \delta$ with the magnetic meridian. Then, the apparent dip θ_2 is given by

$$\tan\theta_2 = \frac{V}{H\cos(90^\circ+\delta)} = -\frac{V}{H\sin\delta}.$$

$$\text{Hence } \operatorname{Cot}^2\theta_1 + \operatorname{Cot}^2\theta_2 = \frac{H^2\cos^2\delta}{V^2} + \frac{H^2\sin^2\delta}{V^2}$$

$$= \frac{H^2}{V^2} = \operatorname{Cot}^2\theta. \quad \ldots \quad (17)$$

where θ is the true dip. [Vide (16)]

Thus measuring θ_1 and θ_2, θ may be determined.

If the plane of the vertical circle AA be brought perpendicular to the magnetic meridian the horizontal component H (which acts along the magnetic meridian) acts perpendicularly to the plane AA and has therefore no effect on the needle. The needle in this position being effectively acted on only by the vertical component V, becomes vertical. Hence the following procedure is adopted :—

Second method

Rotate the vertical circle AA until the needle is vertical. The plane of the vertical circle is now perpendicular to the magnetic meridian. It is rotated further through an angle of 90° ; the plane now comes in the magnetic meridian. The inclination of the needle to the horizon is then read off from the vertical scale : this gives us the dip.

Errors

There are several sources of errors which must be eliminated before the correct dip is obtained.

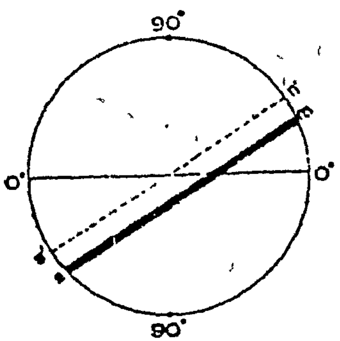


Fig. 30

(*a*) *The centre of the needle may not coincide with the centre of the vertical circle.* Thus with the needle in the position *ns* the correct dip is *n′* or *s′* ; the reading *n* is too low, whereas the reading *s* is too high to an equal extent. The error is therefore eliminated by noting the readings of both the ends *n* and *s* and taking the mean of these two readings.

(*b*) *The 0° – 0° line may not be horizontal.* Let *a′b′* be the 0° – 0° line slightly inclined to the correct horizontal line *ab*. If *NS* represents the position of the needle the readings *N* and *S* are both too small. This error is eliminated by rotating the vertical circle through 180°. The 0° – 0° line *a′b′* now takes the position *a″b″*. The position of the needle which is controlled by Earth's total intensity however remains unchanged. The readings *N* and *S* are now both too high to an equal extent. The mean of the previous and the present readings gives the true dip.

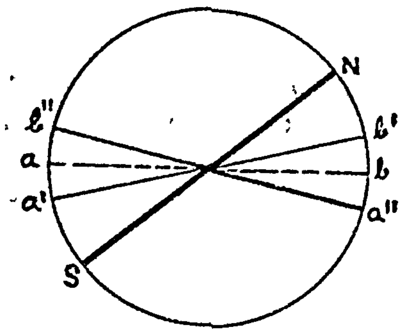


Fig. 31

(*c*) *The magnetic axis of the needle may not concide with the geometric axis.* Let *n′s′* represent the magnetic axis of

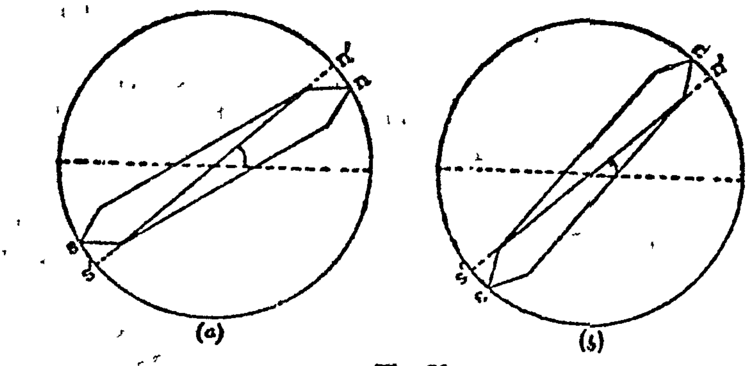


Fig. 32

the needle whose geometric axis is *ns*. The correct readings should be those of *n′* and *s′*. The readings *n* and *s* are therefore both too small [Fig. 32 (*a*)]. The needle is now reversed in its bearings. The position of the magnetic axis *n′s′* relative to the dip circle remains unchanged. But the geometric axis

is now inclined the other way to $n's'$ [Fig. 32 (*b*)]. The readings *n* and *s* are now too high. The error is therefore eliminated by taking the mean of the previous and the present readings.

(*d*) *The centre of gravity of the needle may not coincide with the axis of rotation of the needle.* Let the C. G. be situated a

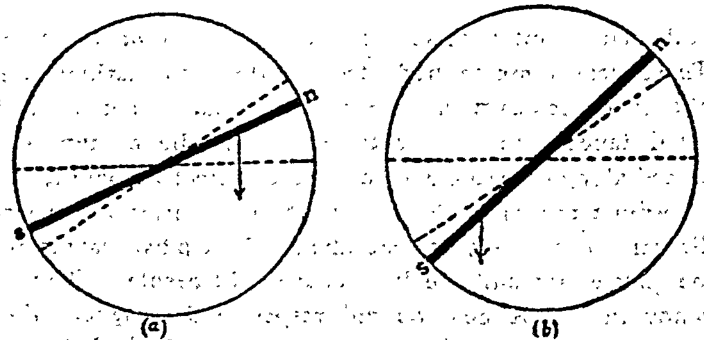


Fig. 33

little towards the north pole of the needle [Fig. 33 (*a*)]. Then the weight of the needle brings the north pole a little downwards. The readings are therefore too low. This error is eliminated by re-magnetising the needle so that the polarity is reversed. The end nearer the C. G. now becomes a south pole which is consequently brought a little downwards [Fig. 33 (*b*)]. The readings are therefore too large. The mean of these two sets of readings gives the true dip.

The C. G. may also be displaced perpendicular to the axis of the needle. But this error is automatically eliminated during the procedure (*c*) when the needle is reversed in its bearings.

Art 15. Earth's Horizontal Component.

The measurement of H consists of two separate experiments, first by Deflection Magnetometer and secondly by Vibration Magnetometer.

Deflection Magnetometer.

It essentially consists of a short magnetised needle pivoted at the centre of a graduated circular scale; a long pointer usually made of aluminium is rigidly attached at right angles

to the needle. The movement of this pointer along the graduated scale gives the angle of deflection of the needle. The base board is prolonged on opposite sides of the circular

Fig. 34

scale and a centimetre scale is fixed on each of these arms. These scales are so graduated that the 'zero' marking of each of them coincides with the centre of the magnetic needle. Thus the distance from the centre of the needle of any magnet placed along the scale may be read off directly from the scale.

Adjust the magnetometer so that its arms are perpendicular to the magnetic meridian. Place a bar magnet along the scale at any suitable distance from the needle. The needle is now in 'Tan A' position with respect to the magnet. If θ be the angle of deflection of the needle we have by (12) [Page 25].

$\frac{M}{H} = \frac{(r^2 - l^2)^2}{2r} \tan\theta$, where r is the distance of the centre of the magnet from the needle and $2l$ is the length of the magnet. Hence $\frac{M}{H}$ can be determined.

N. B. (1) The arms of the magnetometer can be placed in the magnetic meridian also. In this case the bar magnet is to be placed perp. to the arm; the magnetic needle is in Tan B position with respect to the magnet and the formula is

$$\frac{M}{H} = (r^2 + l^2)^{\frac{3}{2}} \tan\theta$$

(2) Whether we use tan A or tan B position the deflecting magnet must always be *perp.* to the magnetic meridian so that at the centre of the needle the intensity due to the Earth and that due to the magnet are at right angles to each other. Otherwise in either of the two positions if the magnet be placed along the magnetic meridian (*i. e.* perpendicular to the scale when the arms of the magnetometer are East-west or along the scale when the arms are North South) the two intensities lie along the same direction and the needle is not deflected at all.

(3) Strictly speaking, $2l$ is *not* the length of the magnet. it is the distance between the poles of magnet. There is

therefore a small error If greater accuracy is wanted l may be eliminated by placing the magnet with its centre at two different distances r_1 and r_2 and noting the corresponding deflections θ_1 and θ_2 of the magnetic needle. The equation is then $\frac{M}{H}=\frac{1}{2}\cdot\frac{r_1^5\tan\theta_1-r_2^5\tan\theta_2}{r_1^2-r_2^2}$ in 'tan A' position and

$\frac{M}{H}=\frac{r_1^5\tan\theta_1-r_2^5\tan\theta_2}{r_1^2-r_2^2}$ in tan B position. [Vide Appendix A].

Vibration Magnetometer.

In this instrument a stirrup is suspended by a fine string within a glass vessel. At first a heavy weight is placed on the stirrup so that the string is completely untwisted. The heavy weight is now removed and the bar magnet is placed on the stirrup and as the magnet swings to and fro the period of oscillation is measured with a stop watch. The moment of inertia of the magnet being calculated from its mass, length and breadth, the product MH can be determined by (10) [Page 21].

Hence, knowing $\frac{M}{H}$ from the deflection experiment and MH from the vibration experiment, the value of H can be found out.

N. B. From the values of $\frac{M}{H}$ and MH thus found out the value of M can also be determined; and if $2l$ the distance between the poles be known, m the strength of either pole can be calculated.

If the frequency of a magnet oscillating horizontally is 60 per minute at a place where the dip is 60° and 40 per minute at another place where the dip is 45° compare the total intensities at the two places.

If F_1 & F_2 be the total intensities and H_1 & H_2 the horizontal components at the two places $F_1\cos 60° = H_1$ and $F_2\cos 45° = H_2$

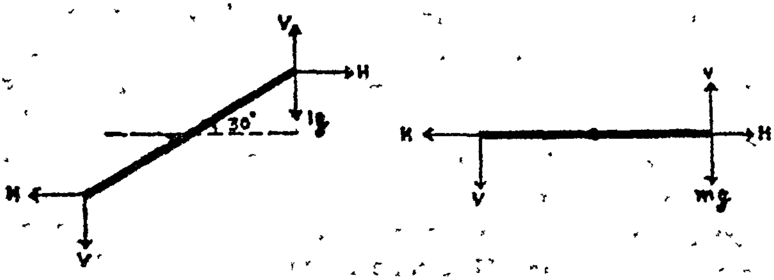
From (10) [Page 21] $2\pi\sqrt{\frac{I}{MH_1}}=\frac{60}{60}=1$

and $2\pi\sqrt{\frac{I}{MH_2}}=\frac{60}{40}=\frac{3}{2}$

$\therefore$ by division $\sqrt{\frac{H_1}{H_2}}=\frac{3}{2}$ $\therefore$ $\frac{H_1}{H_2}=\frac{9}{4}$ or $\frac{F_1 \cos 60^\circ}{F_2 \cos 45^\circ}=\frac{9}{4}$

$$\therefore \quad \frac{F_1}{F_2}=\frac{9\sqrt{2}}{4}=3{\cdot}18.$$

At a certain place the angle of dip is 60°. When a weight of one gm is attached to the upper end of the dip needle the inclination is 30°. What weight attached to the same point would make the needle horizontal?



(1) Fig. 35 (2)

In Fig. (1), taking moments about the centre

$$V.\ 2l \cos 30^\circ = H.\ 2l \sin 30^\circ + 1g.\ l \cos 30^\circ$$

or $\quad 2\sqrt{3}.V=2H+g\sqrt{3} \quad \ldots \quad \ldots \quad (a)$

In Fig. (2), taking moments about the centre

$V.\ 2l=mg.l \quad$ or $\quad V=\frac{1}{2}mg$

where m is the required mass.

Substituting this value of V in (a) $\sqrt{3}.mg=2H+g\sqrt{3}$.

$\therefore \ H=\frac{1}{2}g\sqrt{3}\ (m-1)$. But $\tan 60^\circ=\frac{V}{H}=\frac{m}{\sqrt{3}(m-1)}$

or $\sqrt{3}=\frac{m}{\sqrt{3}(m-1)}$ $\therefore$ $3(m-1)=m$ $\therefore$ $m=1{\cdot}5$ gms.

Art 16 Kew magnetometer

The Kew form of magnetometer is a combined apparatus for measuring the declination D and the horizontal component H. It essentially consists of the magnet M_1 and the telescope T_1 for measuring D and the magnet M_2 and the telescope T_2 for determining H. A graduated brass bar S_1S_2 fixed perp. to the axis of the telescope T_2 carries

the deflecting magnet M_3. The upper portion of the instrument containing these magnets and telescopes can be rotated round a vertical axis, the amount of rotation being

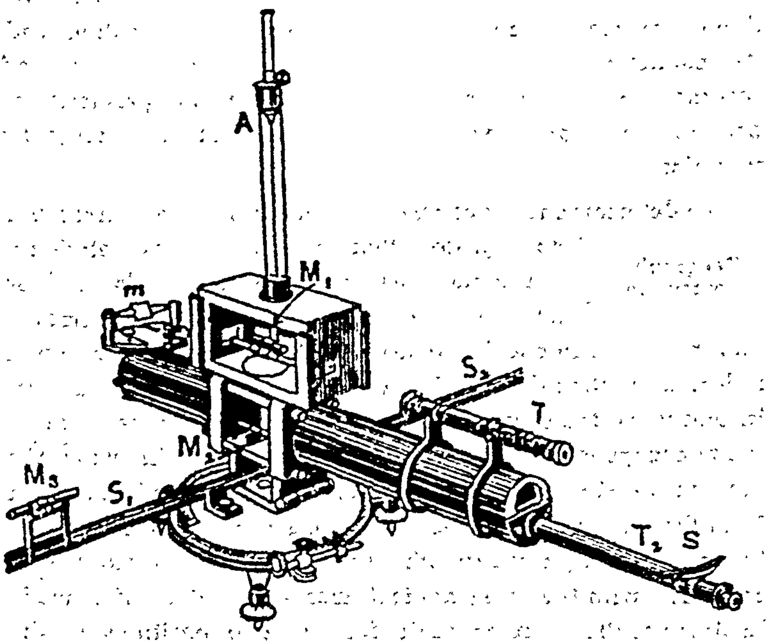


Fig. 36

given by a graduated circular scale at the base. The whole instrument rests on three levelling screws.

Determination of D by Kew magnetometer.

Magnetic meridian

To find the declination D the magnet M_1 and the telescope T_1 are used, all other magnets being removed from the apparatus. The magnet M_1 suspended from A by a single fine wire is a magnetised hollow cylinder at one end of which a scale on glass and at the other end a convex lens are fixed, the lens being at the end nearer to the telescope T_1. The length of the magnet being equal to the focal length of the lens the scale (illuminated by light reflected from the mirror m) is clearly seen by the telescope T_1 focussed for infinity. The

telescope is rotated until the central marking of the scale coincides with the vertical cross wire of the telescope. Readings of the circular scale at the base are taken to note down the position. In order to remove any possible error due to the geometric axis of the magnet not coinciding with its magnetic axis, the magnet is turned upside down and readings are again taken. The mean of the previous and present readings gives the direction of the magnetic meridian.

Geographic meridian

To determine the geographical meridian the mirror *m* is so adjusted that the axis of its rotation is horizontal and perp to the axis of the telescope T_1. The magnet M_1 is removed and the instrument is rotated until an image of the Sun as reflected by the mirror *m* is seen in the telescope. The exact instant when the centre of the Sun crosses the cross wire of the telescope is noted by a chronometer. Then knowing the latitude of the place the azimuth of the Sun can be obtained from the National Almanac. The telescope is further rotated through this angle of azimuth ; the axis of the telescope now coincides with the geographical meridian. The difference in readings of the circular scale for the two positions of the telescope pointing magnetic and geographical meridians, gives us the declination D of the place.

Determination of H by Kew Magnetometer.

To determine H, as described previously the instrument is rotated until the axis of the telescope T_1 lies along the magnetic meridian. The telescope T_2 being always parallel to T_1 is also thus brought in the magnetic meridian ; the brass bar S_1S_2 being perpendicular to the axis of the telescope T_2, is now in the East-West position. The magnet M_1 is removed and the magnet M_2 is suspended by a fine wire from the same point A. A small mirror is attached to the end of the magnet M_2 nearer to the telescope T_2. A small scale S at the top of the telescope T_2 is reflected by this small mirror and is seen by the telescope T_2. With the deflecting magnet M_3 removed, the

marking of the scale coinciding with the cross wire of the telescope, is first observed. The same observation is repeated when the magnet M_3 is brought in its position. From the difference in the two readings, the deflection of the magnet M_2 is known and hence $\frac{M}{H}$ for the magnet M_3 can be calculated by (12) [Page 25]. Te find the product MH the deflecting magnet M_3 is placed in the position of M_1, all other magnets beIng removed. As it oscillates it is observed by the telescope T_1 and the time period is determined by a chronometer. Thus T is determined. Hence measuring I, MH can be determined from (10) [Page 21].

Art 17 Continuous records

It has been observed that all the three elements of Earth's magnetism at any place are never constant; they undergo small but systematic changes. It is a matter of great importance to observe and record these changes continuously. An instrument known as magnetograph has been devised to record these changes automatically and continuously. For this purpose the three elements chosen are the declination D, the horizontal component H and the vertical* component V.

Since the direction of geographical meridian at any place is always fixed the variation in the declination D is entirely due to changes in the magnetic meridian at the place. To get a continuous record of this change a magnet suspended from a fixed point by a fine wire is enclosed in a hollow, rectangular copper box; As the magnet oscillates induced † currents generated in the copper of the box serve to damp the oscillations. The magnet always lies along the magnetic meridian and the direction of the

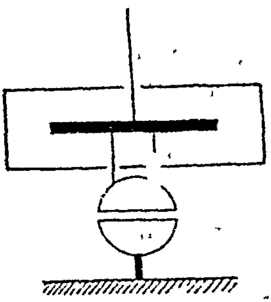


Fig. 37

* It may be seen that the element inclination or dip can be easily found out if both H and V are known.

† Vide Chapter XVI.

vertical mirror is attached to one end of the magnet. A beam of rays is reflected by this mirror and is afterwards concentrated to a point by a convex lens. A photographic paper wound on a rotating drum receives this point image; the photographic paper in this case moves in the horizontal direction and a continuous record of the variation of V is made on the paper.

The three records for the variations of D, H and V may conveniently be on the same photographic paper.

Art 18 Magnetic variations. All the three magnetic elements are found to undergo steady and systematic variations. These variations can broadly be classified under three headings.

(1) Secular Variation. This is a long period variation. The declination in London was 11° 15′ E in 1580, 5° 30′ E in 1600 and 0° in 1659. The declination then turned westward becoming 10°30′ W in 1709 and reaching the maximum value 24°30′ W in 1820. It has since been slowly decreasing. It is expected that it will again be zero about the year 2139. The magnetic system is thus found to rotate slowly from east to west, making a complete revolution in 960 years The magnetic north pole describes a small circle of radius equal to 17° as can be seen from Fig 38.

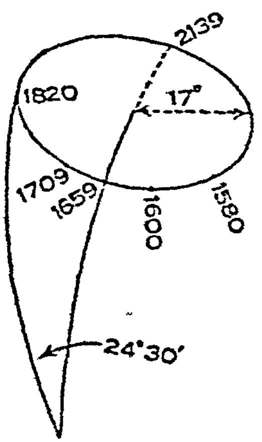


Fig. 38

Similar secular variations in inclination and horizontal component are also observed. In London inclination reached the maximum value 74° 42′ N in 1723. Since then it has been slowly decreasing at an average rate of about 1·1′ per year. The horizontal component on the other hand has been found to be steadily increasing at about 0·00002 unit per year. Complete cycles in these two cases cannot yet be deduced.

(2) **Diurnal Variation.** The elements are also found to undergo diurnal variations. Variation in the declination in London is shown by the curve A in Fig 39. Just before 8 A. M. this reaches an extreme value of about 4′ east of its mean position; afterwards at about 1 P. M. it reaches the other extreme value 5′ West.

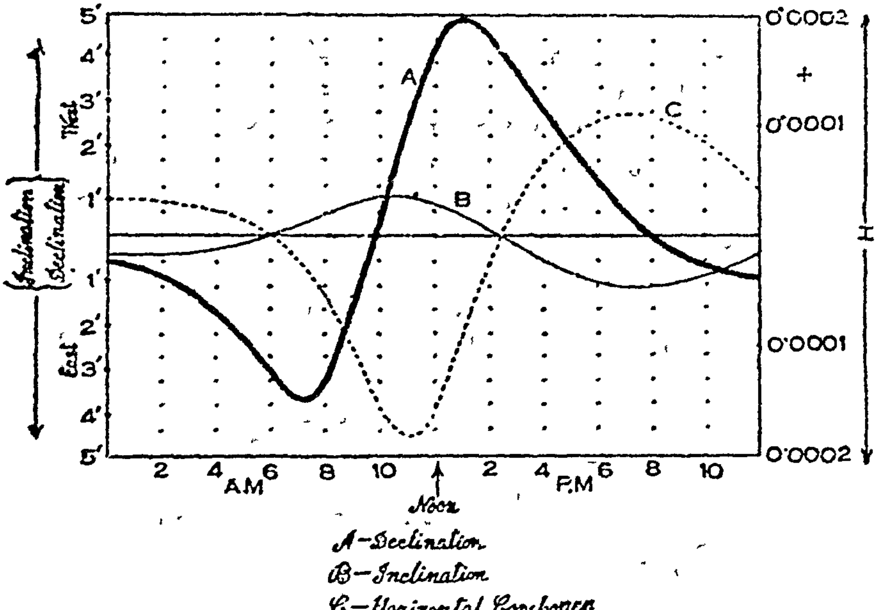


Fig. 39

Similarly variation in the inclination represented by the curve B has two extreme values, one approximately at 11 A. M. and the other at 7 P. M. But variations are much smaller in this case. In the case of the Horizontal component (curve C), the maximum and minimum also occur at about 7 P. M. and 11 A. M. respectively.

(3) **Annular variation.** An annular variation in declination has also been observed. In London the maximum easterly deviation occurs in August and the westerly in February, the amplitude of variation being about 2·2′. In Northern and southern hemispheres these variations occur simultaneously but in opposite directions.

Beside these variations there are occasional sudden and violent fluctuations; these are known as magnetic storms.

Magnetic storm

The theory of terrestrial magnetism is not yet fully developed. Although according to Gauss the magnetic effect at any place is primarily due to some source within the Earth Schuster and others are of opinion that at least diurnal variations and similar other effect owe their origin to some source external to the Earth. It is now definitely known that round the Earth at a height of a few miles there are several layers of electronic belts. The concentration of electrons in these belts depends—partly at least—upon the solar radiation and therefore undergoes a diurnal fluctuation. Electrons are also in constant motion producing electronic currents. It is believed that these electronic currents produce corresponding effect on the magnetic elements of the Earth.

Sunspots frequently seen on the surface of the Sun are found to be closely associated with the magnetic effect on the surface of the Earth. The frequency of sunspots is found to undergo a complete cycle in an eleven-year period. Variations in the magnetic elements of the Earth are strongly correlated with this eleven year cycle.

Eleven year period.

Art 19 Mariner's Compass.

Out in the open sea the captain of a ship finds the direction with the help of what is known as a *Mariner's Compass.* A number of short magnets is fixed parallel to one another on a circular card board pivoted at the centre. The circumference of the cardboard is marked by 32 directions which are known as the points of the compass. The N-S direction is specially indicated by a crown. The vertical pivot on which the cardbord rests is fixed to a base B. It is essential that the base B is always horizontal even when the ship is being tossed by the waves of the sea. This is done by what is known as *Gymballs* arrangement. The base B is attached to a ring AA at two diametrically opposite points about which it can freely rotate. The ring AA again

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is attached to another outer ring CC at two diametrically opposite points about which it can freely turn. The axis of rotation of B is *perpendicular* to that of AA. The ring CC is attached to some suitable stand on the ship. When the ship is tossed by the waves the outer ring CC of course turns though various angles along with the ship. But obviously the base B remains unaffected.

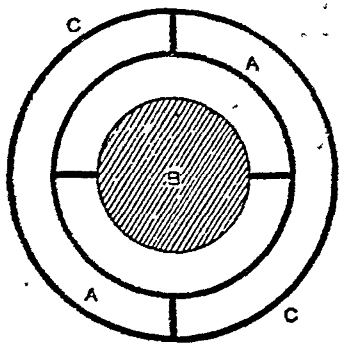

Fig. 40

The Mariner's compass indicates the direction of the magnetic North pole. The direction of the geographical North pole makes with this an angle equal to the declination of the place. Measuring the latitude and longitude of the place by usual methods the declination can be found out from the Nautical Almanac and hence the direction of the geographical North pole can be determined.

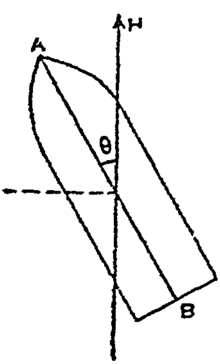

Fig. 41

The permanent magnets and soft iron of the ship however affect the compass. To find the effect due to permanent magnets we suppose that the permanent magnets are equivalent to a single magnet of moment M placed along the keel AB of the ship. At any time when this makes an angle θ with the magnetic meridian the component of M perpendicular to the magnetic meridian is $M \sin \theta$ and the couple tending to rotate the compass magnets is proportional to $M \sin \theta$. Clearly this changes its sign when θ passes through 180°. The effect is therefore called *semi-circular deviation.* This is corrected by suitably placing small permanent magnets near the Mariner's Compass.

To find the effect due to the soft iron we suppose that the soft iron in the ship is equivalent to a single piece of soft iron placed along the keel AB of the ship [Fig 41]. This is magnetised by Earth's Horizontal component H. If at any instant the keel AB makes an angle θ with H the magnetic moment induced in the soft iron is proportional to H cos θ and hence the couple due to this on the Mariner's Compass is proportional to H cos θ sin θ, *i.e.* to $\frac{1}{2}$ H sin 2θ. Obviously this changes sign when 2θ passes through 180°, *i.e.* when θ changes by 90°. This is therefore called *Quadrantal deviation.* This is corrected by placing suitable hollow spheres of soft iron near the Mariner's compass.

Exercise III

1. A compass needle is in tan A position with respect to a short magnet placed in East-West direction. In a certain place a certain deflection is produced when the distance between them is 25 cms. At another place, in order to produce the same deflection the magnet has to be pushed 5 cms towards the needle. Compare the horizontal forces at the two places.

Ans. 64 : 125.

2. What is meant by the horizontal intensity of Earth's magnetic field ? Explain what observations are necessary for the determination of total intensity of earth's magnetic field at any given place C. U. 1947.

3. How is the horizontal component of the earth's magnetic field determined ?

At a place where the angle of dip is 45° and the total intensity is 0·4 a magnet vibrating horizontally makes 10 oscillations per minute. Calculate the number it would make at a place where the dip is 60° and the total intensity is 0·5.

Ans. 9·40.

4. Explain how the magnetic meridian at any place may be determined with the help of a Dip Circle.

The apparent dip indicated by a Dip circle in any position is 60°. If the Dip circle be rotated through 90°, the apparent dip changes to 45°. Find the true dip at the place. Ans. 40° 54′

5. Explain why, in determining the dip at any place by a Dip circle, it is necessary to reverse the polarity of the needle.

6. Describe the Dip Circle and explain how you would determine by means of it (*a*) the magnetic meridian (*b*) the dip.

What are the various errors which may arise in the determination of dip by means of a Dip circle and show how they are eliminated.

7. If θ' be the apparent dip when the plane of the Dip circle makes an angle α with the magnetic meridian, prove that the true dip θ is given by the equation

$$\tan \theta = \tan \theta' . \cos \alpha$$

8. Explain what you mean by a magnetic map? How is it prepared?

Describe an accurate method for finding the declination of a place. How would you arrange for the observation of its daily variation?

9. Describe a Mariner's Compass and explain by what arrangement the Compass needles are made to remain always horizontal even when the ship is tossed by the waves.

Explain how the compass is affected by permanent magnets and soft iron of the ship and indicate in a general way how these are corrected.

C. U. Questions.

1962. (*a*) Describe a method of determining the horizontal component of the earth's magnetic field.

(*b*) A horizontally suspended magnet makes 30 oscillations in 2 min 30 sec at a place where the dip is 60° and the total intensity 0·5. Calculate the number of oscillations it will make in 3 min at another place where the dip is 45° and the total intensity is 0·57. Ans. 45·71

Hints :— $\frac{150}{30} = 2\pi\sqrt{\frac{I}{M \times 0{\cdot}5 \cos 60^\circ}}$

and $\frac{180}{n} = 2\pi\sqrt{\frac{I}{M \times 0{\cdot}57 \cos 45^\circ}}$ Hence find n.

1964. (1) Briefly describe a method of determining the horizontal component of earth's magnetic field at a place.

(2) Write notes on (*a*) dip circle (*b*) magnetometer.

1965. Describe the magnetometer working on the equation $T = 2\pi\sqrt{\frac{I}{MH}}$ and show how this can be used to compare the earth's magnetic field at two different places.

1966. (Special) Explain what is meant by magnetic moment.

In an experiment to determine M/H using a deflection magnetometer in the end on position the centre of the bar magnet was placed 20 cms from the magnetometer compass. The deflection was 18° and the length of the magnet is 10 cms. If H = 0·18 C. G. S. unit calculate the moment of the bar magnet. **Ans.** 205·5

1970. What do you mean by dip, declination and horizontal component of Earth's magnetic field.

Describe a Dip Circle and explain how with the help of this instrument you can measure the dip at a place on the Earth's surface. Briefly indicate the corrections required to obtain accurate result.

1972, 1975. Describe briefly the constructions of a Deflection and a Vibration Magnetometer. Explain how with their help the Horizontal Intensity of the Earth's magnetic field may be determined at any locality.

STATICAL ELECTRICITY

CHAPTER IV

FUNDAMENTAL THEORY

Art 20 When a glass rod is rubbed with silk it acquires a peculiar property—it has now the power of attracting light bodies, such as small pieces of paper. The same property is exhibited when an ebonite rod is rubbed with flannel or with cat's skin. According to modern ideas all substances contain electrons or negatively charged particles. Whenever two bodies are rubbed together some of the electrons pass from one body to the other. The body which now contains electrons in excess is negatively charged and the other body in which there is deficiency of electrons is positively charged. Thus any two bodies when rubbed together become charged by friction. In the case of a glass rod rubbed with silk electrons pass from glass to silk. So the glass rod becomes positively* charged and silk is negatively charged. In the case of ebonite rubbed with flannel or cat's skin reverse is the case. Electrons pass from the flannel (or cat's skin) to the ebonite so that the ebonite rod and the flannel (or cat's skin) acquire negative and positive charges respectively. A charged body has the power of attracting uncharged bodies. This is why light particles are attracted by the glass rod or the ebonite rod when they are rubbed respectively with silk or flannel. Theoretically whenever any two bodies are rubbed together both of them become charged and therefore acquire the power of attracting uncharged bodies. But the effect is marked in a few cases only.

When a positively charged glass rod is brought near a light pith ball suspended by a fine silk thread it is found that

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* *If a glass rod be rubbed with flannel the glass becomes negatively* charged and flannel positively charged.

the pith ball is at first attracted by the glass rod. But as soon as it touches the rod it is repelled and it flies away from the rod. This is because the uncharged pith ball is at first attracted by the charged glass rod; but on touching the rod the pith ball shares some of the charges on the rod, *i. e.* the pith ball also becomes positively charged. It is then repelled by the glass rod If now a negatively charged ebonite rod is brought near the positively charged pith ball attraction again takes place. As the pith ball however touches the ebonite rod the small amount of positive charge on the pith ball is more than neutralised by the negative charge on the rod so that the pith ball becomes negatively charged; it is then immediately repelled by the ebonite rod. Thus

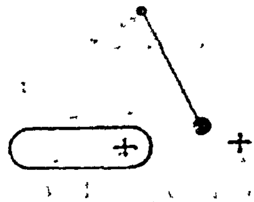


Fig. 42

(1) A body is charged by friction

(2) A charged body attracts an uncharged body

and (3) Like charges repel and unlike charges attract.

Art 21
Induction

If an uncharged body B be brought near a charged body A an opposite charge is induced on the face of B near to A and a similar charge is repelled as far off as possible, *i. e.* it is accumulated on the opposite face of B. The former is known as bound charge and the latter free charge. If B be now momentarily touched* by the hand, *i.e.* momentarily connected to the Earth the free charge—which is repelled as far off as possible—passes on to the Earth. If A be now

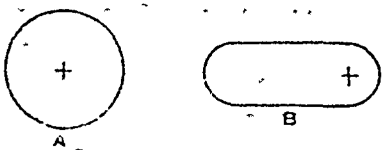


Fig. 43

* It is not necessary to touch B on the remote face. Even if B be touched on the face *nearer* to A the *free* charge passes on to the Earth while the bound charge remains on the face of B.

removed the bound charge on B becomes free and distributes itself over the entire face of B. This is known as charging by induction.

Insulators and Conductors Substances may, generally speaking, be divided into two classes—conductors and insulators. Substances through which charge may pass very easily are known as conductors and substances through which charge cannot pass are called insulators. Thus silk is an insulator while copper is a conductor. All metals are good conductors. Solutions of inorganic salts in water are also good conductors. The best non-conductors (or insulators) are ebonite, silk, sulphur, glass, shellac, paraffin, etc. Pure water is s non-conductor ; but water generally contains dissolved substances which make water conducting. There is however no sharp boundary line between conductors and insulators. There are substances which are neither good conductors nor good-insulators ; they are known as semi-conductors. Wood, paper and cotton are examples of such semi-conductors. Their conductivity depends upon the amount of moisture present in them. When dry they are fairly good insulators. But with a little of moisture they become conductors to some extent.

Art. 22 Gold leaf electroscope A gold leaf electroscope usually consists of a metal box with glass panes on the front and at the back. A metallic rod R carrying two gold leaves at the lower end passes into the box through a non-conducting stopper S. At the upper end a metallic plate P is attached to the rod. If a positively charged body B be brought near the plate P negative charge is induced on P and free positive charge passes on to the gold leaves. As both the leaves become similarly charged they mutually repel and therefore diverge. If the plate be momentarily touched by the hand the free positive charge on the leaves passes on to the Earth and the leaves collapse. If now the body B be taken off the negative charge on P becomes free and spreads over the entire system. The leaves being now

both negatively charged they diverge again. Thus by bringing a *positively* charged body B the electroscope is charged *negatively* and vice versa.

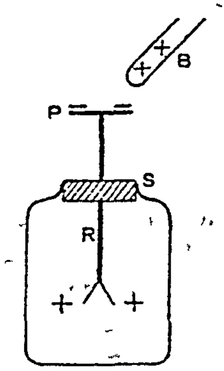


Fig. 44

If an ebonite rod rubbed with flannel be brought near the electroscope the leaves at once diverge showing the existence of charge on the rod. If instead of the rod, the flannel be brought near the electroscope then also there is divergence. But if both the rod and the flannel be brought together near the electroscope no divergence is produced. This proves that by friction equal and opposite charges are produced on the two bodies.

That equal and opposite charges are also produced by induction was proved by Faraday by his ice-pail experiment. On the plate B of an electroscope a metallic can is placed and into the can a charged metallic ball suspended by a silk thread is introduced. As the ball is gradually lowered into the can the leaves begin to diverge. Divergence becomes maximum when the ball is well within the can. The can is now momentarily touched by the hand. The free charge on the leaves goes away and the leaves collapse. There remain now the inducing charge on the ball and the induced charge on the can. They are obviously of opposite signs. To test whether they are also equal or not the metal ball is now made to touch the can. If the two charges are exactly equal they neutralise each other completely and the leaves show no divergence. On the other hand if one of the charges be in excees there will be some residual charge. Part of this charge will flow to the leaves and there will be some divergence. When the experiment is actually performed no divergence is observed in the

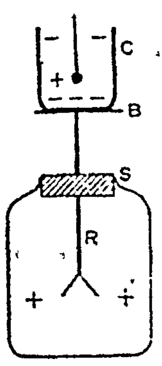


Fig. 45

leaves. Thus it is proved that the induced charge is equal to the inducing charge.

If two charges Q_1 and Q_2 be separated by a distance r each is acted on by a force $\frac{Q_1Q_2}{kr^2}$ where k is a constant depending upon the nature of the medium in which the charges are placed and is called the Specific Inductive Capacity (S. I. C.) of the medium.

Art 23
Fundamental formula

If the two charges are like the force is of repulsion; if they are unlike the force is of attraction. In either case the magnitude of the force between the charges is given by the above expression.

For air the S. I. C is unity; hence if the charges be placed in air the force between them is $\frac{Q_1Q_2}{r^2}$. As in Magnetism the unit charge is therefore defined as follows :—

Unit charge

If two charges of equal strength are placed in air at a distance of one cm. apart and if the force between them be one dyne then each of the two charges is said to be a unit charge.

This is known as an electrostatic unit (E. S. unit) charge. There is another unit known as electromagnetic unit (E. M. unit). The practical unit of charge is a Coulomb (Vide Chap IX).

One Coulomb $= 10^{-1}$ E. M. unit of charge, and one E. M. unit of charge $= 3 \times 10^{10}$ E. S. units of charge.

Hence one Coulomb $= 3 \times 10^9$ E. S. units of charge.

Electric Intensity

The electric intensity at a point is defined to be the *force* experienced by a unit positive charge placed at that point.

Thus if we consider a point at a distance r from a charge Q the electric intensity at the point is given by $F = \frac{Q.1}{kr^2} = \frac{Q}{kr^2}$

If the medium be air the intensity is $\frac{Q}{r^2}$. It follows from the above definition of the electric intensity that if a charge q be placed in a field of intensity F the force on the charge is qF.

It is to be noted that the electric intensity is a vector quantity, *i. e.* it has both magnitude and direction. The magnitude is given by $F=\frac{Q}{kr^2}$ and the direction is indicated by the definition that it is the force experienced by a unit *positive* charge. It is obvious that the force on a unit *negative* charge is of equal magnitude but is directed in the opposite direction.

The electric intensity is also known as the strength of the field or field strength ; the word 'field' alone often conveys the same idea.

Potential The electric potential of a body is the electric condition of the body which determines the flow of electricity from or towards the body and is measured by the work done in carrying a unit positive charge from infinity up to the body.

The electric potential at any point is thus numerically equal to the work done in carrying a unit positive charge from infinity up to the point.

The potential difference between two points is thus the work done in carrying a unit positive charge from one point to the other.

Hence the work done in carrying a charge Q from one point to another at a potential difference V is equal to QV. If both Q and V are measured in E. S units the work done is obtained in ergs. Thus if an electron (charge $=4{\cdot}80\times10^{-10}$ E. S. unit) falls through a potential difference of one volt $\left(\frac{1}{300}\text{ E. S. unit}\right)$ work done $=4{\cdot}80\times10^{-10}\times\frac{1}{300}=1{\cdot}60\times10^{-12}$ ergs. This energy is known as one electron volt. Atomic energies are nowadays measured in electron volts.

Art 24 Consider two points A and B very close to each other on the X axis at distances x and $x+dx$ from an arbitrary origin O. Let the potential at A be V and that at B be $V+dV$. The potential

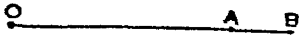


Fig. 46

difference $V_A - V_B = V - (V + dV) = -dV$. This is equal to the work done in carrying a unit positive charge from B to A. If F be the electric intensity at A* along X axis the force on a unit charge is F; and since the distance traversed is AB $= dx$ we have

$$F dx = -dV.$$

$$\text{or} \quad F = -\frac{dV}{dx} \quad \ldots \quad \ldots (18)$$

Equation (18) gives us the relation between the electric intensity and the electric potential at a point. In this equation F measures the intensity along X axis. If the actual intensity be in any oblique direction the component along X axis is $-\frac{dV}{dx}$; similarly, the component along Y axis is $-\frac{dV}{dy}$ and that along Z axis is $-\frac{dV}{dz}$.

It is to be remembered that unlike electric intensity electric potential is a scalar quantity *i. e.* it has magnitude but no direction.

Consider a point A on the X axis at a distance x from the point charge Q. Then the intensity at A is $\frac{Q}{kx^2}$

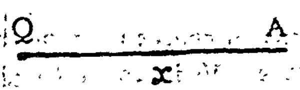


Fig. 47

$$i.e. \quad -\frac{dV}{dx} = \frac{Q}{kx^2}$$

$$\text{or} \quad dV = -\frac{Q}{kx^2} dx$$

$\therefore \quad V = -\int \frac{Q}{kx^2} dx + C = \frac{Q}{kx} + C$ where C is the constant of integration. Since the potential at infinity is zero, *i. e.* V=0 when $x = \infty$, we have C=0. Hence $V = \frac{Q}{kx}$. If the medium be air $V = \frac{Q}{x}$

---

* The distance AB($=dx$) being infinitesimally small the electric intensity at A may be supposed to be the same as at B or at any point in between.

N. B. (1) Throughout the remaining portion of the book we shall always assume the medium to be air unless otherwise stated.

(2) It should be noted that so far magnetism and statical electricity are exactly analogous to each other.

The capacity of a conductor is the charge necessary to raise the potential of the conductor by unity.

Hence if C be the capacity the charge Q necessary to raise the potential to V units, is CV, *i. e.* $Q=CV$.

If Q and V be in E. S. units, C also is in E. S. units. If however both Q and V be expressed in practical units C also is in practical units. The practical unit of charge (Q) is a Coulomb, that of the potential *(V)* is a volt and that of the capacity *(C)* is a Farad. The following conversion table should be remembered :—

One Volt $=10^8$ E. M. units of E. M. F.

One E. S. unit of E. M. F. $=3\times10^{10}$ E. M. units of E. M. F.

$=300$ Volts.

Also, one E. M. unit of capacity $=9\times10^{20}$ E. S. units of capacity.

One Farad $=10^{-9}$ E. M. units of capacity.

$=9\times10^{11}$ E. S. units of capacity.

One micro-farad $=10^{-6}$ Farad

$=9\times10^{5}$ E. S. units of capacity.

It is to be noted that the capacity of a conductor depends upon the shape and size of the conductor. It does not depend on the material of which the conductor is made. Generally speaking, the greater is the area exposed to air the greater is the capacity of the conductor. The capacity of a conductor also depends on the position of the conductor with respect to other neighbouring conductors. It also depends on the specific inductive capacity of the medium in which the conductor is placed. It will be proved later in Art 33 that the capacity of a sphere is equal to its radius.

Art 25 Potential energy

Students must not confuse the potential of a charged conductor with the potential energy of a charged conductor. The former simply means the work that would be done in bringing a unit positive charge from infinity to the conductor *i. e.* work done in giving an additional unit charge to the

conductor. The latter, however, means the work that has already been spent in charging the conductor to the present amount.

Consider a conductor at potential V, carrying a charge Q. Let C be the capacity of the conductor. We suppose that the *total charge Q on the conductor is gradually built up by* successively bringing infinitesimal charges dq from infinity to the conductor. At any stage during this process of charging let the amount of charge on the conductor be q and the potential v. Then $q = Cv$. To bring the next instalment of charge dq to the conductor, the work done $= vdq$ [$\because$ v, the potential, measures the work done in bringing a unit charge to the conductor].

$\therefore$ The total work done (*i. e.* potential energy)

$$\left.\begin{aligned} &= \int_0^Q v\,dq = \int_0^Q \frac{q}{C}\,dq = \frac{1}{2}\frac{Q^2}{C} \\ &\text{And since } Q = CV, \text{ this also} = \tfrac{1}{2}CV^2 = \tfrac{1}{2}QV \end{aligned}\right\} \quad \ldots \quad (19)$$

If two conductors at potentials V_1 and V_2 be electrically connected charge flows from the conductor at higher potential to that at lower potential. It will now be shown that whatever be the values of V_1 and V_2—in all cases there is a loss of energy of the system. Only when $V_1 = V_2$ there is neither any loss nor any gain. Let C_1 and C_2 be the capacities of the two conductors and let V be the final common potential after the conductors are connected electrically. Since the total charge on the two conductors remains the same before and after electric connection, we have

Art 26
Loss of energy

$$C_1V_1 + C_2V_2 = C_1V + C_2V \quad \text{or} \quad V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

But Original energy $= \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 = \frac{1}{2}(C_1V_1^2 + C_2V_2^2)$.

and final energy $= \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 = \frac{1}{2}(C_1 + C_2)V^2$

$$= \tfrac{1}{2}(C_1 + C_2)\cdot\frac{(C_1V_1 + C_2V_2)^2}{(C_1 + C_2)^2} = \tfrac{1}{2}\cdot\frac{(C_1V_1 + C_2V_2)^2}{C_1 + C_2}$$

$\therefore$ Decrease in energy

$$=\tfrac{1}{2}(C_1V_1^2+C_2V_2^2)-\tfrac{1}{2}\frac{(C_1V_1+C_2V_2)^2}{C_1+C_2}$$

$$=\tfrac{1}{2}\,\frac{(C_1+C_2)\,(C_1V_1^2+C_2V_2^2)-(C_1V_1+C_2V_2)^2}{C_1+C_2}$$

$$=\tfrac{1}{2}\,\frac{C_1C_2(V_1-V_2)^2}{C_1+C_2}.$$

In this expression C_1 and C_2 are essentially positive quantities and $(V_1-V_2)^2$ being a perfect square, is also positive. Hence there is always a loss of energy.

It is to be seen that loss of energy vanishes when $V_1=V_2$; but in that case no flow of charge takes place even when the conductors are electrically connected.

According to the principle of conservation of energy energy can never be really lost. The energy which thus appears to be lost reappears partly as heat in the connecting wire and partly as heat, light and sound produced by the sparking between the two conductors.

This loss of energy is in ergs provided both C and V are expressed in C. G. S. units.

Three charges +10, −10 and +10 are placed at the three corners of a square of side 5 cms. Find the potential and intensity at the fourth corner.

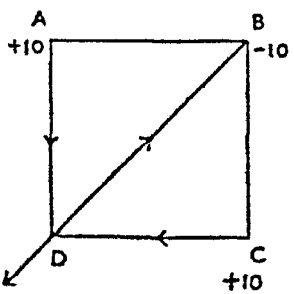


Fig. 48

$BD^2=BC^2+CD^2=2.5^2$

$\therefore\ BD=5\sqrt{2}.$

$$V_D=\frac{10}{5}+\frac{10}{5}-\frac{10}{5\sqrt{2}}$$

$=2+2-\sqrt{2}$ C. G. S. units.

$=4-\sqrt{2}$ C. G. S. units.

To find the intensity we must consider the direction as well. Thus the intensity at D is (1) $\frac{10}{5^2}=\frac{2}{5}$ along AD (2) $\frac{10}{5^2}=\frac{2}{5}$ along CD and (3) $\frac{10}{5^2.2}=\frac{1}{5}$ along DB. The resultant of

(1) and (2) is $\frac{2\sqrt{2}}{5}$ along BD produced. Hence the resultant intensity at D is $\frac{2\sqrt{2}-1}{5}$ C. G. S. unit along BD produced.

Two spheres A and B of diameters 20 cms and 30 cms respectively have charges 50 and 60 units respectively. If they are connected by a wire how much charge flows through the wire? Does the charge flow from A to B or from B to A?

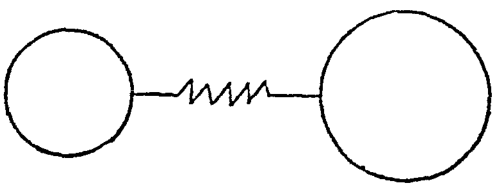


Fig. 49

The capacities of A and B are respectively equal to 10 and 15.

$\therefore\ V_A = \frac{50}{10} = 5$ and $V_B = \frac{60}{15} = 4$.

Since A is at a higher potential charge flows from A to B.

To find the final common potential V we apply the following :—

Total charge before connection = total charge after connection.

$\therefore\ 50 + 60 = 10V + 15V$ or $V = \frac{110}{25} = 4{\cdot}4$.

$\therefore$ Charge on $A = 10 \times 4{\cdot}4 = 44$. Thus $50 - 44 = 6$ units of charge flows through the wire.

A and B are two conductors whose capacities are in the ratio 2 : 3. A receives a charge and shares it with B. Compare the total energy of A and B with that originally possessed by A.

Let the capacities of A and B be respectively 2C and 3C and let Q be the charge received by A. If, after A shares the

charge with B, the final common potential be V,

$$2CV + 3CV = Q \quad \text{or} \quad V = \frac{Q}{5C}$$

Originally the energy of $A = \frac{1}{2}\frac{Q^2}{2C} = \frac{Q^2}{4C}$. Finally the energy of $A = \frac{1}{2}.\ 2C\ V^2 = \frac{Q^2}{25C}$ and the energy of $B = \frac{1}{2}.\ 3C.\ V^2 = \frac{3Q^2}{50C}$.

Hence the total energy of A and $B = \frac{Q^2}{25C} + \frac{3Q^2}{50C} = \frac{Q^2}{10C}$.

$\therefore$ the required ratio $= \frac{Q^2}{10C} \Big/ \frac{Q^2}{4C} = \frac{2}{5}$

Art 27 Inverse Square Law. As in Magnetism the law that the force between two charges varies inversely as the square of the distance is known as Inverse Square Law. The proof of the law is as follows :—

Cavendish's Proof. It is obvious that the force between two charges must depend on the distance. We assume that this force varies inversely as p^{th} power of the distance. We shall see that experimental evidence supports the view that $p = 2$.

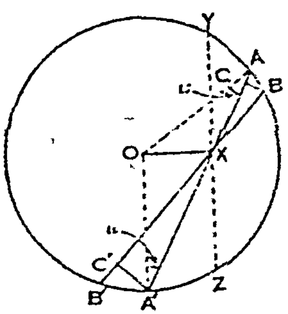


Fig. 50

Consider a hollow spherical conductor with centre O charged uniformly with positive surface density σ. Let X be any point within the sphere. Through X draw two cones AXB and A′XB′ one cone being simply the extension of the other beyond the vertex. Let each of these cones be of a small solid angle $d\theta$. Let the areas on the sphere intercepted by these cones be AB and A′B′ at distances r and r' from X. Join OA, OA′. Let $\angle OAA' = \angle OA'A = \omega$. Through B and A′ draw BC and A′C′ perpendicular sections of the cones. Then each of the $\angle^s$ ABC and B′A′C′ is also equal to ω (since the angle between two lines is equal to that between their perpendiculars).

Thus $\quad AB\cos\omega = BC = r^2 d\theta,$

and $\quad A'B'\cos\omega = A'C' = r'^2\, d\theta$

$\therefore \quad AB = \dfrac{r^2 d\theta}{\cos\omega}$ and $A'B' = \dfrac{r'^2\, d\theta}{\cos\omega}$

$\therefore \quad$ The charge on $AB = \sigma AB = \dfrac{\sigma r^2\, d\theta}{\cos\omega}$

and $\quad$ that on $A'B' = \sigma A'B' = \dfrac{\sigma r'^2 d\theta}{\cos\omega}$

$\therefore$ Intensity at X due to charge on AB

$$= \frac{\sigma r^2 d\theta}{r^p \cos\omega} = \frac{\sigma d\theta}{r^{p-2}\cos\omega} \quad \text{along AX} \quad \ldots \quad (\alpha)$$

and intensity at X due to charge on A'B'

$$= \frac{\sigma r'^2 d\theta}{r'^p \cos\omega} = \frac{\sigma d\theta}{r'^{p-2}\cos\omega} \quad \text{along A'X} \quad \ldots \quad (\beta)$$

In the figure r' is $> r$.

Three cases now arise ;

(1) $p > 2$. In this case since $r' > r$, (α) is $> (\beta)$, *i. e.* the resultant intensity at X due to charges on AB and A'B' is directed along AX, *i. e.* along XA' [Fig 50]. Thus dividing the whole of the sphere into pairs of such cones it is obvious that the resultant intensity due to every one of such pairs is directed towards the left of the plane YZ (drawn perp. to OX). Hence the resultant intensity due to charges on the entire sphere is directed towards the centre O.

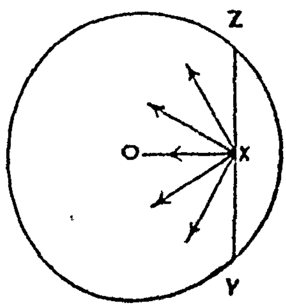


Fig. 51

(2) $p < 2$. In this case (α) is $< (\beta)$, *i. e.* the resultant intensity due to charges on AB and A'B' is directed along A'X *i. e.* along XA [Fig 50] ; arguing as before the resultant intensity due to the total charge on the sphere is directed away from the centre O.

(3) $p=2$. In this case $(\alpha)=(\beta)$. Hence the resultant intensity at X vanishes.

Cavendish placed a conducting sphere S inside another sphere S′ insulated from each other. S′ was charged positively and was connected to S momentarily by a wire. The two were then disconnected. From the preceding discussion it is obvious that positive electricity would flow towards the centre and hence S would be positively charged if $p > 2$, positive electricity would flow away from the centre and therefore S would be negatively charged if $p < 2$; if however, $p=2$ there will be no flow of electricity and hence S would remain uncharged. In the actual experiment S was tested and was found to be uncharged. Cavendish therefore concluded that $p=2$. Later,

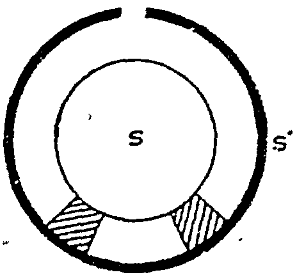


Fig. 52

Maxwell repeated the experiment with improved apparatus and came to the conclusion that the difference in the value of p from 2, can hardly be greater than $\frac{1}{21600}$.

Art 28
Torsion balance

The torsion balance may also be used to prove the inverse square law. It essentially consists of a cylindrical glass vessel round the side of which a scale is graduated. A silver wire carrying a horizontal insulating rod AB (of shellac) is suspended from a torsion head which can be rotated and the amount of rotation can be read off from another circular scale at the top. A small pith ball is attached to one end A of the insulating rod. Another

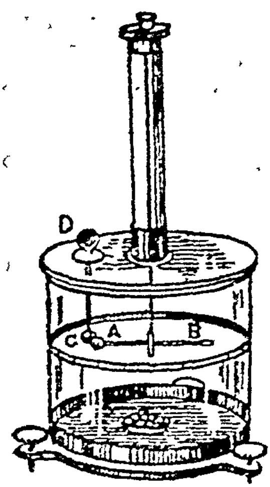


Fig. 53

insulating rod CD carrying a second pith ball at the lower end C, is placed in the vertical position. The torsion head is adjusted so that in the equilibrium position the two pith balls are just in contact. The pith attached to the vertical rod is now removed and taken outside; it is then given a certain charge and is again placed in its own position at the bottom of the vertical rod. When the other pith ball comes in contact with this pith ball charge is shared by both of them and repulsion takes place. The rod AB rotates and the suspension wire is twisted. Equilibrium takes place

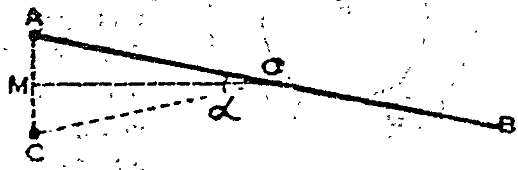


Fig. 54

when couple due to torsion in the suspension wire balances that due to electric repulsion. The torsion head is now turned through an angle β in the opposite direction so that the angular separation between the pith balls is reduced to α. Then the torsion in the wire is $\alpha+\beta$. Let O be the centre of the horizontal rod AB carrying the pith ball at its end A. Then if C be the fixed ball $\angle AOC=\alpha$. Join OC, AC and drop OM perp. to AC. Let F be the force of repulsion between A and C; then the moment about O of this force $=F.\ OM$ $=F.\ l\cos\frac{\alpha}{2}$ where $l=OA=$ half the length of the horizontal rod. And the couple due to torsion $=\mu(\alpha+\beta)$ where μ is the couple per unit twist.

$$\therefore\ F\,l\cos\frac{\alpha}{2}=\mu(\alpha+\beta)\qquad\therefore\quad F=\frac{\mu(\alpha+\beta)}{l\cos\frac{\alpha}{2}}$$

$$\therefore\ F.\ AC^2=\frac{\mu(\alpha+\beta)}{l\cos\frac{\alpha}{2}}\left(2l\sin\frac{\alpha}{2}\right)^2$$

$$\left[\because\ AC=2AM=2l\sin\frac{\alpha}{2}\right]$$

$$=4\mu l(\alpha+\beta)\sin\frac{\alpha}{2}\tan\frac{\alpha}{2}\qquad\ldots\qquad\ldots\qquad(a)$$

If inverse square law be true F will vary inversely as AC^2, *i. e* $F.AC^2$ will remain constant. Hence from (*a*) $(\alpha+\beta)\sin\frac{\alpha}{2}\tan\frac{\alpha}{2}$ will remain constant. The torsion head head is rotated through different angles so that sets of values of α and β are obtained. It is then shown experimentally that for all sets,

$$(\alpha+\beta)\sin\frac{\alpha}{2}\tan\frac{\alpha}{2}=\text{const}$$

Thus Inverse Square Law is verified.

N. B, (1) If α be small, $\sin\frac{\alpha}{2}=\frac{\alpha}{2}=\tan\frac{\alpha}{2}$. In that case $(\alpha+\beta)\sin\frac{\alpha}{2}\tan\frac{\alpha}{2}=\frac{1}{4}(\alpha+\beta)\alpha^2$; thus the approximate relation $(\alpha+\beta)\alpha^2=$ const. may be used.

(2) The torsion balance may also be used to verify inverse square law in the case of magnetism. In that case the two rods carrying the pith balls are replaced by two magnets. Repulsion takes place between two similar poles which now occupy the positions of the two charged pith balls. Otherwise calculations are exactly identical.

Exercise IV

1. Three charges +10, +4, +4 are placed at the three corners A, B, C of an equilateral triangle of side 2 cms. Find the potential and intensity at the middle point of BC.

Ans. 13·77 ; 3·33 away from A.

2. Explain the term electric potential. A charge of 5 units is taken from a point at potential −15 to another at V. If the work done thereby be 200, find V. Ans. 25

3. A spherical conductor of radius 5 cm and charged to a potential of 48 E. S units, is placed at a great distance from another of radius 8 cm and charged to a potential of −30 E. S. units. Calculate the potential at a point midway between the centres of the spheres. Ans zero. C. U. 1932.

4. Describe experiments which have established the Inverse Square Law of force between charged bodies. Give the theory.

Charges 1, 2 and -3 units are placed at the three corners A, B and C of an equilateral triangle, taken in order. If the length of each side of the triangle be 10 cms find the force, in magnitude and direction, at the middle point of AB.

Ans. 0·057 making 45° with AB C. U. 1941.

5. Define the term potential as used in electrostatics.

Two spheres of radii 5 and 10 cms. respectively have equal charges of 60 units each. They are then joined by a thin wire so as to be able to share the charges between them. Calculate the total energy of the conductors before and after sharing. What becomes of the difference of energy ? C. U. 1944.

Ans. 375 ergs ; 333·3 ergs.

6. Define electric potential and electric intensity. What is the relation between these quantities ?

A conductor of capacity 15 is raised to a potential of 40 ; it is then connected to an uncharged sphere and the common potential drops to 30. Find the diameter of the sphere. Find also the loss of energy when the charge is shared.

Ans. 10 cms ; 3000 ergs.

7. The radius of the Earth is 6400 Kilometres. Calculate its capacity in microfarads. [One micro-farad $= 9 \times 10^5$ E. S. units of capacity]. Ans. 711 mfd.

8. Two spheres of diameters 8 and 12 cms are respectively charged with 24 and 60 units of charge. Find the loss of energy when they are connected by a wire. Ans. 19·2 ergs.

9. Two spheres of diameters 6 and 10 cms placed at a distance from each other are charged respectively with 5 and 12 units of positive electricity. They are then connected by a fine wire. Does any spark pass ? If so, how much energy is dissipated ? Ans. 0·50 ergs. C. U. 1949.

10. Two equal raindrops charged with equal quantities of positive electricity are combined so as to form one large drop. Compare the potentials before and after union. Compare also the surface densities. Ans. $1 : 2^{\frac{2}{3}}$ and $1 : 2^{\frac{1}{3}}$

11. What is the inverse square law as applied to the force due to an electric charge ?

Give an account of the experiments of Cavendish and Maxwell to prove the validity of the inverse square law.

12. Find the kinetic energy acquired by an electron (charge $4{\cdot}77 \times 10^{-10}$ E.S. units) in falling through a P.D. of one volt ($\frac{1}{300}$ E S. unit) [This energy is known as one electron-volt]

If the mass of the electron be $8{\cdot}9 \times 10^{-28}$ gm, find the velocity acquired. [This is known as the velocity of one volt].

Ans. $1{\cdot}59 \times 10^{-12}$ ergs ; $59{\cdot}77 \times 10^{6}$ cms per sec.

[Hints: Kinetic energy=work done $= 4{\cdot}77 \times 10^{-10} \times \frac{1}{300}$.

Kinetic energy is also$= \frac{1}{2} \times 8{\cdot}9 \times 10^{-28} \times v^2$. Hence find v]

13. An electron starts from rest from a point on one conductor and reaches a second conductor with a velocity of 10^9 cms. per sec. Calculate the P. D. between these conductors in volts. Which of them is at the higher potential ? Ans. 279·9 volts. The second conductor.

14. In an electron tube the two electrodes at a difference of potential of 100 volts are 0·6 cms apart. How long would an electron take, starting from rest at one electrode, to reach the other ? Ans. $2{\cdot}01 \times 10^{-9}$ secs.

C. U. Question.

1964. Define electric potential and electric field intensity.

A conducting plate is charged to a potential of 4000 volts. A second metal plate charged to a potential of 1000 volts is brought near the first to a distance of 10 cms. What is the field intensity at any point between the plates ? Ans. 1 E.S.U.

1965 Two conductors having different capacitances are charged separately to different potentials. Deduce an expression for the loss of energy when these two conductors are made to share their charges.

A sphere of radius 5 cms and charged with 50 E. S. U. is connected by a wire of negligible capacitance with another sphere with the same charge but double the radius. Calculate the loss of energy. Ans 41'7 ergs

1966. Define (*a*) electric intensity and (*b*) difference of potential. How are these quantites related ?

1969. Distinguish between the potential and potential energy of a charged conductor. Prove that there is always a loss of energy when two conductors at different potentials are joined together.

1972. Define electrostatic potential at a point in an electric field. Derive an expression for the potential at a distance r cm from an isolated point charge having q electrostatic units of charge.

CHAPTER V

LINES OF FORCE, GAUSS'S THEOREM AND ITS APPLICATIONS

Art. 29 **Line of force.** An electric line of force is a line such that at any point on it the electric intensity is tangential to it.

Fig. 55

Thus in Fig 55, at points A, B,··· on the line of force the intensity is tangential to the line and is directed away from the positive charge.

If the potential be the same at all points on a surface the surface is said to be equipotential.

As in Magnetism it can be similarly proved that two lines of force cannot intersect each other and that a line of force cuts an equipotential surface at right angles (Vide Art 6).

The surface of a conductor is always an equipotential surface; for, otherwise, electricity would flow from points at higher potential to those at lower potential. It follows that lines of force coming out of a charged conductor are, at the start, perpendicular to the surface of the conductor. Hence the intensity at any point close to a charged conductor must be directed *normally* to the surface.

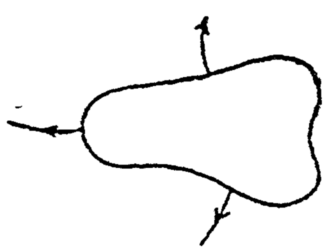
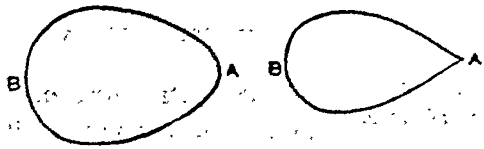


Fig. 56

Although the surface of a charged conductor is an equipotential surface the distribution of charge over the entire surface is generally not uniform. If the surface have different curvatures at different points the concentration of charge is

also different at different points. The more curved a surface is the greater is the concentration of charge there. Thus in Fig 57 (a) charge is more concentrated at A than at B. This is easily proved with the help of a proof plane*. The proof plane is made to touch the conductor at A and is then presented to a gold leaf electroscope; the amount of divergence of the leaves is noted.



(a) Fig. 57 (b)

Next after discharging the proof plane it is again made to touch the conductor at B. If it is now presented to the gold leaf electroscope it is noticed that the divergence is now less. This proves that charge is more concentrated at A than at B.

If the surface at A becomes more and more curved concentration of charge at A also becomes greater and greater Ultimately when the surface at A becomes pointed as in Fig. 57 (b) an interesting phenomenon occurs. It is obvious that air molecules must always be striking the surface of a charged conductor and taking away charge from the conductor. Generally speaking the removal of charge in this way is so slow that for a fairly long time a charged conductor retains its charge. If however the conductor be pointed as in Fig 57 (b) the concentration of charge at A is extremely large so that air molecules coming in contact with the conductor at A take away considerable amount of charge. Thus in a very short time almost all the charge is taken away from the conductor and the conductor becomes practically discharged in no time. Thus a pointed conductor cannot retain its charge.

A collecting comb in an electrostatic machine [Vide Art 40] depends for its action upon this property of a pointed conductor. A group of pointed rods mutually connected with one another is known as a collecting comb. Let us suppose that

* A proof plane is a small conductor (usually, a circular metal piece) provided with an insulating handle.

this is in metallic connection with a Leyden* jar L. If now a charged body be brought near the collecting comb opposite charge is induced on the points and similar charge passes on to the Leyden jar. But charge on the points dissipates away into air in no time and is attracted by the opposite charge on the charged body and ultimately the charge on the body is neutralised. Thus the net result is that by the action of the collecting comb the charge on the charged body is transferred to the Leyden jar L.

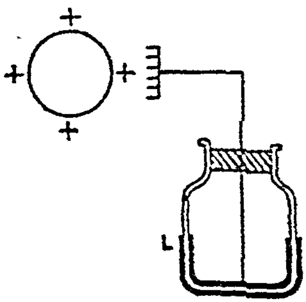


Fig. 58

Lightning Conductor

The property of a pointed conductor has also been ultilised in the case of a lightning conductor. First suggested by Benjamin Franklin in 1749 it is nothing but a pointed † metallic rod placed vertically over the top of a tall building. The pointed end projects above the highest point of the building and the rod is in metallic connection with the Earth.

During a thunderstorm clouds become heavily charged and there is a chance of a lightning flash passing between a cloud and a tall building. This chance is however minimised by the presence of lightning conductors. The charge on the cloud induces opposite charge on the lightning conductor and similar charge passes on to the Earth. But the lightning conductor being pointed it cannot retain its charge. The induced charge on the lightning conductor is therefore quickly dissipated away into air and is attracted towards the cloud. Thus the charge on the cloud is diminished. Sometimes even after this the cloud may still have sufficient charge so that a flash may ultimately pass. But the lightning conductor being nearest to the cloud the flash passes on to it and as it is

* What is a Leyden jar will be explained in Art 44.

† There may be more than one pointed end in a lightning conductor.

metallically connected to the Earth the charge passes on to the Earth through the metallic connection and the building remains safe.

Art 31
Tube of force

Lines of force coming out of a charge may be divided into a number of groups, each group being called a tube of force. Obviously, such grouping may be done in a variety of ways. According to Faraday grouping is so made that Q tubes come out of a charge Q, where as, according to Maxwell, $\frac{4\pi Q}{k}$ tubes emanate from a charge Q.

Thus, Q Faraday Tubes $=\frac{4\pi Q}{k}$ Maxwell tubes.

Or one Faraday tube $=\frac{4\pi}{k}$ Maxwell tubes.

Throughout the remaining portion of this book whenever we shall speak of tubes of force we shall always mean Faraday tubes.* We shall also use the expression "lines of force" when tubes of force are actually meant. Thus we speak of 50 lines of force emanating out of a charge of +50.

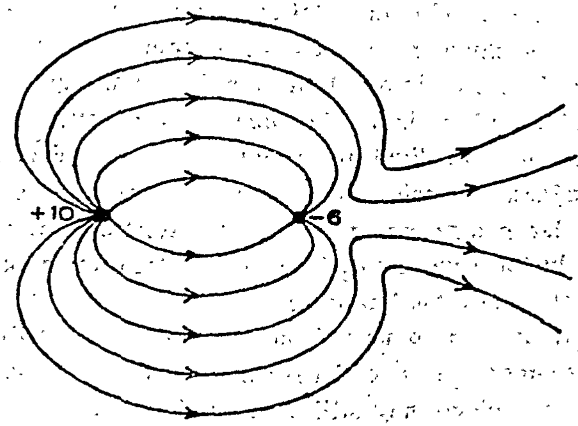


Fig. 59

* It is be noted that in Magnetism Maxwell tubes are always used instead of Faraday tubes [Vide Art 7]

Lines of force start from positive charges and end on negative charges. If we have two charges +10 and −6, ten lines of force come out of the charge +10. Of these six go to the charge −6 and the remaining four pass on to infinity. [Vide Fig. 59]

It is obvious that within a charged hollow conductor there cannot be a line of force and hence there cannot be any intensity; for otherwise, if a line of force exists joining two points of the conductor some work will have to be done in moving a unit charge along this line from one end to the other, *i. e.* there will be some difference of potential between the end points. But this is impossible, as all points of a conductor are at the same potential.

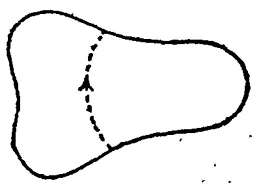


Fig. 60

Since there is no intensity within a hollow conductor the potential at all points throughout the interior must necessarily be the same. This potential is the same as that of the conductor itself.

Art 32 Normal Induction

If we consider an elementary area dS of a surface placed in an electric field and if F be the electric intensity at dS, making an angle θ with the normal then normal intensity is F $\cos\theta$ and normal induction over dS, is $kF \cos\theta\, dS$ where k is the S. I. C. of the medium. The normal induction over an entire surface may be found by integrating the above expression.

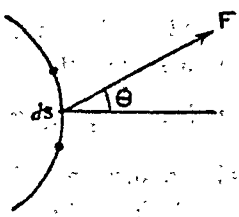


Fig. 61

Gauss's Theorem

Statement. The total normal induction over a *closed* surface is equal to 4π times the charge inside the surface.

Proof. Let AB represent an elementary area dS of a closed surface completely surrounding a charge Q at O. Let

D be the central point of AB. Then the intensity at dS is $\frac{Q}{k.OA^2}$ acting along OD produced. [Since AB is an infinitesimal area it is immaterial whether we take OA or OD as the distance of BA from O]. If this makes an angle θ with the normal DN, the normal induction over dS is

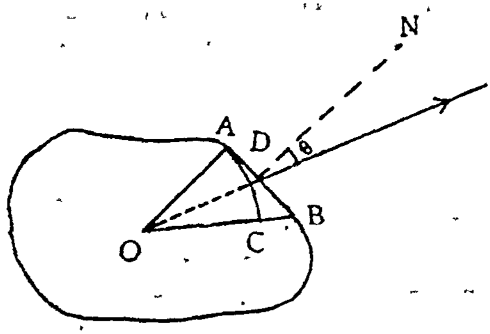


Fig. 62

$$k.\frac{Q}{k\,OA^2}.\quad \cos\theta.\,AB = Q.\quad \frac{AB\cos\theta}{OA^2}$$

Draw AC perp to OD. Then $\angle BAC$ is also θ. Therefore $AB\cos\theta$ = area AC. Hence

$$\text{Normal induction over } dS = Q.\frac{\text{area AC}}{OA^2} = Q\,dw$$

where dw is the solid angle AOB subtended by AB at O. Hence the total normal induction over the entire closed surface $= \int Q\,dw = Q \times$ total solid angle subtended by the closed surface at O $= 4\pi Q$.

If there be two or more charges inside the surface the normal induction over the surface, contributed by each of these charges, is 4π times the corresponding charge; hence total normal induction due to all the charges is equal to 4π times the sum of all the charges.

If there be any charge or charges outside the surface they do not contribute towards the total normal induction over the surface; for, if a cone with a small solid angle dw, be drawn with one of the charges Q as the vertex, it will in general meet the closed surface in two elementary areas. The normal inductions over these two areas will be equal,

each being equal to $Qd\omega$ but they will be of opposite signs, for one will be directed along the inward normal and the other along the outward normal. They will therefore cancel each other. Similarly, the whole of the surface being divided into pairs of such elementary areas, the total normal induction over the entire surface will be zero.

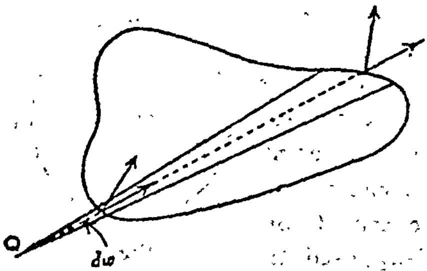


Fig. 63

If there be no charge within the surface the total normal induction over the surface is zero.

Art 33. Uniformly charged sphere

Consider a sphere of radius a, charged uniformly with Q units of charge.

Let F be the intensity at A due to this charge, A being a point outside the sphere at a distance r from the centre O. With centre O and radius equal to OA ($=r$) describe a sphere. From symmetry, the intensity at every point on the sphere is F directed normally to the sphere. Let us apply Gauss's theorem over the surface of this sphere. We have

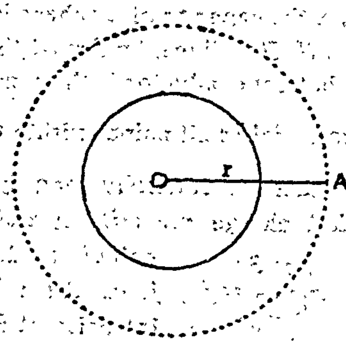


Fig. 64

$$\text{Normal induction} = kF \times \text{area} = kF.\ 4\pi r^2.$$

$\therefore$ by Gauss's Theorem, $kF.\ 4\pi r^2 = 4\pi Q$.

$$\therefore \quad F = \frac{Q}{kr^2}.$$

If the whole of the charge on the sphere be concentrated at O, the intensity at A is also equal to $\dfrac{Q}{kr^2}$ [Art 23].

Thus for any point outside the sphere the whole of the

charge may be supposed to be concentrated at the centre. The potential at A is therefore equal to $\frac{Q}{r}$.

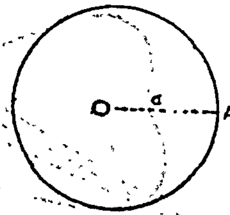


Fig. 65

Now, let the point approach the sphere until it just reaches the surface of the sphere. The distance of the point A from the centre is now equal to the radius of the sphere. The charge Q on the sphere may still be supposed to be cencentrated at the centre. Thus the potential at A is $V=\frac{Q}{a}$, A now being a point on the sphere the potential of the sphere is thus equal to $\frac{Q}{a}$. But if C be the capacity of the sphere, $Q=CV=C\frac{Q}{a}$. Thus $C=a$ *i. e.* the capacity of a sphere is equal to its radius.

N. B. Since, by Art 31 the potential at all points within a hollow conductor is the same as that of the conductor the potential at all points within the sphere is $\frac{Q}{a}$

Art 34 Cylindrical charge

Let an infinitely long cylinder be charged uniformly so that charge *per unit length* of the cylinder is Q. Let P be a point (outside the cylinder) at a distance r from the axis of the cylinder. To find the intensity at P we imagine a co-axial closed cylinder of length L, passing through P and apply Gauss's theorem over this cylinder. From symmetry the intensity at all points of the curved surface of this cylinder is F and is directed normally to the surface. The normal induction over this curved surface is therefore equal to $kF.2\pi rL$. The plane faces of this cylinder do not contribute anything towards the total normal induction

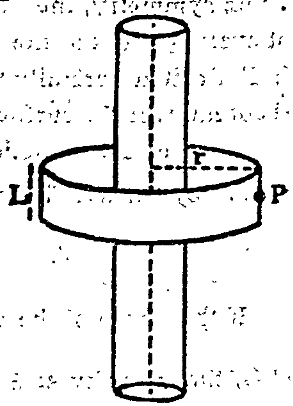


Fig 66

because (from symmetry) intensity at any point on the plane faces lies along the plane and the normal intensity is zero. And since a length L of the charged cylinder is enclosed within the imaginary closed cylinder the charge on this portion is QL. We have therefore $kF.2\pi rL = 4\pi QL \therefore F = \frac{2Q}{kr}$ (20)

Let S_1 and S_2 be two sections of a tube of force. Let us apply Gauss's theorem to this portion of the tube bounded by S_1 and S_2. Since intensity is always tangential to the tube there is no component normal to the curved surface of the tube and hence the normal induction over the curved surface is zero. If F_1 and F_2 be the intensities at the faces S_1 and S_2 the normal induction over these two sections $= F_2S_2 - F_1S_1$ the intensity at S_1 being directed inwards, is taken to be negative. Since there is no charge inside the tube $F_2S_2 - F_1S_1 = 0$

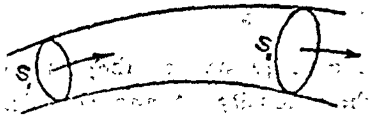


Fig. 67

$$\therefore F_2S_2 = F_1S_1$$

If two sections S_1 and S_2 are equal, *i. e.* if the lines of force are parallel, $F_1 = F_2$. *i. e.* the field is uniform.

Art 35 Coulomb's Theorem. Let ab be an elementary area ds of a charged conductor and let P be a point just outside the conductor and close to ds. Let F be the intensity at P. Imagine a closed rectangular surface ABCD such that (1) P lies on AB (2) AB is just outside and CD just inside the conductor; but each of them is equal and parallel to ab $(= ds)$. and (3) AD and BC just enclose ds and are perpendicular to ds.

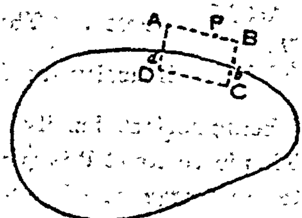


Fig. 68

In applying Gauss's theorem over this closed surface we notice that since there is no intensity within a conductor (Vide Art 31), the portion $aDCb$, being within the conductor,

contributes nothing towards the total normal induction. Of the remaining portions again since the intensity near a charged conductor is perpendicular to the surface (Vide Art 29). there is no *normal* induction over Aa and Bb. Hence we are left with only AB over which the normal induction is obviously equal to $kFds$. If σ be the surface density over ds, the charge enclosed is $\sigma\, ds$. Hence

$$kFds = 4\pi\sigma ds \quad \therefore \quad F = \frac{4\pi\sigma}{k}.$$

N. B. Since tubes of force diverge out as they recede away from the conductor, the intensity at any considerable distance from the conductor, is *not* equal to $\frac{4\pi\sigma}{k}$, but falls off inversely as the cross section of the tube of force (Vide Art 34). If however in any special case the tubes of force are all parallel the intensity even at a considerable distance from the conductor is equal to $\frac{4\pi\sigma}{k}$.

[Compare the case of a parallel plate condenser ; Art 48].

Art 36
Electric Pressure

Let P be a point close to but just outside an elementary area ds of a charged conductor. Then the intensity at P is $\frac{4\pi\sigma}{k}$ σ being surface density over ds. This intensity at P is partly due to the charge on ds and partly due to the charge on the remaining portion of the conductor. If f and f' be these two components we have

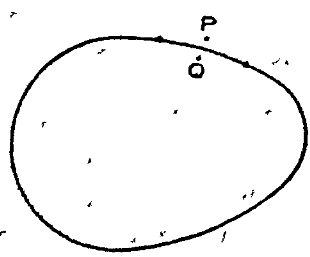


Fig. 69

$$f + f' = \frac{4\pi\sigma}{k} \quad \ldots \quad \ldots \quad \ldots \quad (a)$$

If we now imagine a point Q close to ds but just inside the conductor the intensity at Q due to charge on ds is numerically equal to f but reversed in direction, *i. e.* equal to $-f$; but that due to charge on the remaining portion of the

conductor, is practically the same as at P, *i. e.* equal to f'. But Q being a point within the conductor we know that the resultant intensity at Q is zero. Hence

$$-f+f'=0 \quad \ldots \quad \ldots \quad (\beta)$$

$$\therefore \text{ from } (\alpha) \text{ and } (\beta) \quad f=f'=\frac{2\pi\sigma}{k}.$$

Thus in the region in which ds is situated the intensity due to charge on the remaining portion of the conductor, is $f'=\frac{2\pi\sigma}{k}$. The charge σds on the elementary area ds, may therefore be regarded as placed in the field f'. Hence the force on ds = charge × intensity $=\sigma ds\times f'=\frac{2\pi\sigma^2}{k}ds$.

Thus the electric pressure or force *per unit area is* $\frac{2\pi\sigma^2}{k}$.

Since $F=\frac{4\pi\sigma}{k}$ the electric pressure $\frac{2\pi\sigma^2}{k}=\frac{k}{8\pi}\left(\frac{4\pi\sigma}{k}\right)^2=\frac{kF^2}{8\pi}$.

If a soap bubble be given a certain amount of charge there is an outward electric pressure at every point of the surface; as a result the bubble expands slightly.

An insulated soap bubble of radius 8 cms receives a charge of 40 E. S. units. Find approximately the increase in radius due to electric pressure. (Atm. Press. = 10^6 dynes per sq. cm. Term containing surface tension may be neglected).

Since surface tension is to be neglected we may suppose that initially the pressure inside the bulb = 1 Atm = 10^6. After the charge is given to the bubble let the radius be increased by a small amount r. Since Press × Vol = Const., if P be the new pressure

$$10^6\times 8^3=P\,(8+r)^3 \quad \therefore \quad P=10^6\left(\frac{8}{8+r}\right)^3$$

$$=10^6\left(1+\frac{r}{8}\right)^{-3}=10^6\left(1-\frac{3r}{8}\right).$$

Thus the new pressure P inside the bubble is less than the

outside atmospheric pressure by $10^6 - P = 10^6 \times \frac{3r}{8}$. This is balanced by the electric pressure in the outward direction.

Now, surface density $\sigma = \frac{40}{4\pi 8^2} = \frac{5}{32\pi}$ very approximately.

$\therefore$ Electric pressure $2\pi\sigma^2 = 2\pi . \frac{25}{32^2 \pi^2} = \frac{25}{16 \times 32\pi}$.

Hence $10^6 \times \frac{3r}{8} = \frac{25}{16 \times 32\pi}$ $\therefore$ $r = \frac{8 \times 25}{3 \times 16 \times 32\pi} \times 10^{-6}$

$= 4{\cdot}14 \times 10^{-5}$ cm.

Art 37
Faraday's Conception

We know that unlike charges attract and like charges repel. But what is the exact mechanism by which a charged body exerts a force on another, although separated by some distance? According to Faraday tubes of force associated with a charged

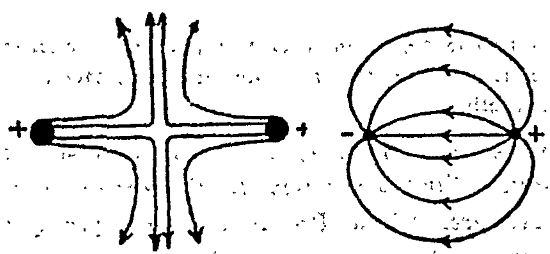


Fig. 70

body are not imaginary but have got real existence in space. In order to explain the action at a distance Faraday assumed two properties of tubes of force, *viz.* tubes of force are always in a state of tension and (2) tubes of force exert lateral pressure on one another. If two charges are unlike several tubes of force connect them and due to the tension of these tubes the two charges are attracted towards each other. If two charges are like no tube of force connects them; but the tubes of force from one of them exert lateral pressure on those from the other. It is due to this lateral pressure that

the two charges are separated from each other, *i. e.* they repel each other. [Vide Fig 70].

According to this idea the electric pressure over any area of a charged conductor is due to the tension of the tube of force emanating from the area. It follows

Tension of tube

that the tension of a tube *per unit area of cross section* is $\frac{kF^2}{8\pi}$. It can also be proved that the lateral pressure exerted by a tube on another is also equal to $\frac{kF^2}{8\pi}$.

If a rubber tube be pulled at two ends the tube becomes elongated and we say that the tube is in a state of tension.

Energy in the medium

Obviously stresses are set up in the tube and a strain is produced. In a similar way since a tube of force is in a state of tension stresses must necessarily be produced in the medium. Again just as a rubber tube in a state of tension possesses potential energy similarly there must also be energy in the medium when a tube of force exists there.

If we suppose that the cross-section ds of a tube of force is moved through a small distance δx against the tension of the tube the work done thereby $= \frac{kF^2}{8\pi} ds \times \delta x$. This energy is

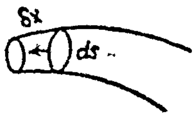


Fig. 71

stored up as energy in the space now occupied by the tube of force. As the volume of this space is $ds \times \delta x$ the energy per unit volume is $\frac{kF^2}{8\pi}$.

Let a line of force pass from one medium to another

Art 38
Refraction of lines of force

of specific inductive capacities k_1 and k_2 respectively and let it make angles θ_1 and θ_2 with the normal to the surface of separation. Let F_1 and F_2 be the intensities in the two media acting along the lines of force.

Consider a small closed rectangular surface ABCD such that (1) AB is in the upper medium and DC in the lower one (2) AB and DC are equal and are each parallel to the surface of separation (3) AD and BC are infinitesimally* small and are perpendicular to the surface of separation.

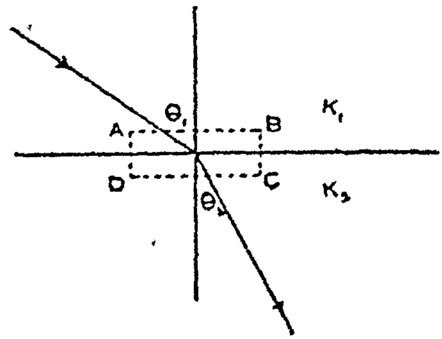


Fig. 72

Since A and D are infinitely close to each other V_A may be taken to be equal to V_D; similarly $V_B = V_C$

$$\therefore \quad V_A - V_B = V_D - V_C$$

Since difference of potential is equal to intensity × distance and since AB = DC, it follows that

intensity along AB = intensity along DC

$$\text{Or} \qquad F_1 \sin \theta_1 = F_2 \sin \theta_2 \qquad \ldots \qquad (a)$$

i. e. tangential component of the intensity is continuous (*i. e.* same in both the media).

Again, since there is no charge within ABCD, the total normal induction over the surface is zero. AD and BC being infinitely small normal induction over them is also negligibly small. Hence

Normal induction over AB = Normal induction over DC

$$\text{or} \qquad k_1 F_1 \cos \theta_1 = k_2 F_2 \cos \theta_2 \qquad \ldots \qquad (b)$$

i. e. normal component of induction is continuous.

Hence, dividing (*a*) by (*b*), $\frac{1}{k_1} \tan \theta_1 = \frac{1}{k_2} \tan \theta_2$

or $\quad \frac{\tan \theta_1}{\tan \theta_2} = \frac{k_1}{k_2}$ a const. for a given pair of media.

* AB and DC are quite small but AD and BC are much smaller.

Art. 39 We shall now describe a few electric machines whereby charge may be generated. All electric machines generally depend upon the principle of induction. Electrophorus—one of the simplest of such machines—consists of an ebonite plate P resting on a metallic disc S known as *sole*. Another metallic disc C known as *cover* is provided with an insulating handle H. A metallic knob K is also attached to the cover C. To use the instrument the ebonite plate P is vigorously rubbed with cat's skin so that negative charge is generated on its surface. The cover C is now placed on P. As the ebonite plate is rough contact takes place between P and C at few points only. We may therefore suppose that there is a layer of air in-between P and C. Hence by induction positive charge is generated on the lower face of C and the free negative charge passes on to the upper face. If C be momentarily touched with the hand the free negative charge passes on to the Earth. The cover may now be taken off by the insulating handle H. It is now positively charged and a spark may be obtained from the knob K. The cover may again be placed on P and the proceedure may be repeated so that by once charging the ebonite plate by friction electric energy may be obtained a number of times. The repeated supply of electric energy is really obtained from the work done against the attraction between positive and negative charges (on C and P respectively) when the cover C is raised from the ebonite plate P. The negative charge on P also induces positive charge on the upper surface of the sole S, the free negative charge passing away to the Earth. This induced positive charge on S keeps the negative charge on P bound; dissipation of the negative charge (on P) is therefore prevented. Thus the function of the sole S is simply to prevent the charge on P from being dissipated away into air.

Electrophorus

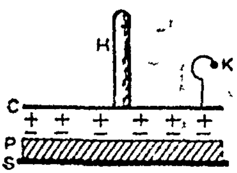


Fig. 73

Art. 40
Voss machine

Although an electrophorus is satisfactory so far as it goes it can never produce a large amount of charge. To do this various machines have been designed. In the Voss machine there are two circular glass discs of equal* size placed side by side in the vertical position. One of these is fixed and the other can be rotated. A number of small tinfoils is pasted

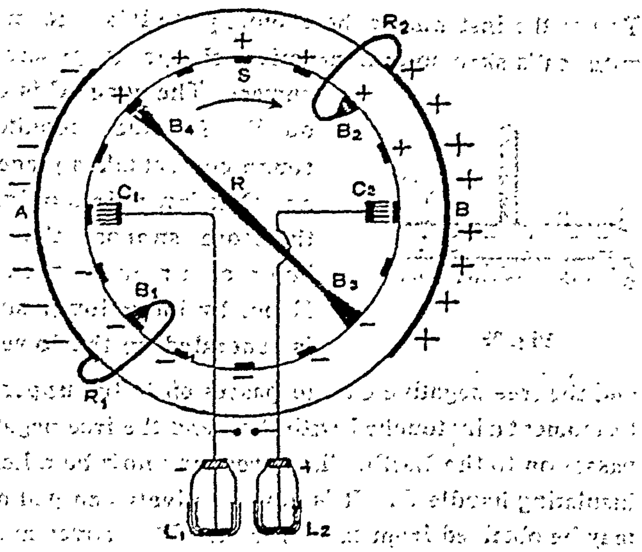


Fig. 74

at regular intervals on the revolving disc. These are known as *studs* or *carriers*. Two fairly large tinfoils A and B are pasted on the fixed disc. These are known as *armatures*. Two bent metallic rods R_1 and R_2 are connected respectively to the armatures A and B; at the other ends they end in metallic brushes B_1 and B_2 which touch the carriers as they pass. Another metallic rod R fixed in position ends in two metallic brushes B_3 and B_4 which also touch the studs as they are rotated. Two *collecting combs* [Vide Art 29] C_1 and C_2 are placed as shown in the diagram at small distances from the

* In the diagram one disc is shown to be of greater size; this is done so that the diagram may be easily understood.

studs; they are connected respectively to Leyden jars* L_1 and L_2,

In the atmosphere due to the action of Sun's rays and for various other reasons small amounts of charge are always present. Let us suppose that one of the studs S is slightly charged positively by this atmospheric electricity. As the disc is rotated in the direction of the arrow the stud S rotates and when it comes under the brush B_2 it becomes momentarily connected to the armature B and hence most of its charge passes on to the armature. It retains however a very small amount of positive charge and when it comes near the collecting comb C_2 its charge is collected by the comb and transferred to the Leyden jar L_2 and the stud becomes uncharged. In this uncharged condition the stud comes in contact with the brush B_3. Due to the positive charge on the armature B negative charge is induced on the stud and free positive charge passes on to the opposite stud in contact with B_4; this positively charged stud at B_4 rotates and the same action is repeated as before. As the disc is rotated the negatively charged stud at B_3 comes in contact with the brush B_1 and most of its negative charge passes on to the armature A. The little negative charge remaining on the stud is transferred to the Leyden jar L_1 by the collecting comb C_1. In this uncharged condition it comes in contact with the brush B_4 and becomes positively charged as explained before. Ultimately the studs in different positions are charged as shown in the diagram. Every time a stud comes under the brush B_4 another stud comes under the brush B_3. Positive charge is induced on the former and negative charge on the latter—and free negative and positive charges destroy each other. Thus as different charged studs come near the collecting combs more and more positive charge is collected in Leyden jar L_2 and more and more negative charge in Leyden jar L_1. The Leyden jars L_1 and L_2 therefore become gradually heavily charged.

* What is a Leyden jar will be explained in Art 44.

Art 41
Wimshurst machine

In the Wimshurst machine also there are two equal circular glass discs. In this case however both the discs can be rotated. But they are so geared that they rotate in opposite directions. Equal number of tinfoils are pasted at regular intervals on each of the two discs. These are known

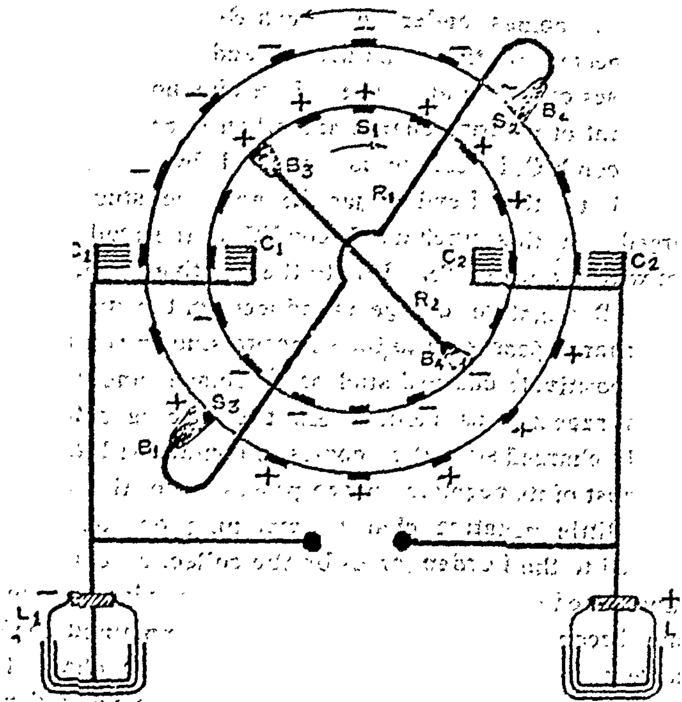


Fig. 75

as studs or *carriers*. Two metallic brushes B_1 and B_2 attached to the metallic rod R_1 touch two opposite studs on one of the discs and metallic brushes B_3 and B_4 attached to the metallic rod R_2 touch two opposite studs on the other disc. As shown in the diagram pairs of collecting combs C_1 and C_2 are placed at small distances from the studs in the two discs. These are connected to the Leyden jars L_1 and L_2.

To understand the action of the machine we suppose that

one of the studs—say, S_1 on the first disc is charged* with a small amont of positive electricity. As the two discs are rotated in the directions indicated by the arrows the stud S_1 soon comes before the brush B_2. The positive charge on the stud induces negative charge on the opposite stud S_2 in contact with B_2 and free positive charge passes on to the remote stud S_3 (in contact with the brush B_1). Afterwards when the stud S_1 comes before the collecting comb C_2 its positive charge is transferred to the Leyden jar L_2. In this uncharged condition the stud S_1 next comes in contact with the brush B_4. At the same instant the studs S_2 and S_3 on the second disc previously charged by induction with negative and positive charges, come in front of the brushes B_3 and B_4. Induction again takes place; positive charge is therefore induced on the stud in contact with the brush B_3 and negative charge on the stud in contact with the brush B_4. Free negative and positive charges moving along the rod R_2 destroy each other. These studs on the first disc thus charged by induction with positive and negative charges afterwards come in front of the brushes B_2 and B_1 respectively. Induction again takes place on both sides of the rod R_1 and free positive and negative charges cancel each other. The negative charges on the stud (before the brush B_1) is afterwards transferred to the Leyden jar L_1 by the collecting comb C_1. Ultimately the studs in different positions become charged as shown in the diagram. By the collecting combs C_1 negative charge is transferred to L_1 and by the collecting combs C_2 positive charge is collected in L_2. The Leyden jars L_1 and L_2 therefore gradually become heavily charged with negative and positive electricity respectively.

Art 42
Van de Graaff generator

In the machines described in the previous articles charge is generated and multiplied by induction. But in a Van de Graaff generator charge is rather collected than generated. G is a large metal globe placed at some height on a suitable

* This may be due to atmospheric electricity.

insulating stand. A and B are two cylinders capable of rotation round their respective axes. A band of silk or rubber or some other insulating substance passes round the cylinders so that when the cylinders are set into rotation the band revolves in the direction indicated by the arrow. C is a pointed metallic rod fixed in position at a small distance from the moving band. C is connected to a source of constant positive potential so that positive charge comes to C. As however C is pointed the charge cannot reside there; it moves away from the rod and becomes attached to the band. This charge moves upwards along with the band. When the charge ultimately comes before another pointed rod D connected to the globe it is immediately transferred to the globe (by the action of the point at D). Thus as the band rotates more and more positive charge is collected on the globe. By a suitable modification of the apparatus another globe may simultaneously be charged with negative electricity. The potential difference between the globes may be extremely high. Van de Graaff used two globes 15 ft in diameter placed on insulating towers 22 ft high and obtained a potential difference of 5 million volts.

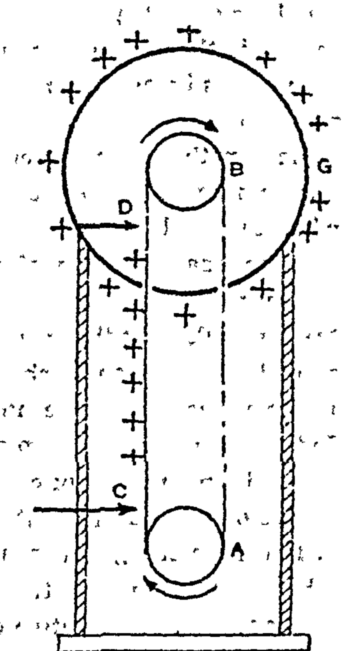


Fig. 76

Exercise V

1. Two charges +120 and −60 are situated at points A and B 10 cms apart, Find a point on AB such that at this point

the potential is zero. Is there any such second point on AB produced ? Determine a few other points in the field at which the potential is zero ; hence draw the equipotential surface for zero potential. Show that this is a sphere ; find its centre and radius.

Ans. The points of zero potential are C in AB where AC = $6\frac{2}{3}$ cms and D in AB produced where AD = 20 cms. The equipotential surface is the sphere drawn with CD as diameter. Centre O of this sphere is at a distance of $13\frac{1}{3}$ cms from A.

2. Explain in general terms how the forces of attraction and of repulsion between charged conductors may be explained. Obtain an expression for the tension of a tube of force.

3. Define potential and potential energy of a charged conductor. Show that potential inside a hollow conductor is everywhere the same.

A metallic sphere of diameter 20 cms is charged with 60 units of electricity. Find the potential at a distance of (*a*) 5 cms. (*b*) 10 cms, and (*c*) 15 cms. from the centre.

Ans. (*a*) 6 (*b*) 6 (*c*) 4.

4. Obtain an expression for the intensity at any point close to a charged conductor. In what way does it vary at different points near the conductor, when the conductor is irregular in shape ?

5. A very long cylindrical conductor has a charge Q per unit length. Show that the field at a distance *d* from the axis of the cylinder is $\frac{2Q}{d}$, *d* being greater than the radius of the cylinder.

6. Calculate the potential in electrostatic units to which a spherical conductor of unit radius has to be raised in order that the electrical pressure may be equal to the normal atmospheric pressure, viz. 10^6 dynes per sq. cm. C. U. 1939

Ans. 5013 C. G. S units.

7. Obtain an expression for the force per unit area of the surface of a charged conductor.

A soap bubble is given a charge so that the pressure inside is the same as that outside. Find the amount of this charge if the surface tension be S and the soap bubble be of 2 cms radius.

Ans $16\sqrt{\pi S}$

8. If the radius and surface tension of a spherical soap bubble be r and T respectively, show that the charge of electricity required to expand the bubble to twice its linear dimensions, would be $4\sqrt{\pi r^3(12T+7rP)}$, where P is atmospheric pressure.

9. Show that in passing from one medium to another lines of force are refracted. Deduce the law of this refraction.

C. U. Questions

1963. Write notes on (*a*) Equipotential surface (*b*) Van de Graff generator.

1964 (1) Write short notes on (*a*) Wimshurst machine (*b*) Van de Graaff generator.

(2) Derive an expression for the energy stored in a unit volume of a dielectric medium placed in an electrostatic field.

1965 Explain what is meant by total normal induction over a surface. State what relation this bears to the total electrostatic charge enclosed by the surface. Find the intensity at a point outside an isolated charged conducting sphere at a distance r from the centre.

1966, 1974. State and prove Gauss' theorem.

Deduce an expression for the outward force per cm^2 on the surface of a uniformly charged conductor.

A metal ball of radius 5 cm is given a charge of 100 electrostatic units. Find the force acting on it per unit area.

1968. State and prove Gauss' theorem in electrostatics and use it to determine the electric intensity very near the surface of a closed charged conductor.

1970. (1) Write short notes on "Van de Graaf generator".

(2) State and prove Gauss' theorem on total normal induction.

1971. Define total normal induction and an equipotential surface. Derive an expression for the mechanical force per unit area acting on an insulated conducting surface having a surface density of charge σ.

A metal ball of radius 5 cm. is given a charge of 100 electrostatic units. Find the force acting on it per unit area.

1973. State and prove Gauss' theorem on total normal induction.

Find the value of the field intensity at a point outside a long charged cylinder.

1976. State and prove Gauss' theorem on total normal induction. Determine the electric intensity at a point close to a plane charged sheet of conductor.

CHAPTER VI

CONDENSERS

Art 43
Theory of a condenser

Let an insulated conductor A be connected to the positive* terminal of an electric machine. Charge flows to A from the machine until A acquires the same potential as that of the machine. If now another conductor B—earth connected—be brought near A a negative charge is induced on B This negative charge on B reduces the potential of A so that a fresh supply of charge comes to A from the machine. This again induces a greater amount of negative charge on B. This in its turn further reduces the potential of A, and so on. Finally a fairly large amount of charge accumulates on A.

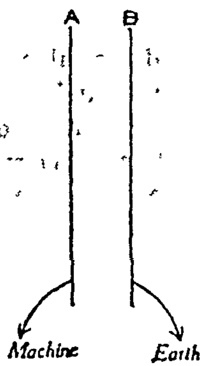


Fig. 77

A pair of such conductors separated by a nonconducting medium (known as dielectric) constitutes a condenser.

Electricity is thus apparently condensed in A; hence the name condenser.

If the conductors A and B are parallel plates separated by a dielectric the condenser is said to be a parallel plate condenser; if they are concentric spheres the condenser is called a spherical condenser and finally if they are co-axial cylinders the combination is known as a cylindrical condenser.

If B be not earthed negative charge is induced on the face

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* If the conductor be connected to the negative terminal, negative charge comes to A. This induces positive charge on B and the arguments still hold good as in the article.

of B nearer to A but free positive charge instead of passing away to earth remains on the other face of B. This positive charge counterbalances to a great extent the effect of the induced negative charge in lowering the potential of A. Thus the potential of A is not so effectively lowered and therefore the accumulation of charge on A is not so large as when B is earthed.

The capacity of a condenser is given by the usual relation $C=\frac{Q}{V}$ where Q is the charge on the conductor (called positive plate) connected to the machine and V is the difference of potential between the conductors. If the second plate (called negative plate) be earth-connected V simply means the potential of the first one.

The energy of a condenser $=\frac{1}{2}QV=\frac{1}{2}\frac{Q^2}{C}=\frac{1}{2}CV^2$.

The proof is exactly the same as in the case of a single conductor. [Vide Art 25].

Art 44
Leyden Jar

Leyden jar is a practical form of a condenser. It consists of a glass vessel coated inside and outside with tinfoils P_1, P_2. These two tinfoils act as the two plates of a condenser and glass is the dielectric between the two plates. A metallic rod R passes through an insulating stopper at the mouth of the jar. At the upper end the rod ends in a metallic knob K and at the lower end a metallic chain is attached; this chain is in contact with inner tinfoil.

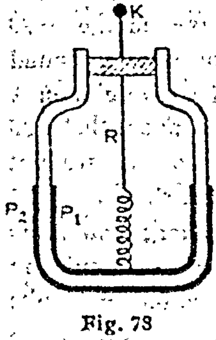

Fig. 78

To charge the Leyden jar it is held by the hand and the knob is made to touch the machine. Thus the inner coating is connected to the machine (through the knob) and the outer coating to the Earth (through the hand). Leyden jar is thus charged.

To discharge the Leyden jar a pair of discharging tongs is

used. It consists of two mutually connected bent rods R_1 and R_2 attached to an insulating handle H. The knobs at the ends of the rods are made to touch—one to the outer coating and the other to the knob connected to the inner coating of the Leyden jar. Thus the outer coating is connected to the inner coating through the discharging tongs and the condenser is discharged.

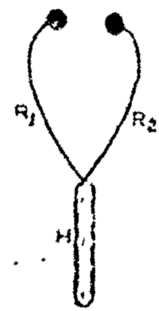

Fig. 79

There is another process—a slow process, by which also a condenser may be discharged. Suppose one of the plates A of the condenser has a positive charge amounting to 100 units. This induces negative charge on the other plate B. Now the induced charge is equal to the inducing charge only when B *completely surrounds* A. In this case since B does not completely surround A the amount of negative charge on B will be somewhat less. Let us suppose that the charge induced on B is 98 units. Positive and negative charges on A and B, by their mutual attraction, generally tend to bind each other. In this case since none of A and B completely surrounds the other, charge on one of them binds somewhat smaller amount of charge (of opposite sign) on the other. The negative charge −98 units on B is of course bound by + 100 units on A. But the entire amount of + 100 units on A cannot be bound by −98 units on B. Let us suppose that + 96 units are bound. Thus + 4 units of charge on A are not bound—they are free. If we therefore touch A these free + 4 units pass on to the Earth. As soon as the charge on A is reduced to + 96 units the entire amount of −98 units on B is no longer bound. If −94 units are now bound −4 units are free. If we now touch B this −4 units pass on to the Earth. With the reduction of the charge on B some charge again becomes free on A. Thus by alternately touching A and B the condenser may be slowly but steadily discharged.

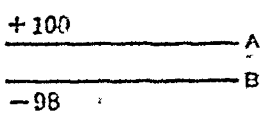

Fig. 80

Art 45
Displacement current

Let us consider the charging of a condenser a little more in detail. In the circuit shown in Fig 81 when the key K is closed positive charge begins to flow from the battery to the plate P_1 of the condenser in which air or some other substance is the dielectric between the plates. This induces negative charge on P_2 and free positive charge flows back to the battery. This goes on until the condenser is fully charged. Thus there is a temporary current in the wires connecting the condenser to the battery. Apparently there is no current through the dielectric between the plates of the condenser *i. e.* there is a discontinuity in the space between the plates. Maxwell however supposed that here also there is no discontinuity. According to modern ideas every atom consists of a positively charged nucleus surrounded by a number of rotating electrons. Thus every molecule consisting of two or more atoms of a substance contains both positive and negative charges and these charges are equal in amount so that as a whole the molecule is neutral.

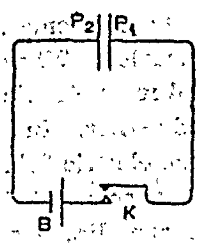

Fig. 81

In the case of a conducting substance some of the electrons at least are loosely bound with the nucleus so that by the application of an electric field they become detached and begin to move. In fact it is the movement of these free electrons that constitutes the current in a conductor. In the case of atoms in a dielectric however the electrons are not so loosely bound to the nucleus and ordinarily they cannot be detached from the nucleus. Nevertheless by the action of an electric field positive and negative charges in a molecule are somewhat displaced within the molecule relative to each other. Each molecule is then a dipole or an electric doublet. An electric doublet is exactly analogous to a small magnet (or a magnetic doublet) with two poles at the two ends. If l is the distance separating the effective charges $+q$ and $-q$ the electric moment of the

doublet is ql. At any neighbouring point it produces an electric potential and an electric field exactly in the same way as a small magnet produces a magnetic potential and a magnetic field at any external point. Thus as the plates P_1 and P_2 become gradually charged there is a displacement of charges within the molecules of the dielectric. This is equivalent to a current known as *displacement current.*

In the case of some dielectrics, however, even without any external electric field the electric charges within the molecules are so permanently displaced that each molecule is in reality a permanent dipole: such molecules are known as **polar molecules.** In the case of other dielectrics there is no such permanent displacement of charges within the molecules. Molecules in this case are called **non-polar** molecules. When a dielectric with non-polar molecules is placed in an electric field the molecules become dipoles as explained earlier. In a dielectric with polar molecules with no external electric field the dipoles, *i.e.* the molecules of the dielectric are so oriented at random that the resultant electric effect in any direction is nil. When an electric field is applied the dipoles are gradually aligned* in the direction of the electric field, the degree of alignment depending on the strength of the electric field. The electric moment of each molecule may also be affected by the external electric field.

A dielectric placed in an electric field is analogous to a magnetic substance placed in a magnetic field. As we shall see in Art 188 the magnetic moment per unit volume of the magnetic substance is known as Intensity of Magnetisation. In a similar way the total electric moment (of the doublets) per unit volume of the dielectric is called *Polarisation.*

Art 46
Dissecting Leyden jar

In one form of Leyden jar the different parts can be separated from one another. Thus in Fig 82 P_1 and P_2 are the two metallic plates separated by glass. In the usual way

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* This is exactly analogous to the orientation of atoms in a magnetic field (Vide Art 142).

by holding P_2 with the hand and by connecting the knob K to the machine the condenser is charged. P_1 and P_2 are now separated with the help of an insulating handle and are tested by a gold leaf electroscope; they are found not to possess any charge. On the other hand the glass when tested is found to be charged. They are again re-assembled so as to from the original condenser; a spark may now be obtained by connecting the plates P_1 and P_2. This shows that charge lies not on the plates but in the dielectric.

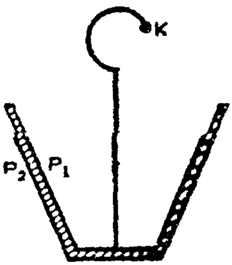

Fig. 82

Residual charge

The energy of the condenser is due to the strain caused in the dielectric by the electric field. When the two plates of a charged condenser are connected a spark is obtained and the strain is removed. It is however found that after a spark has been obtained from a charged condenser a second (and possibly, a third) spark may also be obtained before charging the condenser again. This shows that strain is not removed completely by once discharging the condenser. There remains a residual strain in the medium, *i. e.* a *residual charge on the condenser. This corresponds to residual magnetism in* magnetic substances. [Vide Art 139]

Art. 47

We now consider the effect of joining condensers in parallel and also in series.

Condensers in parallel

Let C_1, C_2, C_3 be the capacities of three condensers in parallel; one set of plates of these condensers is connected to the machine and the other set to the Earth. Let Q_1 Q_2, Q_3 be the charges on the positive plates of the three condensers. Since the condensers are in parallel the potential difference V is the same in all of them. Thus $Q_1 = C_1V$, $Q_2 = C_2V$ and $Q_3 = C_3V$.

Now let us replace these condensers by a single condenser of capacity C so that conditions remain the same, *i. e.* the

positive plate receives the total charge $Q_1+Q_2+Q_3$ and the difference of potential between the plates, is V.

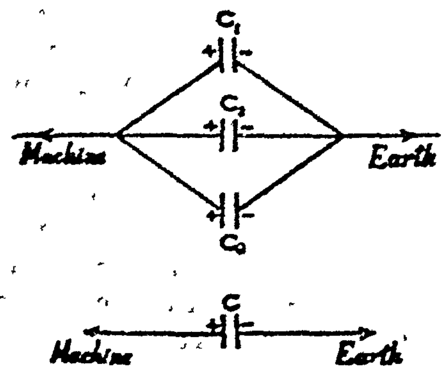

Fig. 83

$$\text{Then} \quad Q_1+Q_2+Q_3=CV$$
$$\text{or} \quad C_1V+C_2V+C_3V=CV.$$
$$\therefore \quad C_1+C_2+C_3=C \qquad \ldots \qquad (21)$$

Condensers in series

Let C_1 C_2 C_3 be the capacities of three condensers connected in series. Let the positive plate of the first condenser receive a charge Q from the machine. This induces $-Q$ on the next plate and the free positive charge Q passes on to the positive plate of the second condenser and so on. If the last plate be connected to the Earth the final free positive charge Q passes on to the earth. Let V_1, V_2, V_3 be the differences of potential between the plates in the three condensers.

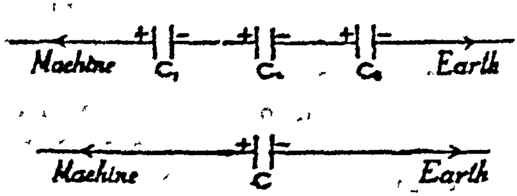

Fig. 84

Then obviously, the potential of the machine is $V_1+V_2+V_3$

$$\text{and} \qquad Q=C_1V_1=C_2V_2=C_3V_3 \qquad \ldots \qquad (a)$$

As before let us replace the three condensers by a single one of capacity C so that conditions remain the same, *i. e.* the positive plate receives the charge Q and the difference of potential is equal to that between the first plate of the first condenser and the last plate of the last condenser, *i. e.* equal to $V_1+V_2+V_3$.

$$\therefore \quad Q=C(V_1+V_2+V_3)$$

$$=C\left(\frac{Q}{C_1}+\frac{Q}{C_2}+\frac{Q}{C_3}\right) \qquad \text{from } (a)$$

$$\therefore \quad \frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3} \qquad \ldots \qquad (22)$$

N. B. It should be noted that these results are just the reverse of those for resistances in series and resistances in parallel [Vide Art 67].

Art 48
Parallel plate Condenser

Consider a condenser consisting of two parallel plates charged with opposite kinds of electricity. In this case lines of force between the two plates are parallel. The intensity at all points between the plates is constant and is equal to

Fig. 85

$\frac{4\pi\sigma}{k}$ by Coulomb's Theorem (Art 35), where σ is the surface density of the positive plate and k is the S. I. C. of the dielectric. Hence if a unit positive charge be moved from the negative plate to the positive work done = Intensity × distance $=\frac{4\pi\sigma}{k}d$ where d is the distance between the plates; by definition the work done is equal to the difference of potential V between the plates.

$$\text{Thus} \qquad V=\frac{4\pi\sigma}{k}d.$$

But the charge Q on the positive plate $=A\sigma$, where A is the area of the plate.

$$\therefore \quad C=\frac{Q}{V}=A\sigma\times\frac{k}{4\pi\sigma d}=\frac{kA}{4\pi d} \qquad \ldots \qquad (23)$$

If the medium be air, the capacity is given by

$$C=\frac{A}{4\pi d} \quad \ldots \quad (23a)$$

From (23) and (23a) it is evident that in the case of a parallel plate condenser the capacity is increased k times when the medium air between the plates is replaced by some dielectric whose S. I. C. is k. This is true for all kinds of condensers. The specific inductive capacity of a substance may therefore be defined* as follows:—

The specific inductive capacity of a substance is the ratio of the capacity of a condenser where the substance is the dielectric to the capacity of an identical condenser where air is the dielectric.

Guard-ring Condenser

In the preceding discussion we have assumed that the lines of force between the two plates are parallel. This is certainly true near the central portion of the plates. But near the edge lines of force are bent due to repulsion of the neighbouring lines of force, which lie on one side only (see Fig. 85). To avoid this difficulty the positive plate P_1 is made circular and is surrounded by another concentric plate P_2 of the shape of a ring (Vide Fig. 86). This ring—or guard ring, as it is called—is also maintained at the same potential as the central plate. The negative plate P_3 covers the total area below P_1 and P_2. The lines of force from P_1 are now very nearly parallel even near the edge because they are now pressed from both sides by neighbouring lines of force. Such a

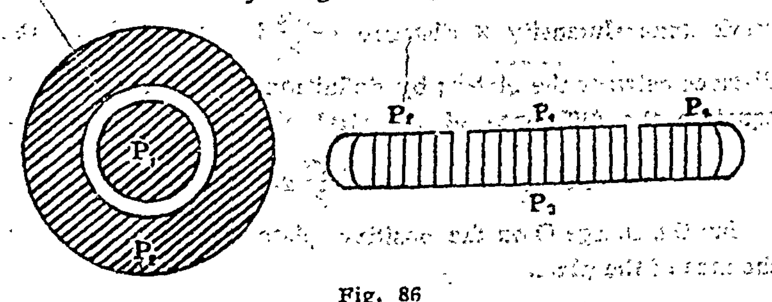

Fig. 86

* Compare the definition given in Art. 28.

condenser is known as a guard-ring condenser. The capacity of this condenser is also $\frac{kA}{4\pi d}$ but A now represents the area of the central plate P_1 only.

More accurately, A represents the area of the plate P_1 *plus* half the area of the annular opening between P_1 and P_2.

In the formula for the capacity A represents the area of the over-lapping portion of the plates. If one plate be slided or ex-centrically rotated in its own plane, keeping the other plate fixed, the area of the over-lapping portion is diminished and therefore the capacity of the condenser is also correspondingly diminished. The capacity of a condenser can also be adjusted by moving one of the plates parallel to itself. The distance between the plates is in this case altered and the capacity is also correspondingly changed. A condenser whose capacity can be varied by any of these means is called a variable condenser.

Variable Condenser

Art 49

Condenser with compound dielectric

In article 48 the entire space was supposed to be filled with a dielectric of S. I. C. k. Let us now suppose that the space is partly filled with a dielectric slab (S. I. C. $= k$) of thickness t, the remaining portion being filled with air. If the total distance between the plates be d, thickness of the air portion is $d - t$. The intensity within the dielectric is $\frac{4\pi\sigma}{k}$ and that within air is $4\pi\sigma$. Hence

Difference of potential V

$=$ Work done in carrying a unit positive charge from the negative plate to the positive one

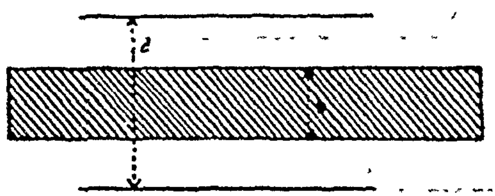

Fig. 87

$$=\frac{4\pi\sigma}{k}.t+4\pi\sigma(d-t)$$
$$=4\pi\sigma\left\{d-t+\frac{t}{k}\right\}$$

As before, the charge on the positive plate $Q=A\sigma$

$$\therefore \quad C=\frac{Q}{V}=\frac{A}{4\pi\left\{d-t+\frac{t}{k}\right\}} \tag{24}$$

N. B. (1) Since k is usually much larger than unity, the capacity is increased by the introduction of the dielectric slab.

(2) It is to be noted that a dielectric slab of thickness t is equivalent to air of thickness $\frac{t}{k}$. Hence in the preceding case the total equivalent air thickness $=d-t+\frac{t}{k}$. This therefore takes the place of d in (23a).

In the preceding section the dielectric slab was supposed to cover the entire area of the plates. Let us now suppose that the dieletric is introduced partly between the two plates. In this case we have really two condensers connected in parallel. If A_1 be the area of the plate covered by the dielectric and A_2 the area of the remaining portion of the plate the capacities of the two portions are $\frac{A_1}{4\pi\left\{d-t+\frac{t}{k}\right\}}$ and $\frac{A_2}{4\pi d}$. Hence the resultant capacity

$$=\frac{A_1}{4\pi\left\{d-t+\frac{t}{k}\right\}}+\frac{A_2}{4\pi d}$$

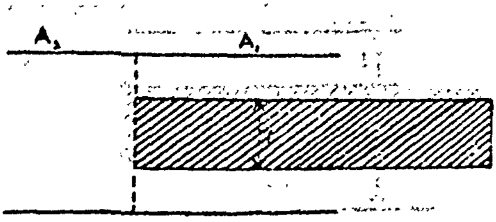

Fig. 88

Art 50 Energy of a condenser.

Let us now compare the energies of a parallel plate condenser when the medium is entirely air and when a dielectric slab is introduced between the plates. We suppose that the medium is first of all entirely air, the condenser is charged by a machine and then the dielectric slab is introduced between the plates. Two cases now arise—when we introduce the slab, do we keep the condenser connected to the machine or separate it from the machine? If we separate it the charge on the condenser is kept constant. Since the capacity C increases by the introduction of the slab and since $Q=CV$, the potential V must necessarily diminish. On the other hand if we keep the condenser connected to the machine the potential of the positive plate remains constant *i. e.* continues to be the same as that of the machine; in this case, by the introduction of the slab C increases and hence Q also must increase *i. e.* more charge comes from the machine to the plate. We shall discuss these two cases by using different expressions for the energy in the two cases.

Case I. Charge remains constant.

Energy $= \frac{1}{2}\frac{Q^2}{C}$. Since C increases with the introduction of the dielectric slab the energy decreases.

Case II. Potential remains constant.

Energy $= \frac{1}{2}CV^2$. In this case energy obviously increases with the introduction of the dielectric slab.

Force on the dielectric slab

Now it is a well-known law in mechanics that the potential energy of a system always tends to diminish. Hence when charge remains constant, since by the introduction of the slab energy diminishes electric forces tend to pull the slab in between the plates. On the other hand when potential remains constant, since the energy increases by the introduction of the slab, electric forces tend to push the slab out of the plates.

[For the attraction between the plates, Vide Art 55]

Art 51
Spherical Condenser

Let A and B be two concentric spheres of radii a and b respectively. Let the inner sphere A receive a charge Q and let the outer sphere B be connected to the Earth.

1st Method

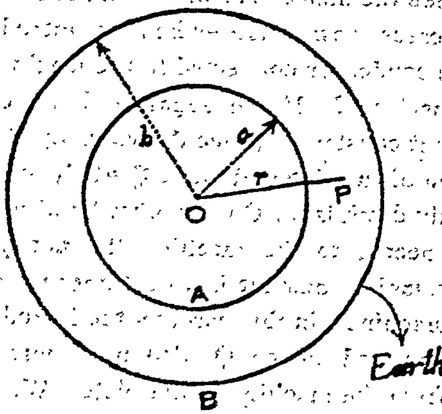

Fig. 89

In the space between the two spheres let P be a point at a distance r from the common centre O. The point P being within B, the intensity at P is due to the charge on A alone and is therefore equal to $\frac{Q}{kr^2}$ k being the S. I. C. of the medium between the spheres. Hence $\frac{dV}{dr} = -\frac{Q}{kr^2}$. [From equation (18) Page 62.]

$$\therefore \quad dV = -\frac{Q}{kr^2}\,dr$$

$$\therefore \quad V = \int_b^a -\frac{Q}{kr^2}\,dr = \frac{Q}{k}\left[\frac{1}{r}\right]_b^a$$

$$= \frac{Q}{k}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{Q}{k}\,\frac{b-a}{ab}$$

$$\therefore \quad C = \frac{Q}{V} = \frac{kab}{b-a}$$

Second Method

If Q be the charge on A the induced charge on B is $-Q$ since all the Q lines of force coming out of A, meet B.

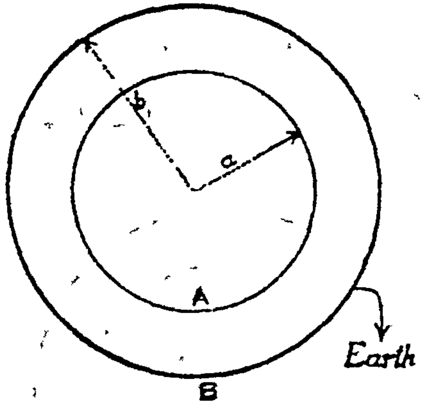

Fig 90

The potential V (of A)

$$= \frac{Q}{ka} \qquad + \qquad \left(-\frac{Q}{kb}\right)$$

(due to its own charge) (Vide Art 33) (due to the charge on B) (Vide Art 33 Note)

$$= Q\,\frac{b-a}{kab}$$

$\therefore$ The capacity $C = \dfrac{Q}{V} = \dfrac{kab}{b-a}$ ... ... (25)

If the medium be air $C = \dfrac{ab}{b-a}$ ... ... (25a)

The entire space between the two spheres has so long been supposed to be filled up by a dielectric of S. I. C. k. Let us now suppose that between the two spheres there is a concentric dielectric spherical shell of inner and outer radii r_1 and r_2, the remaining portion being air. At a distance r from the centre intensity within air $= \dfrac{Q}{r^2}$ and that within the dielectric $\dfrac{Q}{kr^2}$.

$\therefore$ within air $\dfrac{dV}{dr} = -\dfrac{Q}{r^2}$ $\therefore$ $dV = -\dfrac{Q}{r^2}dr$

and within the dielectric $\dfrac{dV}{dr} = -\dfrac{Q}{kr^2}$ $\therefore$ $dV = -\dfrac{Q}{kr^2}dr.$

Hence potential difference between the two spheres

$$V = \int_b^{r_2} -\frac{Q}{r^2}dr + \int_{r_2}^{r_1} -\frac{Q}{kr^2}dr + \int_{r_1}^{a} -\frac{Q}{r^2}dr.$$

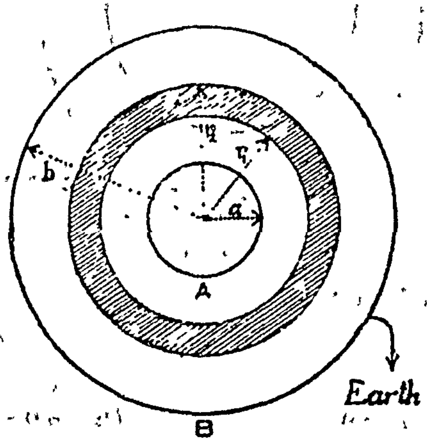

Fig. 91

$$= Q\left[\frac{1}{r}\right]_b^{r_2} + \frac{Q}{k}\left[\frac{1}{r}\right]_{r_2}^{r_1} + Q\left[\frac{1}{r}\right]_{r_1}^{a}$$

$$= Q\left(\frac{1}{r_2} - \frac{1}{b}\right) + \frac{Q}{k}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + Q\left(\frac{1}{a} - \frac{1}{r_1}\right)$$

$$= Q\left(\frac{1}{a} - \frac{1}{b}\right) - Q\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \frac{Q}{k}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$= Q\left\{\left(\frac{1}{a} - \frac{1}{b}\right) - \left(1 - \frac{1}{k}\right)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)\right\}$$

$$= Q\left\{\frac{b-a}{ab} - \frac{k-1}{k}\cdot\frac{r_2 - r_1}{r_1 r_2}\right\}$$

$$= Q\,\frac{kr_1r_2(b-a) - (k-1)ab(r_2 - r_1)}{k\,ab\,r_1r_2}$$

$$\therefore\quad C=\frac{Q}{V}=\frac{k\,ab\,r_1r_2}{kr_1r_2(b-a)-(k-1)ab(r_2-r_1)}\quad\cdots\quad(26)$$

It is to be noticed that in the above expression if we put $k=1$, *i. e.* if the dielectric shell be replaced by air the above expression becomes dentical with (25*a*).

Art 52 Inner sphere earthed

Let us now suppose that the inner sphere A is connected to the Earth. Let the outer sphere B receive a charge Q. In this case the induced charge on A is *not* equal to $-Q$; for only a fraction of the lines of force from B passes inwards and reaches A; other lines proceed outwards and pass on to infinity. Let Q' be the induced charge on A. Let the medium between the spheres be air.

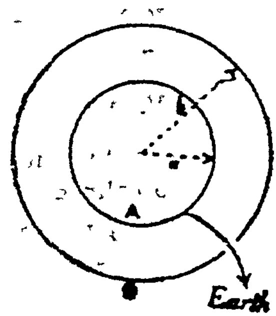

Fig. 92

Then the potential of A

$$=\frac{Q'}{a}\qquad+\qquad\frac{Q}{b}$$

(due to the charge on A) (Vide Art 33) (due to the charge on B) (Vide Art 33 Note)

But since A is earthed its potential is zero.

$$\therefore\quad\frac{Q'}{a}+\frac{Q}{b}=0\quad\therefore\quad Q'=-Q\frac{a}{b}.$$

Hence finally the potential V of B is given by

$$V=\frac{Q'}{b}\qquad+\qquad\frac{Q}{b}$$

(due to charge on A) (due to charge on B)

$$=-Q\frac{a}{b^2}+\frac{Q}{b}=\frac{Q}{b}\left(1-\frac{a}{b}\right)=Q\frac{b-a}{b^2}$$

$$\therefore\quad C=\frac{Q}{V}=\frac{b^2}{b-a}\quad\cdots\quad\cdots\quad(27)$$

Art 53 Cylindrical Condenser Let A and B be two co-axial cylinders of radii a and b respectively. Let the outer cylinder B be connected to the earth. Let P be a point between the cylinders at a distance r from the common axis. If Q be the charge *per unit length* on A the intensity at P $=\frac{2Q}{kr}$ [Vide Art 34], k being the S. I. C. of the medium between the two cylinders.

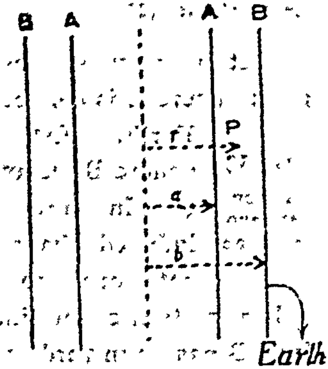

Fig. 93

$$\therefore \frac{dV}{dr}=-\frac{2Q}{kr}$$

or $$dV=-\frac{2Q}{kr}dr.$$

$$\therefore V=\int_b^a -\frac{2Q}{kr}dr=-\frac{2Q}{k}\Big[\log_e r\Big]_b^a=\frac{2Q}{k}\log_e\frac{b}{a}$$

$$\therefore C=\frac{Q}{V}=\frac{k}{2\log_e\frac{b}{a}} \quad \ldots \quad \ldots \quad (28)$$

Since Q is the charge *per unit length*, the above expression gives us the capacity of the condenser *per unit length*. For a length l of the cylinders the capacity

$$C=\frac{kl}{2\log_e\frac{b}{a}} \quad \ldots \quad \ldots \quad (28a)$$

A submarine cable is a practical example of the cylindrical condenser. The inner core—a conductor—takes the place of A. The inner core is covered by some non-conducting material which represents the space between A and B. The sea water which surrounds the insulating coating serves the purpose of the Earth-connected cylinder B. Thus a and b in equation (28a) are the inner and outer radii of the insulating coating of the submarine cable.

Art 54 Numerical example

In the preceding articles if numerical values in C. G. S. units are substituted in the expressions for the capacity, the capacity is obtained in Electrostatic units (E. S. units). To convert this to the practical unit, viz., Farad, the following table should be remembered :—

One Farad $= 10^{-9}$ Electromagnetic units (E. M. units) of capacity.

One E. M. unit of capacity.

$= v^2$ E. S. units of capacity.

where $v =$ velocity of light

$= 3 \times 10^{10}$ cms per sec.

$\therefore$ One Farad $= 10^{-9} \times 9 \times 10^{20}$ E. S. units

$= 9 \times 10^{11}$ E. S. units of capacity.

A Farad is however too large a unit for ordinary purposes; a micro-farad ($= 10^{-6}$ Farad) is therefore generally used as the practical unit.

One micro-farad $= 9 \times 10^{5}$ E. S. units of capacity.

Problems

The conductor within a submarine cable is of diameter 80 mm If the diameter of the guttapercha coating be 100 mm find the capacity of a cable 1000 meters long. (S. I. C. of guttapercha = 4·2).

Here the inner radius $a = 40$ mm $= 4$ cm.

the outer radius $b = 50$ mm $= 5$ cm.

$\therefore$ from (28a)

$$C = \frac{4{\cdot}2 \times 1000 \times 100}{2 \log_e \frac{5}{4}} = \frac{2{\cdot}1 \times 10^5}{\log_{10} 1.25 \times \log_e 10}$$

$$= \frac{2{\cdot}1 \times 10^5}{{\cdot}0969 \times 2{\cdot}303} = 7{\cdot}474 \times 10^5 \text{ E. S. units}$$

$$= \frac{7{\cdot}474 \times 10^5}{9 \times 10^5} = 0{\cdot}83 \text{ micro-farads.}$$

A parallel plate condenser is made up of 21 circular metal plates each of diameter 10 cms, separated by sheets of mica of dielectric constant 6 and thickness 0·2 mm. Calculate its capacity in micro-farads if alternate plates are connected together.

Here there are 20 condensers in parallel for each of which

$A = \pi.5^2$, $d = 0.2$ mm $= 0.02$ cms and $k = 6$

$\therefore$ from (23)

$$C = \frac{20 \times 6 \times \pi \times 5^2}{4\pi \times 0.02} = 37500 \text{ E. S. units}$$

$$= \frac{37500}{9 \times 10^5} \text{ micro-farads}$$

$$= 0.042 \text{ micro-farads.}$$

Exercise VI

1. What is a condenser and why is it so called ?

Obtain an expression for the capacity of a parallel plate condenser. Is the expression quite accurate ? If not explain the reason why it is inaccurate. What modified arrangement has been made to remove the inaccuracy ?

A condenser consists of two circular parallel plates of diameter 20 cms and 0.5 mm apart in air. If the plates are at a difference of potential of 300 volts calculate the charge on the condenser. [1 Volt $= \frac{1}{300}$ E. S. units of P. D.]

Hints :— $C = \dfrac{\pi\,10^2}{4\pi \times \cdot 05} = 500$ and V $= 300$ volts $= 1$ E. S. unit.

$\therefore$ Q = CV = 500 E. S. unit.

2. What is a guard ring condenser ? Why is it used instead of ah ordinary condenser ?

A dielectric of thickness t and of S. I. C. k is inserted between the plates of a parallel plate condenser. If the distance between the plates be d find through what distance must one of the plates be moved so that the capacity of the condenser remains unchanged. Ans. $t - \dfrac{t}{k}$

3. What is meant by the capacity of a condenser ?

A parallel plate condenser is made up of 51 plates (each of size 8 cm × 5 cm.) separated by sheets of mica of S. I. C. 6 and thickness 0.2 mm If the alternate plates are connected together

calculate the capacity of the condenser in microfarads. [one microfarad $= 9 \times 10^5$ E. S. units of capacity.] Ans. 0·063 mfd.

4. A condenser consists of 201 circular sheets of tinfoils separated by mica of S. I. C. 6 and thickness 0·5 mm, alternate plates being connected together. If the capacity of the condenser be 0·4 microfarad, find the radius of the tinfoils. Ans. 7·75 cms.

5. A parallel plate condenser of one micro farad capacity is to be constructed, using paper sheets of 0·05 mm thickness as the dielectric. Find how many sheets of circular metal foils of diameter 20 cms will be needed for the purpose. Dielectric constant for paper is 4·0. Ans. 46.

6. Find an expression for the potential energy of an insulated charged conductor.

Two circular plates of a parallel plate condenser, each of diameter 10 cms., are at a distance of 8 mm. apart. The plates are charged to a P. D. of 10; after disconnecting from the source of P. D, a glass slab 5 mm thick is introduced between the plates. Calculate the loss of energy produced by the introduction of the glass slab. Find also the final capacity of the condenser and the potential difference between the plates, (S. I. C. of glass = 6). Ans. 203·4 ergs ; 16·3 ; 4·79.

7. A charged oil drop of radius 0·00013 cm is prevented from falling under gravity by the vertical electric field between two horizontal plates charged to a difference of potential of 8300 volts. The distance between the plates is 1·6 cms and the density of oil is 0·92 gm per c. c. Calculate the magnitude of the charge on the drop. (One E. S. unit of P. D. = 300 volts and one coulomb = 3×10^9 E. S. units of charge). Ans. $4·80 \times 10^{-10}$ E.S. unit $= 1·60 \times 10^{-19}$ coulombs.

Hints :—P. D $= \frac{8300}{300} = 27·7$ E. S. unit. Hence upward force on the drop $= Q \times \frac{27·7}{1·6} =$ weight of the drop $= \frac{4}{3}\pi (0·00013)^3 \times 0·92 \times 981$. Hence find Q.

8. Two brass plates are arranged horizontally, one 2 cms above the other and the lower plate is earthed. The plates are

charged to a difference of potential 6000 volts. A drop of oil with an electronic charge of 4.774×10^{-10} E. S. U. is in equilibrium between the plates, so that it neither rises nor falls. If the density of oil be 0.92 find the radius of the drop.

Ans 0.000108 cm

9. Explain the action of a condenser and define (*a*) the capacity of a condenser and (*b*) the specific inductive capacity of dielectric.

The thickness of the air layer between the two coatings of a spherical air condenser is 2 cm. The condenser has the same capacity as that of a sphere of 120 cm. diameter. Find the radi of the surfaces of the air condenser. Ans 10 cms : 12 cms.

10. Find the capacity of a spherical conenser when the inner sphere is connected to the Earth.

Two exactly similar condensers are connected by a wire and a charge of 500 units is given to them. If turpentine of S. I. C. 2.16 be poured into one of them find how much charge flows from one condenser to another. Ans 91.8 units.

11. A submarine cable consists of copper wire of diameter 4 mm surrounded by guttapercha of thickness 6 mm. If the S. I. C. of guttapercha be 4.2, find the capacity (in micro-farads) of 30 Kilometers of the cable. [one micro-farad $= 9 \times 10^5$ E S. units of capacity]. Ans. 5.05 mfd.

C. U. Questions.

1962. Find the capacity of a condenser consisting of two concentric metallic spheres the inner of which is charged and the outer one earthed.

1962. Define the capacitance of a condenser. What is the practical unit for it ? Deduce an expression for the capacitance of a parallel plate condenser with a two-component *compound* dielectric. Distinguish between the terms polar dielectrics and non-polar dielectrics.

1963 Write notes on "Dipole".

1964 What do you mean by 'dielectric constant' ?

Write short notes on 'Dielectric polarisation'.

1965. Explain what is meant by the statement that the capacitance of a condenser is one microfarad.

Three capacitors each of 6 micro-farads capacitance are connected in series and a battery of 100 volts applied across the combination. Calculate the charge taken from the battery and the energy stored in the capacitors.

Ans. (*a*) 2×10^{-4} coulombs (*b*) 10^{-2} Joules.

1966. Define the capacity of an electrical condenser. Find the capacity per unit length of a condenser consisting of two co-axial cylinders of radii r_1 and r_2 $(r_1 > r_2)$ the space between the two cylinders being filled up with a material of specific inductive capacity k and the outer cylinder connected to the Earth.

1967. Find the capacitance of a parallel plate capacitor with compound dielectric.

Two capacitors of capacitances $3\mu F$ and $6\mu F$ are connected in series and the resultant combination is connected across 1000 volts. Calculate (a) the equivalent capacitance of the condenser (b) the charge taken by each condenser.

Ans. $2\mu F$; 2×10^{-3} coulomb.

1969. Find the capacity of a parallel plate condenser with compound dielectric.

1970. Find the capacity per unit length of a cylindrical condenser, the outer cylinder being earthed.

A metal wire 1 mm. in diameter is stretched along the axis of a conducting cylinder whose internal radius is 1 cm. Calculate the capacity of the structure per unit length in microfarad.

1971, 1974. Define a condenser in electrostatics. Find an expression for the capacity of a parallel plate condenser with air as dielectric.

Find the capacity of two circular parallel metal plates, each of radius 10 cm. separated by an air resistance of 1 mm. Express the result in microfarads.

1973. Find an expression for the capacity of a parallel plate condenser with air as dielectric.

A parallel plate condenser consists of two plates of area

500 sq cm each, separated by a sheet of mica 0·075 mm thick. Find its capacitance in microfarads, if dielectric constant for mica be 6·5.

1975. Define capacity of a condenser.

Find an expression for the energy of a charged condenser. A charged condenser of capacity C is made to share its charge Q with another uncharged condenser of capacity $2C$ being connected in parallel. Find the sum of the energies of the two condensers before and after sharing. Account for the loss of energy in the process.

CHAPTER VII

MEASUREMENT OF POTENTIAL, CAPACITY AND SPECIFIC INDUCTIVE CAPACITY

Electrometers are instruments by which the potential difference between two bodies may be measured.

Art 55. Attracted Disc Electrometer.

This is also sometimes called Kelvin's Absolute Electrometer.

The action of this instrument depends upon the force of attraction between the plates of a parallel plate charged condenser. Let us find an expression for this force.

Let V_1 and V_2 be the potentials of the two plates of a parallel plate condenser, the plates being separated by a distance d.

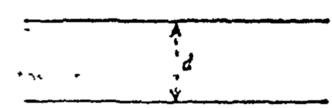

Fig. 94

The intensity at any point between the plates, is therefore equal to $\dfrac{V_1 - V_2}{d}$. By Coulomb's Theorem if σ be the surface density of the positive plate the intensity is also equal to $4\pi\sigma$

$$\text{Hence} \qquad 4\pi\sigma = \frac{V_1 - V_2}{d} \qquad \therefore \qquad \sigma = \frac{V_1 - V_2}{4\pi d}$$

Again $2\pi\sigma^2$ is the electric pressure or force per unit area, on either of the plates. Hence if S be the area of a plate the force with which the two plates are attracted towards each other, is

$$F = S.\, 2\pi\sigma^2 = 2\pi S\left(\frac{V_1 - V_2}{4\pi d}\right)^2 = \frac{S}{8\pi d^2}(V_1 - V_2)^2$$

$$\therefore \qquad V_1 - V_2 = d\sqrt{\frac{8\pi F}{S}} \qquad \ldots \qquad \ldots \qquad (a)$$

connected to a potential V_1. The two plates being thus at a difference of potential there is a force of attraction acting between the two. The position of C is slowly adjusted until due to this force of attraction A comes in flush with B. We then have from (a)

$$V - V_1 = d_1 \sqrt{\frac{8\pi mg}{S}}$$

where d_1 is the distance between A and C and S is the area of the plate A.

C is discharged and is again connected to a second potential V_2. By adjusting the position of C A is again made flush with B.

Then $$V - V_2 = d_2 \sqrt{\frac{8\pi mg}{S}}$$

where d_2 is the new distance between A and C.

$\therefore$ By subtraction $$V_2 - V_1 = (d_1 - d_2) \sqrt{\frac{8\pi mg}{S}}$$

$d_1 - d_2$ is the distance through which C is moved between the first position and the second and can therefore be accurately measured by the micrometer screw. $\sqrt{\frac{8\pi mg}{S}}$ is a constant for the given instrument and can be easily determined from a knowledge of the values of m and S.

Thus $V_2 - V_1$ is known. If $V_1 = 0$. *i. e.* if C be first connected to the Earth and then to V_2, the actual value of V_2 can also be determined. Since the value of V_2 can be determined from a knowledge of the different constants involved in the equation the instrument is known as an *absolute* electrometer.

A condenser consists of two circular plates of 20 cms radiv. separated by an air gap of 5 mm. If the plates are at a difference of potential of 10 E. S. units find the force between the plates. Calculate also the work done in separating the plates from the present position to a distance of 1 cm, the potential difference being maintained constant.

$$4\pi\sigma = \text{Intensity} = \frac{10}{0{\cdot}5} = 20 \qquad \therefore \qquad \sigma = \frac{5}{\pi}$$

Hence force of attraction = Area $\times 2\pi\sigma^2$

$$=\pi.\,20^2.\,2\pi.\frac{25}{\pi^2}=20{,}000 \text{ dynes.}$$

Again at any stage if x be the distance between the plates

$$4\pi\sigma=\frac{10}{x} \quad \text{or} \quad \sigma=\frac{5}{2\pi x}$$

$\therefore$ Force of attraction $=\pi.\,20^2.\,2\pi\,\dfrac{25}{4\pi^2x^2}=\dfrac{5000}{x^2}$

Hence required work done $=\displaystyle\int_{0\cdot5}^{1}\frac{5000}{x^2}\,dx=-\left[\frac{5000}{x}\right]_{0\cdot5}^{1}=5000$ ergs.

Art 56. Kelvin's Quadrant Electrometer

The Quadrant electrometer essentially consists of four hollow quadrants made out of a flat cylindrical hollow brass box, each quadrant being supported by a glass rod. Opposite quadrants are electrically connected. The two pairs AA and BB thus formed are connected to potentials V_1 and V_2 whose difference is to be measured by the quadrant electrometer. A

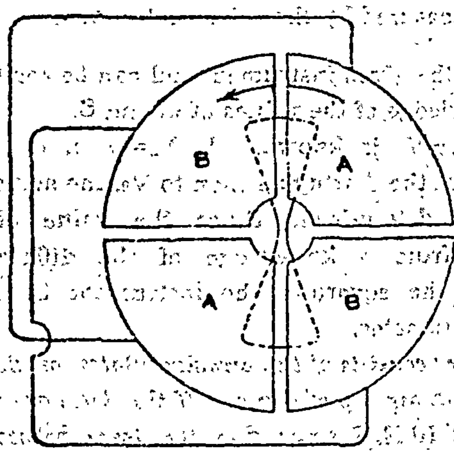

Fig. 96

light aluminium needle of the shape as shown in the figure is suspended by a fine silver wire, so that it hangs inside these quadrants. The suspension wire is attached to a torsion

head at the top; this is adjusted so that initially the needle lies symmetrically among the four quadrants. It is obvious that each quadrant together with the portion of the needle within the quadrant forms a *pair* of condensers, the upper face of the needle together with the upper portion of the quadrant forming one condenser and the lower face of the needle together with the lower portion of the quadrant forming the other condenser of the pair. If the needle rotates in any direction the portion of the needle within one diagonal pair of quadrants increases and that within the other diagonal pair decreases. The needle is maintained at a constant potential V. When the needle rotates the capacity and hence the energy of the condensers formed by one diagonal pair of quadrants increase and the capacity and energy of the condensers formed by the other pair decrease.

Due to electric forces the needle begins to rotate and as it rotates the suspension wire gets twisted. Equilibrium therefore takes place when the couple due to electric forces is balanced by that due to torsion.

If θ be the final deflection of the needle (—say, in the direction of the arrow, see Fig 96) and if μ be the coefficient of torsion of the wire the torsional couple is $\mu\theta$. This is also therefore the electric couple in the equilibrium position.

Let us now suppose that the needle rotates through a further small angle $d\theta$. Since in this case all the conductors are maintained at constant potentials, for the small rotation the work done by the electric couple is equal to the gain in electric energy of the system. The former is obviously equal to $\mu\theta d\theta$. We now proceed to find the latter.

Let C be the *change* in capacity of any diagonal pair of condensers when the needle rotates through a unit angle (*i. e.* one radian). Thus for a deflection $d\theta$ of the needle, the B pair gains in capacity by $Cd\theta$ and the corresponding loss in the A pair is also $Cd\theta$. Since the energy of a condenser is $\frac{1}{2}$ (capacity). (Pot diff.)^2, for the B pair the gain in energy is $\frac{1}{2}\,Cd\theta\;(V-V_2)^2$, and for the A pair the loss in energy is $\frac{1}{2}\,Cd\theta\,(V-V_1)^2$. Thus the net gain in electric energy

$$=\tfrac{1}{2}Cd\theta\{(V-V_2)^2-(V-V_1)^2\}$$

Hence $\mu\theta d\theta=\tfrac{1}{2}Cd\theta\{(V-V_2)^2-(V-V_1)^2\}$

$$\therefore\quad \theta=\frac{C}{2\mu}(V_1-V_2)(2V-V_1-V_2)$$

$$=\frac{C}{\mu}(V_1-V_2)\left\{V-\frac{V_1+V_2}{2}\right\}$$

$$=K(V_1-V_2)\left\{V-\frac{V_1+V_2}{2}\right\} \quad \dots \quad (29)$$

$$\text{where } K=\frac{C}{\mu}$$

There are two ways of using the instrument—(1) Heterostatic and (2) Ideostatic.

(1) **Heterostatic method.** In this method V is made very large; much larger than V_1+V_2. Hence equation (29) reduces to $\theta=KV(V_1-V_2)$.

Thus θ is proportional to V_1-V_2. This is the usual method. In this case since V_1-V_2 is multiplied by V which is very large it is clear that even if V_1-V_2 is small θ will be fairly large. Hence small difference of potential may be measured by this method.

(2) **Ideostatic method.** In this method V is made equal to V_1 or V_2.

If $V=V_1$, equation (29) reduces to

$$\theta=K(V_1-V_2)\left\{V_1-\frac{V_1+V_2}{2}\right\}=\frac{K}{2}(V_1-V_2)^2$$

Thus θ is proportional to the square of the potential difference V_1-V_2 *i. e.* θ is positive even when V_1-V_2 is negative. Hence alternating potential difference may be measured by this method.

It will be seen that the value of the potential difference V_1-V_2 cannot be determined in absolute measure by this instrument; *for,* the constant K in equation [29] cannot be determined in absolute measure—it can only be found out by first using a *known* potential difference and noting the corresponding deflection θ. Quadrant electrometer is *not* therefore an absolute electrometer.

In the Dolezalek form the instrument is very sensitive. The needle is made of paper on which a thin layer of some metal—usually aluminium is deposited. It is suspended by a thin quartz fibre which is made conducting by dipping it into a strong solution of calcium chloride. A small mirror is attached to the quartz fibre. A ray of light reflected by this mirror is incident on a scale. The rotation of the needle is thus measured by the deflection of the spot of light on the scale.

Art 57
Electrostatic Voltmeter

An electrostatic voltmeter is in principle analogous to a Quadrant electrometer used heterostatically. There are however only two quadrants placed diagonally in a vertical plane; they are electrically connected. An aluminium vane (serving the purpose of the needle* in Quadrant Electrometer) is also pivoted in the vertical plane within the quadrants. The vane can rotate about a horizontal axis passing through the centre. To the lower part of the vane, is attached a small projection carrying a horizontal knife edge. A suitable weight placed on this knife edge, serves as the control; when the vane is deflected by electrostatic forces this weight tends to bring back the vane to the original position. The upper part of the vane carries a pointer moving over a graduated scale. When the vane and the quadrants are connected to two different potentials the vane is deflected. If the scale be graduated in volts by previously calibrating the instrument the difference of potential can at once be obtained by noting the deflection of the pointer on the scale. It should be noted here that the deflection is proportional to the square of the difference of

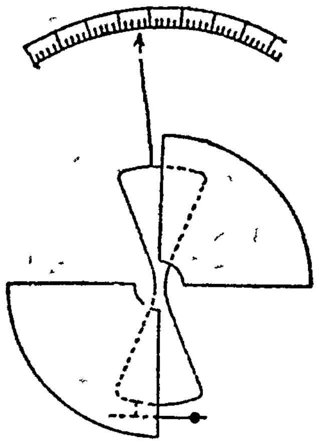
Fig. 97

* The electrometer is here used heterostatically, the needle being connected to a potential different from that of the quadrants.

potential. Hence this instrument can be used for measuring rapidly alternating E. M. F. For measuring high potential difference—1000 volts or more—this is a very suitable instrument, particularly because there being no current through the instrument there is no wastage of power. For different ranges of volts different controlling weights are used.

Art 58 Kelvin's Null Method.

Measurement of capacity

This method is very similar to Wheatstone's bridge method of measuring a resistance. To measure an unknown capacity we require three other known capacities one of which is variable. Four capacities C_1, C_2, C_3, C_4 arranged in a mixed circuit are connected to the two terminals of a battery as shown in Fig. 98. The mid-points B and D of the two parallel circuits are connected to the two pairs of quadrants of the electrometer E. The variable condenser—any of the four condensers may be variable—is adjusted until the electrometer shows no deflection. In that case points B and D are at the same potential.

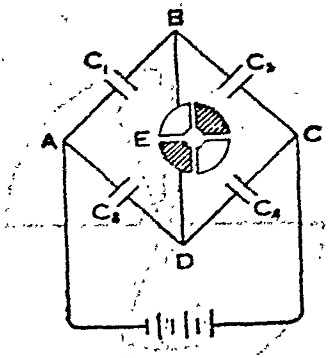

Fig. 98

Let Q_1 and Q_2 be the charges on the positive plates of C_1 and C_3 respectively, $-Q_1$ and $-Q_2$ being the induced negative charges on the corresponding negative plates, free positive charges Q_1 and Q_2 pass on to the positive plates of C_2 and C_4.

Hence $\quad Q_1 = C_1(V_A - V_B) = C_2(V_B - V_C)$

and $\quad Q_2 = C_3(V_A - V_D) = C_4(V_D - V_C)$

Since $V_B = V_D$, we have by dividing the *1st* equation by the *2nd*.

$$\frac{C_1}{C_2} = \frac{C_2}{C_4} \text{ or } \frac{C_1}{C_2} = \frac{C_3}{C_4}$$

Thus of the four capacities if three be known the fourth can be found out.

[Vide also Arts 115 and 155]

Art 59. Determination of S. I. C. of a dielectric.

HOPKINSON'S METHOD

Solid Dielectric

In this method the battery consists of an *even* number of cells, the middle point of the battery being earthed. Hence if $+V$ be the potential of the positive terminal. of the battery, $-V$ is the potential of the negative terminal.

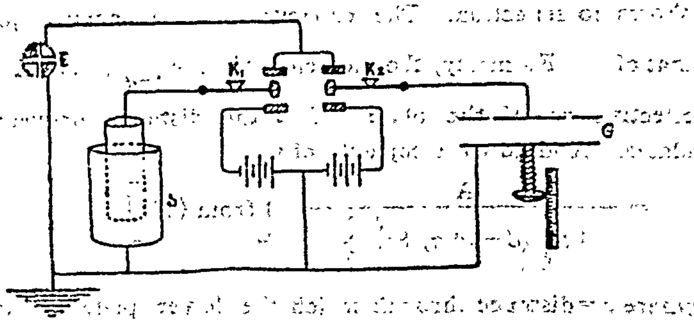

Fig. 99

K_1 and K_2 are two levers which can be made to touch the upper or lower studs as desired. E is a quadrant electrometer: one pair of opposite quadrants is connected to the upper studs (which are mutually electrically connected) and the other pair is earthed. When K_1 and K_2 are in contact with the lower studs, the two terminals of the battery are connected to the two plates of the condensers S and G; the other two plates of these condensers are earthed. S is a sliding coaxial cylindrical condenser, the capacity of which can be varied by sliding the inner cylinder inside the outer one. G is a guard ring air condenser, the lower plate of which can be moved up or down parallel to itself. S and G being connected to potentials equal but opposite in sign ($+V$ and $-V$) the charges on S and G are certainly opposite; they are also equal provided S and G have equal capacities. In that case on making the keys K_1 and K_2 touch the upper studs, these charges neutralise each other completely and the electrometer E shows no deflection. Hence the proceedure is this :—the sliding condenser S is adjusted until on making the levers K_1 and K_2 *first* touch the

lower studs and then the upper ones, there is no deflection in the electrometer. We then conclude that the capacity of S is equal to that of G.

Next a slab of the dielectric whose S. I. C. is to be determined is placed on the lower plate of G. S is no more disturbed but the lower plate of G is slowly lowered until on repeating the previous operations, the electrometer again shows no deflection. The capacity of G is again equal to that of S. Formerly, the capacity of $G = \frac{A}{4\pi d}$ where A is the effective area of the plate and d the distance between the plates. And now the capacity of G

$$= \frac{A}{4\pi\left\{(d-t+x)+\frac{t}{k}\right\}} \quad \left[\text{from (24)}\right]$$

where x = distance through which the lower plate is moved,

t=thickness of the dielectric slab,

k=S. I. C. of the dielectric.

And, since in each case the capacity of G is equal to that of S, we have

$$\frac{A}{4\pi d} = \frac{A}{4\pi\left\{(d-t+x)+\frac{t}{k}\right\}}$$

$$\therefore d = d - t + x + \frac{t}{k}$$

$$\therefore \frac{t}{k} = t - x \quad \therefore k = \frac{t}{t-x}$$

Since x is the distance through which the lower plate is *moved* it can be measured very accurately There is no difficulty about measuring t accurately. Hence k can be determined accurately by this method.

Art 60 Arons and Cohn's method.

Liquid Dielectric An apparatus designed by Silow was used by Arons and Cohn for measuring the S. I. C. of liquids. It consists of a cylindrical

glass vessel along the inside of which four vertical strips of tinfoils A, B, C, D, are pasted. The opposite pairs A, C and B, D are electrically connected. A horizontal arm E of platinum carrying two aluminium pieces F, F curved so as

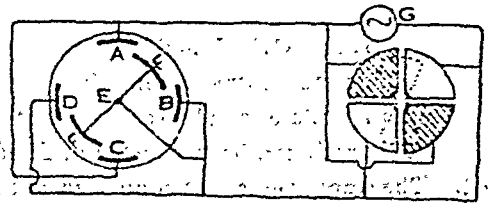

Fig. 100

to be parallel to the sides of the vessel, serves as the needle. This needle is suspended from a torsion head at the top. The needle is also connected to one pair of tinfoils so that this apparatus is completely analogous to a Quadrant Electrometer used ideostatically. In order to measure the S. I. C. of a liquid Silow's apparatus and a Quadrant Electrometer (also used ideostatically) are connected to a generator G of high frequency alternating potential as shown in the figure. The needles in both the apparatus are deflected. If θ_1 and θ_2 be these deflections and V be the P. D. between the terminals of the alternator, we have

$$\left.\begin{aligned} \theta_1 &= A_1V^2 \text{ (for Quadrant Electrometer)} \\ \text{and } \theta_2 &= A_2V^2 \text{ (for Silow's apparatus),} \end{aligned}\right\} \quad \ldots \quad (\alpha)$$

where A_1 and A_2 are the two constants of the two apparatus.

$$\therefore \quad \frac{\theta_1}{\theta_2} = \frac{A_1}{A_2}$$

Next the liquid whose S. I. C. is to be d[illegible] is poured into Silow's apparatus; the capacity of [illegible] is therefore increased k times where k is [illegible] liquid. Hence if θ_3 and θ_4 be the new [illegible]

$\theta_3 = A_1V^2$ (for Quadrant [illegible]

and $\theta_4 = kA_2V^2$ (for Silow's [illegible]

$$\therefore \frac{\theta_3}{\theta_4} = \frac{1}{k} \cdot \frac{A_1}{A_2} = \frac{1}{k} \cdot \frac{\theta_1}{\theta_2}. \quad \therefore \quad k = \frac{\theta_1}{\theta_3} \times \frac{\theta_4}{\theta_2}.$$

N. B. The P. D. between the terminals of an alternator does not always remain the same, *i. e.* V in (α) and V in (β) are not necessarily equal. Hence $\theta_1 \neq \theta_3$ and $k \neq \frac{\theta_4}{\theta_2}$.

Art 61 BOLTZMANN'S METHOD.

Gaseous Dielectric

The apparatus essentially consists of a parallel plate condenser enclosed in a brass box which can be exhausted or filled with any gas as desired. The upper plate P is connected to the positive

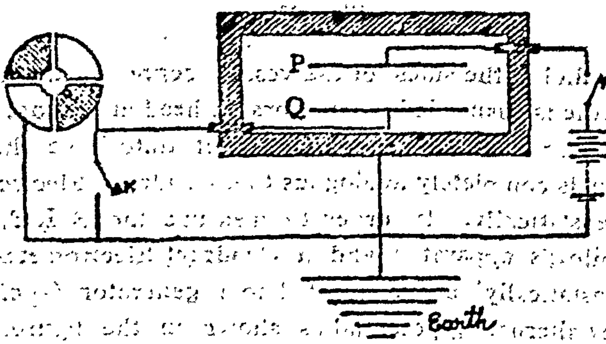

Fig: 101

terminal of a battery through K_1; the lower plate Q is connected to one pair of quadrants of a quadrant electrometer. The other pair of quadrants, the brass box and the negative terminal of the battery are all connected to the *Earth*. The plate Q may also be connected to the Earth by pressing the key K_2. To find S. I. C. of a gas the following operations are successively performed :—

1. *The brass enclosure is exhausted completely.* K_1 and K_2 are both closed. If there be n cells in the battery and if V be the E. M. F. of a single cell the potential of the plate P $= nV$, the plate Q being at zero potential. The electrometer obviously shows no deflection.

2. The key K_1 is opened. The closure is filled with gas. If k be the S. I. C. of the gas the capacity of the con-

denser is increased k times; and since the charge on P remains constant the potential of P is now $\frac{nV}{k}$. K_2 being still closed the potential of Q remains zero and the electrometer also shows no deflection.

3. K_2 is now opened and K_1 is closed. Additional charge comes from the battery to P, so that the potential of P rises to the original value nV. Q now acquires a potential due to this extra charge on P and the electrometer shows a deflection, —say, α. This deflection α is evidently proportional to the increase in the potential of P.

Hence $$\alpha \propto \left(nV - \frac{nV}{k}\right)$$

$$\propto nV\left(1 - \frac{1}{k}\right)$$

4. An additional cell is now included in the battery. The potential of P increases by V. The deflection of the electrometer *increases* by an amount β.

Then $$\beta \propto V$$

$$\therefore \quad \frac{\alpha}{\beta} = \frac{nV\left(1 - \frac{1}{k}\right)}{V} = n\left(1 - \frac{1}{k}\right) \text{ or } 1 - \frac{1}{k} = \frac{\alpha}{n\beta}$$

$$\therefore \quad \frac{1}{k} = 1 - \frac{\alpha}{n\beta} = \frac{n\beta - \alpha}{n\beta} \quad \therefore \quad k = \frac{n\beta}{n\beta - \alpha}$$

Exercise VII

1. An insulated circular metal plate of 8 cm radius is given a charge; it is then found to be attracted towards another exactly similar but earth connected plate placed below the former at a distance of 3 mm. If the force of attraction be equal to 0·4 gm wt, find the charge on the insulated plate.
[$g = 980$ cms/sec^2] Ans. 112 E. S. units.

2. Explain the action of the Attracted Disc Electrometer.
Two plates, each of area 20 sq. cms are maintained at a

difference of potential of 1200 volts. If the distance between the plates be 0·5 cm, find the force of attraction between them. [300 volts = one E. S. unit of P. D.] Ans 50·93 dynes.

3. Describe and explain the action of the Quadrant Electrometer. How would you use it to determine (*a*) the E. M. F. of a cell and (*b*) the strength of a current. How can it be used to measure (*a*) alternating E. M. F. and (*b*) very small difference of potential.

4. The plates of a parallel plate condenser are 2 cms apart. A slab of dielectric of S. I. C. equal to 5 and thickness 1 cm is placed between the plates with its faces parallel to them and the distance between the plates is altered so as to keep the capacity of the condenser unchanged. What is the new distance between the plates ? C. U. 1946. Ans 2·8 cms

Hints :—If x be the new distance $\frac{A}{4\pi.2} = \frac{A}{4\pi(x-1+\frac{1}{5})}$

C. U. Question.

1960. Describe the construction of an attracted disc electrometer and deduce its working formula.

An insulated plate 10 cms in diameter is charged with electricity and supported horizontally at a distance of 1 mm below a similar plate suspended from a balance and connected to Earth. If the attraction is balanced by the weight of one decigram find the charge on the plate. (g = 980 C. G. S. unit).

Ans. 35 E. S. U.

1961. Describe a Quadrant electrometer and give the theory of action. When is it used heterostatically and when ideostatically ?

1962, 1965. Describe an attracted disc electrometer and deduce its working formula. Why is such an electrometer called an absolute electrometer ?

1963. Describe fully any one method of measuring the potential of a charged metallic body. Indicate clearly the precautions to be taken for this measurement.

1966. Give the construction and principle of action of the attracted disc electrometer.

Calculate the force of attraction between the lower and the upper discs of an attracted disc electrometer when a potential difference of 1000 volts is applied between them, given that they are 0·5 cm. apart and of area 10 sq cm. Ans. $\frac{500}{9\pi}$ dynes.

Hints: $V_1 - V_2 = 1000 \text{ volts} = \frac{10}{3}$ E. S. unit.

$$\therefore \text{ Force} = \frac{S}{8\pi d^2}\left(V_1 - V_2\right)^2 = \frac{10}{8\pi(0{\cdot}5)^2}\left(\frac{10}{3}\right)^2 = \text{etc.}$$

1970. Describe and explain the action of an attracted disc electrometer and deduce the working formula. In what sense the *electrometer is called absolute?*

Two metal plates each of area 20 sq. cm. are maintained at a difference of potential of 1200 volts. If the distance between the plates be 0·5 cm. find the force of attraction between them.

1972, 1976. Describe the construction of a Quadrant Electrometer and explain how it is used to measure a difference of potential, clearly mentioning the difference between heterostatic and idiostatic uses.

CURRENT ELECTRICITY

CHAPTER VIII

CELLS, CURRENTS AND RESISTANCES

Art 62
What is a current

If a point A at a certain potential be connected by a metallic wire to another point B at a lower potential electric charge flows from A to B until the potentials at these two points are equalised. If however the difference of potential between A and B be somehow maintained constant charge flows continuously from A to B. Such a continuous flow of charge constitutes what is known as an electric current. A current is thus the rate of flow of charge, *i. e.* $i = \frac{dQ}{dt}$

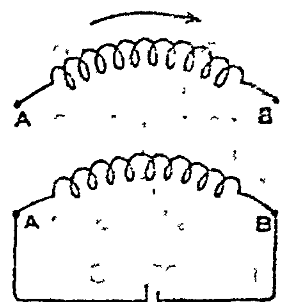

Fig. 102

It was first discovered by Volta that if two *dissimilar* metals are immersed in a fluid a constant difference of potential is maintained between the two metals. This difference of potential is due to what is known as the contact difference of potential at any junction. It is well known that all conductors contain electrons which are more or less loosely bound to the atoms. The concentration of such electrons however is different in different substances so that when a metal is immersed in a fluid there is usually a flow of electrons from one substance to the other. The substance which gains electrons becomes negatively charged and the substance which loses electrons is positively charged. Thus both substances become charged and there is therefore a contact difference of potential between the two. The flow of electrons stops when

the contact difference of potential (which opposes the flow of electrons) is just sufficient to stop the flow.

Thus if two metal plates A and B be immersed in a fluid there is a contact difference of potential between A and the fluid as well as between B and the fluid. If the two metals A and B are of different materials the difference of potential at the contact in the two cases is not the same and there is therefore a difference of potential between A and B.

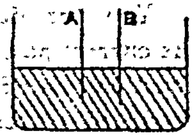

Fig 103

Art. 63
Simple cell and its defects

In a simple cell two plates one of copper and the other of zinc are immersed in dilute sulphuric acid. In this case copper is at a higher potential than zinc. If the plates are connected by a metallic wire an electric current flows from the copper plate to the zinc plate through the wire and from the zinc plate to the copper plate through the liquid. The higher potential plate is called the positive plate and the lower potential one the negative plate and the liquid is known as the exciting liquid. The agent which makes the current flow round the circuit is called the Electromotive Force (or E. M. F.) of the cell. This simple cell suffers from two defects (1) *Local Action* and (2) *Polarisation*. *Zinc* used as the negative plate is commercial* zinc and contains many impurities, such as Arsenic, Carbon, lead and iron. Coming in contact with dil H_2SO_4 these impurities together with zinc form local cells. Local currents are thereby produced and zinc is gradually consumed away without contributing anything to the main current. This defect—known as Local Action—is removed by amalgamating the zinc plate. This is done by rubbing the zinc surface with mercury with the help

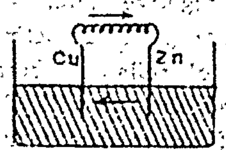

Fig 104

* Pure Zinc is not used because it is costly. Moreover pure Zinc is not acted on by dil H_2SO_4.

of a brush. The zinc amalgam so formed covers up the impurities which are therefore prevented from coming in contact with dil H_2SO_4.

The other defect—Polarisation is due to the fact that when the current passes zinc combines with H_2SO_4 according to the equation $Zn + H_2SO_4 = ZnSO_4 + H_2$. Hydrogen bubbles thus produced travel with the current through sulphuric acid solution and ultimately get deposited on the copper plate. This layer of hydrogen bubbles on the copper plate reduces the current in two ways; first it offers resistance to the current the strength of which is therefore considerably diminished. Further along with zinc plate this layer of hydrogen bubbles form the two plates of a cell and an E.M.F. is generated. This E.M.F.—Polarisation E. M. F.—opposes the main E.M.F. and thereby the strength of the current is reduced. This defect can be removed by oxidising hydrogen (*i.e.* forming water) with the help of an oxidising agent. This oxidising agent, or oxidiser, is also known as a depolariser.

In all kinds of cells Zinc is always the negative plate. By using different substances for the positive plate, the active liquid and the depolariser various kinds of cells have been prepared. In these cells the depolariser and the active liquid are usually kept separated by means of an earthen ware vessel. A list of such cells with their actions is given in Table I.

N. B. (1) Leclanche cell is the only cell where dil H_2SO_4 is *not* the active liquid. In this cell NH_4Cl solution is the active liquid.

(2) Bichromate cell is the only cell where the active liquid and the depolariser are not kept separated by earthen ware vessel; they are mixed up together.

Art 64
Dry cell

Dry cells are nowadays extensively used in torches, high tension batteries and portable testing sets. These are nothing but Leclanche cells. Instead of using a *solution* of NH_4Cl however, a *paste* is made of NH_4Cl (Sal-ammoniac), MnO_2, C (Graphite) and a little water.

Table I The following is the summary of facts relating to different kinds of cells :—

Cell	+ve plate	−ve plate	Active liquid	Depola -riser	E.M.F	Equations	Remarks
Daniell	Cu	Zn	dil H_2SO_4	$CuSO_4$	1·08 Volt	$Zn + H_2SO_4 = ZnSO_4 + H_2$ $H_2 + CuSO_4 = H_2SO_4 + Cu$	Suitable for small but constant current
Bunsen	C	Zn	dil H_2SO_4	Conc HNO_3	1·9 Volt	$Zn + H_2SO_4 = ZnSO_4 + H_2$ $H_2 + 2HNO_3 = 2NO_2 + 2H_2O$	Suitable for strong and constant current. NO_2 fumes are unpleasant Very rarely used.
Grove	Pt	Zn	dil H_2SO_4	Conc HNO_3	1·9 Volt.	Same as for Bunsen Cell	Because of Pt it is costly and is never used
Leclanche	C	Zn	NH_4Cl solution	MnO_2	1·4 Volt	$Zn + 2NH_4Cl = ZnCl_2 + 2NH_3 + H_2$ $H_2 + 2MnO_2 = Mn_2O_3 + H_2O$	Very suitable for small but intermittent current
Bi-chromate	C	Zn	dil H_2SO_4	$K_2Cr_2O_7$	1·8 to 2·2Volt	$K_2Cr_2O_7 + H_2SO_4 + H_2O = K_2SO_4 + 2H_2CrO_4$ $Zn + H_2SO_4 = ZnSO_4 + H_2$ $3H_2 + 2H_2CrO_4 = Cr_2O_3 + 5H_2O$ $Cr_2O_3 + 3H_2SO_4 = Cr_2(SO_4)_3 + 3H_2O$	Suitable for short but strong current

Zinc is the outer wall of a hollow cylinder and this serves as the negative plate. A carbon rod C placed at the centre is the positive plate. The carbon rod is insulated at the bottom by means of a Tar paper washer (TPW). Up to about three-fourth height of the cell the space between zinc cylinder

Zn—Zinc cylinder
C—Carbon rod
TPW—Tar paper washer
Ps—Paste of NH_4Cl, MnO_2, Graphite and H_2O.
SD—Saw Dust
S—Sand
Pt—Pitch
B—Brass cap
Pl—Paper lining

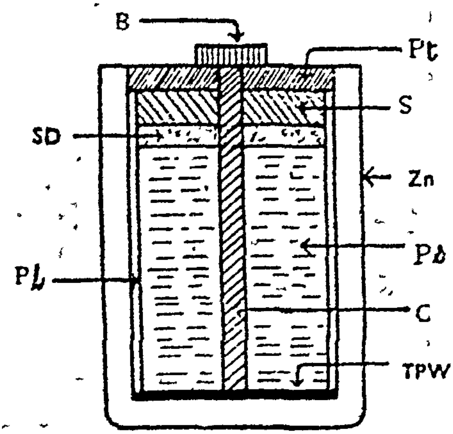

Fig. 105

and the carbon rod is filled up with a paste (Ps) of NH_4Cl, MnO_2, C (graphite) and H_2O. Over the paste there are successively two layers—one of saw dust (SD) and the other of sand (S). Finally the mouth of the cell is sealed with pitch (Pt). A blotting paper lining (Pl) lines up the inside of the zinc cylinder. A brass cap (B) is fitted to the top of the carbon rod ; this serves as the positive pole. A small hole is pierced through the pitch so that gases may come out from within the cell.

Art 65
Standard Cell

The E. M. F. of the cells described so far is never steady. As a current is drawn from one such cell the E. M. F. generally falls off slowly. It is however often necessary to have a source of *constant* E. M. F. which may be used for comparing the E. M. F.'s or for calibrating ammeters, voltmeters etc. The Weston cadmium cell is one such cell and is regarded as an accurate standard

cell. It consists of an H-shaped glass vessel as shown in Fig. 106. In one limb pure mercury (A) is placed at the bottom and this serves as the positive pole. The negative pole is an amalgam (D) of mercury and cadmium and this is placed at the bottom of the other limb. A paste (B) of mercurous

A—Mercury
B—Mercurous sulphate
C—Crystals of $CdSO_4$
D—Amalgam of Hg and Cd
E—Saturated solution of $CdSO_4$

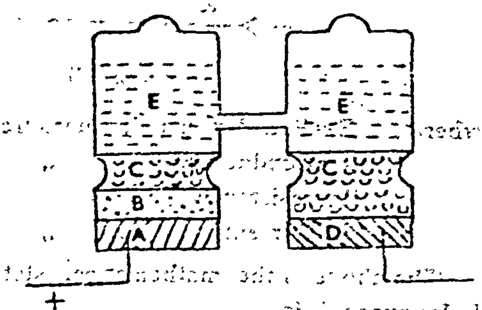

Fig. 106

sulphate is placed over mercury and a saturated solution (E) of $CdSO_4$ is kept in both the limbs, the surface of the solution reaching above the horizontal tube connecting the two limbs. Two wires fused at the bottom of the two limbs are in contact respectively with Hg and the amalgam of Hg and Cd and these serve as the two poles. To keep the solution always in the saturated condition a few crystals (C) of $CdSO_4$ are placed in each limb.

The E. M. F. of this cell is 1·0183 at 20°C. With the rise of temperature the E. M. F. slowly decreases. At any temperature its E. M. F. is given by $E = 1{\cdot}0183 - 0{\cdot}0000406 (t-20)$ Volt. It should be remembered that this cell is used only for comparison purposes. No current should be drawn from this cell even for a short time.

Art 66 Ohm's Law A current always flows from points at higher potential to those at lower potential. If a current passes through a wire different points of the wire are at different potentials. The strength of the current between any two points A and B of the wire is proportional to the potential difference between A and B, the constant of propor-

tionality being known as conductance of the wire (between A and B). Conductance again is the inverse of resistance. Thus

$$i = kE = \frac{E}{R}$$

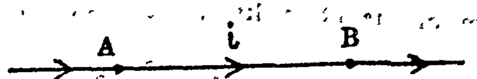

Fig. 107

where E = Pot. diff. usually measured in volts
k = Conductance " " " Mhos
R = resistance " " " Ohms
i = Current " " " Amperes

The above is the mathematical statement of Ohm's Law. In language, it is

Temperature remaining constant the current in any wire is proportional to the potential difference between the terminals of the wire.

The resistance of a wire changes with temperature; hence the necessity of the phrase, "Temperature remaining constant.

Specific resistance At any temperature the resistance of a wire varies directly as its length and inversely as the cross section of the wire; the constant of variation in this case is called the specific resistance of the wire. Thus $R = \rho\frac{l}{A}$ where ρ = specific resistance, l = length, A = area of cross-section. If the wire be of circular cross-section of radius r, $A = \pi r^2$.

The specific resistance of a substance may be defined as the resistance of a wire of that substance of length one cm and of cross-section one sq. cm., *i. e.* it is the resistance of a centimeter-cube of the substance.

N. B. (1) The resistance of a wire depends upon (*a*) the length (*b*) the cross-section and (*c*) the material of the wire. It does not depend on the *shape* of the coil of wire, *i. e.* whether the wire is straight or coiled up, the resistance is the same.

(2) The specific resistance depends *only* on the material of the wire. It does not depend on the length or the cross-section of the wire.

(3) Specific resistance and hence resistance depend on temperature.

Art 67 Grouping of resistances

Resistances in parallel

Let $r_1\ r_2\ r_3$ be three resistances joined in parallel between two points A and B. Let the main current i after coming to A, be divided into $i_1\ i_2\ i_3$ along the three wires. Then $i = i_1 + i_2 + i_3$.

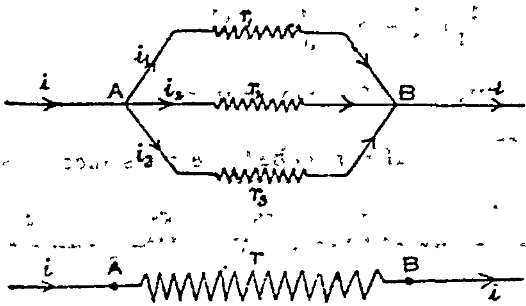

Fig. 108

Let E be the P.D. between A and B. Considering the three wires separately we have $E = i_1 r_1 = i_2 r_2 = i_3 r_3$. The three wires are now replaced by a single wire of resistance r connected between the same two points A and B maintained at the same Pot. diff. E. Then since the whole of the current i passes through this wire

$$E = ir = (i_1 + i_2 + i_3)r = \left(\frac{E}{r_1} + \frac{E}{r_2} + \frac{E}{r_3}\right)r$$

$$\therefore\ \frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} \quad \ldots \quad (30)$$

N. B. (1) If all the wires be of equal resistances the equivalent resistance is equal to the resistance of a single wire, divided by the number of wires in parallel. Thus if there be three resistances in parallel, each of 15 ohms, then the equivalent resistance $= \frac{15}{3} = 5$ ohms.

(2) If two or more resistances be arranged in parallel, the equivalent resistance is always *less* than the *smallest* of the different resistances. For example, if two resistances 10 ohms and 100 ohms be in parallel the equivalent resistance is given by $\frac{1}{r}=\frac{1}{10}+\frac{1}{100}=\frac{11}{100}$

$\therefore\ r=\frac{100}{11}=9\frac{1}{11}$ ohms, *i. e. r.* $<$ 10 ohms.

(3) If one of the resistances be *very small* in comparison to others the equivalent resistance is *practically equal to* this small resistance. For two resistances 0·01 ohm and 100 ohms, the equivalent resistance is given by

$$\frac{1}{r}=\frac{1}{0{\cdot}01}+\frac{1}{100}=100+\frac{1}{100}=\frac{10001}{100}$$

$\therefore\ r=\frac{10}{10001}=0{\cdot}01$ ohm, very approximately.

Resistances in series

If $r_1\ r_2\ r_3$ be three resistances in series

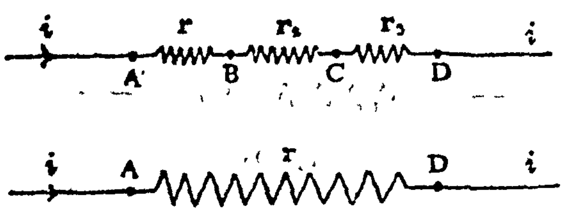

Fig. 109

then the same current i flows through each of them. Hence

$V_A-V_B=ir_1$, $V_B-V_C=ir_2$ and $V_C-V_D=ir_3$.

$\therefore$ adding up, $V_A-V_D=i(r_1+r_2+r_3)$ ... ... (a)

If the three wires be replaced by a single one of resistance r, so that the same current i flows through it and its terminals are at the same potential difference as before we have $V_A-V_D=ir$ ... ... (b)

Hence comparing (a) and (b)

$r=r_1+r_2+r_3$ ... ... (31)

N. B. The results for capacities in series and capacities in parallel, should be contrasted with these. (Vide Art 47)

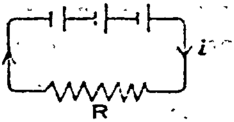

Fig. 110

Cells in series. Three cells, each of E. M. F. E and internal resistance r, are connected in series to send a current through the external resistance R. In this case the three internal resistances and the external resistance are all in series. Hence the total resistance in the circuit is $R+3r$. And the P. D. between the terminals of R is 3E. Hence the current through R is

$$i=\frac{3E}{R+3r}.$$

N. B. If the cells are all of different E. M. F's E_1 E_2 E_3 and of different internal resistances r_1 r_2 r_3, the current through R is given by

$$i=\frac{E_1+E_2+E_3}{R+r_1+r_2+r_3}.$$

Cells in parallel. Three cells each of E. M. F. E and internal resistance r, are connected in parallel between two points A and B. This combination is used to send a current through the external resistance R joined to A and B. The three internal resistances are now in parallel; their equivalent resistance is equal to $\frac{r}{3}$; the total resistance in the circuit is therefore equal to $R+\frac{r}{3}$. The P. D. between A and B is E. Hence the current through R is

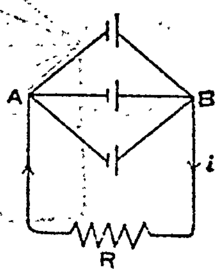

Fig. 111

$$i=\frac{E}{R+\frac{r}{3}}=\frac{3E}{3R+r}$$

N. B. The problem when the cells are of different E. M. F's and of different internal resistances, cannot be solved in this simple way. For this case, Vide Art 72.

Let us now have m rows of cells in parallel, each row containing n cells in series. The resistance of each row of cells is nr. Since there are m rows in parallel their equivalent resistance $= \frac{nr}{m}$. Hence the total resistance $= R + \frac{nr}{m}$. The P. D. between the terminals of R is nE. Hence the current through R is

Cells in mixed circuit

$$i = \frac{nE}{R + \frac{nr}{m}} = \frac{mnE}{mR + nr} \quad \ldots \quad (32)$$

Best grouping

Obviously mn is the total number of cells. Hence for a given total number of cells, the current through the external resistance is maximum when the denominator $mR + nr$ is minimum.

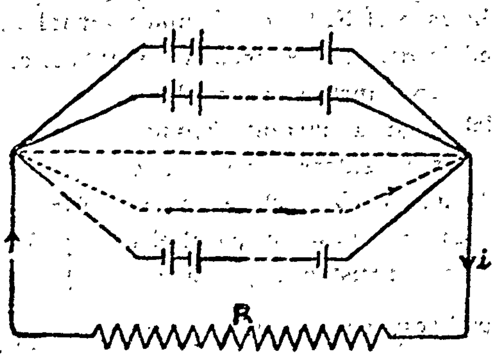

Fig. 112

But $$mR + nr = \left(\sqrt{mR} - \sqrt{nr}\right)^2 + 2\sqrt{mnRr}.$$

The 2*nd* term does not depend on m and n separately but on mn which is constant; the *1st* term being a perfect square its minimum value is zero. Hence the current is maximum when $mR = nr$ or $R = \frac{nr}{m}$ *i. e.* when the external resistance is equal to the effective internal resistance.

Art 69
Shunt

It is often necessary to reduce the strength of the current through a galvanometer. In that case the galvanometer is shunted, *i. e.* a resistance is used in parallel with the galvanometer, so that part of the current passes through the shunt and the remaining portion flows through the galvanometer.

Let S be the resistance of the shunt used with the galvanometer whose resistance is G. Let i be the main current and

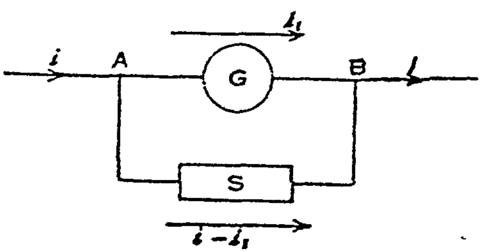

Fig. 113

let i_1 be the current through the galvanometer ; then $i-i_1$ is the current through the shunt. If A and B be the two terminals to which the shunt is connected

$V_A - V_B = i_1 G$ considering the current through the galvanometer,

also, $V_A - V_B = (i-i_1)\,S$ considering the current through the shunt

$\therefore\ i_1 G = (i-i_1)S$ or $i_1(G+S) = iS$

$$\therefore\quad i_1 = i\,\frac{S}{G+S}\quad \ldots\quad \ldots\quad (33)$$

i. e. current through the galvanometer

$$= \text{main current} \times \frac{\text{shunt resistance}}{\text{total resistance}}$$

N. B. In order to *decrease* the current through the galvanometer the shunt resistance has to be *decreased* and *vice versa*.

Problems

1. 24 cells each of E. M. F. 2 volts and of internal resistance 2 ohms, are required to send a current through an external resistance of 3 ohms. Find the best arrangement of cells; find also the maximum current through the external resistance.

Let n cells be connected in a row in series and let there be m such rows in parallel. Then the total number of cells $mn=24$.

Also when the current through the external resistance is maximum the external resistance is equal to the effective internal resistance.

$$i.\ e. \quad 3=\frac{2n}{m} \quad \text{or} \quad n=\frac{3m}{2}$$

$$\therefore \quad m\,\frac{3m}{2}=24 \quad \text{or} \quad m^2=16 \quad \therefore \quad m=4$$

$$\therefore \quad n=\frac{3\times 4}{2}=6$$

Thus we must have 6 cells in series in a row and 4 such rows in parallel.

The current through the external resistance

$$=\frac{4\times 6\times 2}{4\times 3+2\times 6} \text{ [from (32)]}$$

$$=\frac{48}{24}=2 \text{ Amps.}$$

2. A galvanometer of 6 ohms resistance is shunted by a wire of 4 ohms resistance. If this shunted galvanometer be connected in series with a cell of 2 volts E. M. F. and of 1·6 ohms internal resistance find the current through the galvanometer.

If R be the effective resistance of the galvanometer and the shunt we have

$$\frac{1}{R}=\frac{1}{4}+\frac{1}{6}=\frac{10}{24} \quad \text{or} \quad R=\frac{24}{10}=2{\cdot}4 \text{ ohms.}$$

$\therefore$ Total resistance in the circuit $=2{\cdot}4+1{\cdot}6=4$ ohms.

$\therefore$ Current from the battery $=\frac{2}{4}=0{\cdot}5$ Amp.

$\therefore$ Current through the galvanometer

$$=0{\cdot}5\times\frac{4}{4+6} \text{ [from (33)]}$$

$$= \frac{2}{10} = 0{\cdot}2 \text{ Amp.}$$

Multiplying power of a shunt

From equation (33) it is obvious that if the main current i be increased $\frac{G+S}{S}$ times the current through the shunted galvanometer becomes i, *i. e.* main current $\frac{G+S}{S}i$ produces the same deflection in a shunted galvanometer as is produced by the current i in an unshunted galvanometer. $\frac{G+S}{S}$ is therefore called the multiplying power of the shunt.

Again from (33), in order that $\frac{i_1}{i} = \frac{1}{10}$ we have $\frac{S}{G+S} = \frac{1}{10}$ or $10S = G+S$ or $9S = G$ $\therefore$ $S = \frac{G}{9}$.

Thus in order to reduce the current through the galvanometer to $\frac{1}{10}$ of the main current, a shunt of resistance $\frac{G}{9}$ is to be used. Similarly, shunts of resistances $\frac{G}{49}$, $\frac{G}{99}$, etc. should be used when it is necessary to reduce the galvanometer current to $\frac{1}{50}$, $\frac{1}{100}$, etc., of main current.

Art 70 Universal shunt

We have seen in the preceding article that in order to reduce the current through a galvanometer in a desired ratio, shunt of the corresponding resistance is to be used. Hence different galvanometers require different shunts to serve the same purpose. A shunt known as a universal shunt has however been designed, which may be used with galvanometers of all resistances.

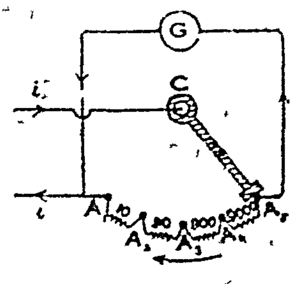

Fig. 114

A universal shunt consists of several steps A_1A_2, A_2A_3,

A_3A_4, A_4A_5 of resistances 10, 90, 900 and 9000 ohms respectively. A handle rotating about C may be made to connect any of these steps. The galvanometer terminals are connected to A_1 and A_5. The leads for the main current are joined to C and A_1. When the handle connects A_5 all the steps are used as the shunt the total resistance being 10,000 ohms. Hence if i be the main current the galvanometer current

$$=i.\frac{10000}{G+10000}.$$

When the handle is rotated so as to touch A_4 the step A_4A_5 is connected in series with the galvanometer. The other steps are now used as the shunt the resistance of which is 1000 ohms. The total resistance (galvanometer circuit resistance + shunt resistance), however, is still G + 10,000 ohms. Hence the galvanometer current is now $=i.\frac{1,000}{G+10000}$ *i. e.* $\frac{1}{10}$ of the previous value.

By connecting the handle to A_3 or A_2 the current through the galvanometer may be reduced to $\frac{1}{100}$ or $\frac{1}{1,000}$ of the former value.

Since these results are independent of the value of G such a shunt may be used with any galvanometer.

N. B. By using other resistances for the different steps or by increasing the number of steps other fractions of the original current may be made to pass through the galvanometer.

Art 71
Kirchoff's Laws

In the simple cases discussed in the previous articles Ohm's Law is sufficient to solve the problems. But in more complicated cases more generalised laws must be used. These are known as Kirchoff's Laws.

Kirchoff's Laws are as follows :—

1st law : In any electric circuit the algebraic sum of currents meeting at a point is zero.

In this law the word *algebraic* is important; it means that in summing up the currents proper attention must be paid to the signs of the currents. The signs are determined by the convention that currents coming towards a point and currents going away from the point, are opposite in signs; it does not matter which we call positive and which negative. This law is more or less self-evident. In Fig. 115 currents i_1, i_2, i_8, i_{13} meet at the point B. Since there can be no accumulation of current at B

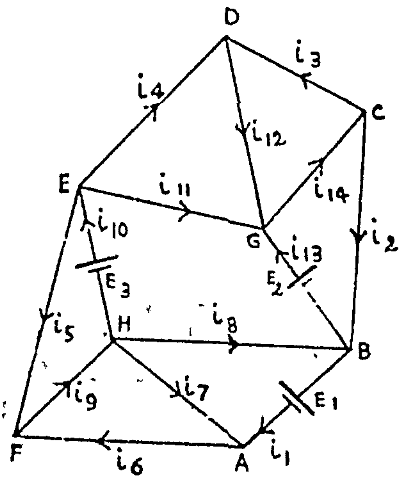

Fig. 115

$$i_1 + i_{13} = i_2 + i_8$$

i. e. $\quad i_1 + (-i_2) + (-i_8) + i_{13} = 0.$

2nd law : In any network of conductors if we consider a closed circuit then the algebraic sum of products of resistances into corresponding currents in the respective branches of the closed circuit, is equal to the total E. M. F. in the circuit.

For the purpose of this law *any* closed circuit may be considered. Thus in Fig. 115 we have a large number of closed circuits, viz., ABHA, ABGEFA, ABGDEFA, ABCDGEFA, CDGEHBC, etc., etc. In any circuit currents in different branches are either clockwise or anticlockwise and hence, are of different signs.

In Fig. 115 let the resistances of the different conductors in the network, be r_1 r_2 r_3 ··· and currents in these conductors, i_1, i_2, i_3, ... Then,

in the circuit ABHA, $\quad i_7r_7 - i_8r_8 - i_1r_1 = E_1$

,, ,, CDEGC, $\quad i_{11}r_{11} + i_{14}r_{14} + i_3r_3 - i_4r_4 = 0$

,, ,, BGEHB, $\quad i_{13}r_{13} - i_{11}r_{11} - i_{10}r_{10} + i_8r_8 = E_2 - E_3$

etc., etc., etc.

N. B. If there be no battery in a circuit the total E. M. F. in the circuit is zero ; if there be two or more batteries in the circuit proper attention must be paid to the way in which they are connected, *i. e.* whether they are in series or in opposition.

Art 72 Applications of Kirchoff's Laws

1. Let three cells A, B, C of E. M. F's E_1, E_2, E_3 and of internal resistances r_1, r_2, r_3 be connected in parallel, so as to send a current i through an external resistance r. Let the currents through the different cells be i_1, i_2, i_3.

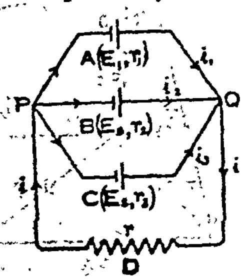

Fig. 116

Applying Kirchoff's 1*st* law to the point P or Q we have

$$i_1+i_2+i_3=i \quad \ldots \quad (i)$$

Applying Kirchoff's 2*nd* law

to the circuit PAQDP, $i_1r_1+ir=E_1$ ... (*ii*)

" " PBQDP, $i_2r_2+ir=E_2$ ... (*iii*)

" " PCQDP, $i_3r_3+ir=E_3$ ... (*iv*)

Dividing the last three equations by r_1, r_2 and r_3 respectively and adding

$$\left(i_1+i_2+i_3\right)+i\left(\frac{r}{r_1}+\frac{r}{r_2}+\frac{r}{r_3}\right)=\frac{E_1}{r_1}+\frac{E_2}{r_2}+\frac{E_3}{r_3}$$

Since from (*i*), $i_1+i_2+i_3=i$, we have

$$i\left\{1+\frac{r}{r_1}+\frac{r}{r_2}+\frac{r}{r_3}\right\}=\frac{E_1}{r_1}+\frac{E_2}{r_2}+\frac{E_3}{r_3}$$

$\therefore$ i is found out.

Hence from (*ii*), (*iii*), (*iv*) i_1, i_2, i_3 may also be determined.

N.B. In this problem the E. M. F's and the resistances are supposed to be known. The currents i_1, i_2, i_3, i are four unknown quantities. We therefore require four equations to determine them. Kirchoff's 1st law gives us one equation. To get the other three equations, we have applied Kirchoff's 2nd law to three circuits. It may be noted, however, that in this problem we have altogether six closed circuits, viz., the

three circuits already considered and three others PAQBP, PAQCP, PBQCP. If Kirchoff's 2nd law be applied to each of these six circuits, we may have six equations; but only three of these equations are independent. It is immaterial which three we take; final result is always the same. But by properly choosing the circuits—as we have done—calculation may be made rather easily.

Wheatstone's Net Let four resistances P, Q, R, S, be connected as in Wheatstone's Net. A galvanometer

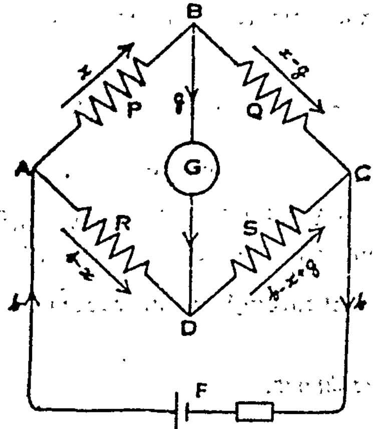

Fig. 117

of resistance G and a battery of E. M. F. E are connected as shown in Fig. 117. Let B be the resistance of the battery circuit and let currents be distributed as noted in the figure.

It will be seen that in assuming the distribution of currents Kirchoff's 1st law is tacitly applied.

Let us now apply Kirchoff's *2nd* law to the three circuits ABDA, BDCB and ABCEA.

From ABDA $\quad Px+Gg-R(b-x)=0$

,, BDCB $\quad Gg+S(b-x+g)-Q(x-g)=0$

,, ABCEA $\quad Px+Q(x-g)+Bb=E$

Re-arranging terms, we have

$$Gg+x(P+R)-bR=0$$

$$(G+S+Q)g-x(S+Q)+bS=0$$

$$-Qg+x(P+Q)+Bb=E$$

$$\therefore \quad g = \frac{\begin{vmatrix} P+R & -R & 0 \\ -(S+Q) & S & 0 \\ P+Q & B & E \end{vmatrix}}{\begin{vmatrix} G & P+R & -R \\ G+S+Q & -(S+Q) & S \\ -Q & P+Q & B \end{vmatrix}} \; *$$

Expanding the numerator in terms of the last column we have

$$\text{Numerator} = E\begin{vmatrix} P+R & -R \\ -(S+Q) & S \end{vmatrix},$$

$$= E\{S(P+R) - R(S+Q)\} = E(PS - QR)$$

Hence numerator is zero when $PS = QR$, *i. e.* when $\frac{P}{Q} = \frac{R}{S}$ the current through the galvanometer is zero.

Miscellaneous Problems.

1. Let twelve wires, each of resistance r, be connected in the form of a cube. It is required to find the effective resistance of this cube when a current enters the cube at one end A and goes out from the diagonally opposite end C′. From symmetry the main current i on arrival at A is divided into three components, each equal to $i/3$ along AA′,

* The solution of simultaneous equations, may be obtained easily with the help of determinants. Thus

$$\text{if} \quad a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

$$x = \frac{+\begin{vmatrix} b_1 & c_1 & d_1 \\ b_2 & c_2 & d_2 \\ b_3 & c_3 & d_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}} \quad y = \frac{-\begin{vmatrix} a_1 & c_1 & d_1 \\ a_2 & c_2 & d_2 \\ a_3 & c_3 & d_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}}. \quad z = \frac{+\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}}$$

AB and AD. Each of these components is again divided equally along corresponding two other wires.

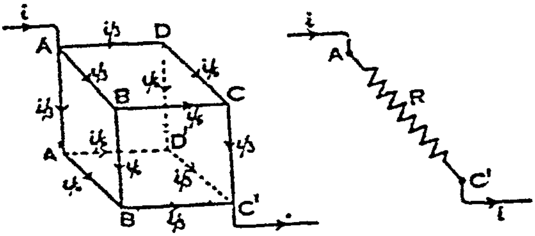

Fig. 118

Hence $V_A - V_{C'} = (V_A - V_{A'}) + (V_{A'} - V_{B'}) + (V_{B'} - V_{C'})$

$$= \frac{i}{3}.r + \frac{i}{6}.r + \frac{i}{3}.r = \frac{5i}{6}.r.$$

If we now replace the cube by a single wire of resistance R joining the same two points A and C′ we have

$$V_A - V_{C'} = iR.$$

Hence $iR = \frac{5i}{6}.r \therefore R = \frac{5r}{6}.$

2. Let a cell of E. M. F. 4 volts have an internal resistance of 6 ohms. Let its terminals be connected to the two ends A and B of a resistance of 10 ohms. Then the current$=\frac{4}{6+10}=\frac{1}{4}$ amp.

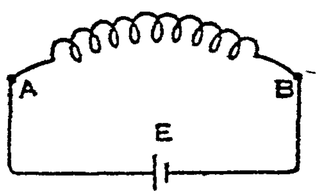

Fig. 119

$\therefore \quad V_A - V_B = \text{current} \times \text{resistance}$

$$= \frac{1}{4} \times 10 = 2{\cdot}5 \text{ volts}$$

Since A and B are connected to the plates of the battery, the P. D. between plates $= 2{\cdot}5$ volts.

If the external resistance be changed the P. D. between the plates also changes.

Thus although the P. D. between the plates of the cell on open circuit (*i. e.* when the plates are not connected by any

external resistance) is 4 volts, that on closed circuit drops down to a lower value depending upon the external resistance. Thus although the E. M. F. of the cell is always the same, the P. D. between the plates of the cell depends upon the external resistance and is therefore different in different cases. It is to be noted that the E. M. F. of the cell is equal to the P. D. between the plates of the cell on *open circuit.* If however the cell has no internal resistance the P. D. of the plates of the cell on open circuit is always the same as that on closed circuit, *i. e.* in this case the E. M. F. of the cell is the same as the P. D. between the plates of the cell, even when the cell is on closed circuit.

When the terminals of a cell are joined by a 5 ohms wire the difference of potential between them is $1\frac{2}{3}$ volts. If a second wire of the same resistance is connected in parallel with the first one the difference of potential is reduced to $1\frac{3}{7}$ volts. What will be the potential difference if a third wire of the same resistance is connected in parallel with the other two ?

Let E be the E. M. F. and r the internal resistance of the cell. When the external resistance is 5 ohms the current is $\frac{E}{5+r}$. Hence the P. D. between the terminals is

$$\frac{E.5}{5+r}=1\frac{2}{3} \quad \text{or} \quad \frac{E}{5+r}=\frac{1}{3} \qquad \dots \qquad (a)$$

When the second wire is joined in parallel the external resistance becomes $\frac{5}{2}$ ohms. In that case P. D. is

$$\frac{E.\frac{5}{2}}{\frac{5}{2}+r}=1\frac{3}{7} \quad \text{or} \quad \frac{E}{5+2r}=\frac{2}{7} \qquad \dots \qquad (b)$$

Solving (a) and (b) $E=2$, $r=1$.

When the third wire is joined in parallel the external resistance becomes $\frac{5}{3}$ ohms. In that case P. D. is

$$\frac{E\frac{5}{3}}{\frac{5}{3}+r}=\frac{5E}{5+3r}=1\frac{1}{4} \text{ volt}$$

A battery of 6 volts E. M. F. and 0·5 ohm internal resistance is joined in parallel with another battery of 10 volts E. M. F. and internal resistance 1 ohm and the combination is used to send

current through an external resistance of 12 ohms. Calculate the current passing through the different parts of the circuit.
C. U. 1955

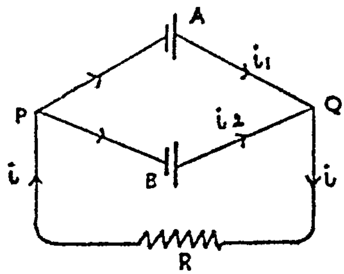

Fig. 120

From Kirchoff's first law

$$i=i_1+i_2 \quad \ldots \quad (a)$$

Applying Kirchoff's second law to the circuit PAQRP

$$0{\cdot}5i_1+12i=6 \quad \ldots \quad (b)$$

and to the circuit PBQRP

$$i_2+12i=10 \quad \ldots \quad (c)$$

Solving (a), (b) and (c)

$i=\frac{22}{37}$ amp $i_1=-\frac{84}{37}$ amp and $i_2=\frac{106}{37}$ amp. The negative value of i_1 indicates that the current through the cell A flows in a direction opposite to that indicated in the diagram.

In a wheatstone's net the four resistances are equal and the galvanometer is replaced by a battery of the same E. M. F. as the one already present. If the resistances of the battery circuits are each the same as those in the other four arms find the currents.
C. U. 1944

Let E be the E. M. F. of each battery and r the resistance of each branch. Let the distribution of currents be as shown in the figure.

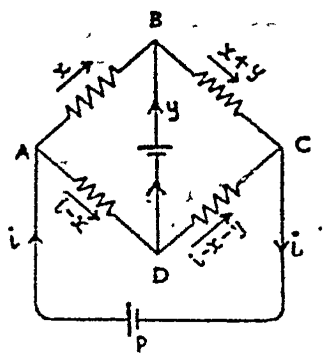

Fig. 121

Applying Kirchoff's second law to the circuits ABDA, BCDB and ABCPA,

$$r(i-x)+ry-rx=E$$

$$\text{or} \quad -2x+y+i=\frac{E}{r} \quad \ldots \quad (1)$$

$$r(x+y)-r(i-x-y)+ry=E$$

$$\text{or} \quad 2x+3y-i=\frac{E}{r} \quad \ldots \quad (2)$$

$$\text{and} \quad rx+r(x+y)+ri=E \quad \text{or} \quad 2x+y+i=\frac{E}{r} \quad \ldots \quad (3)$$

Solving (1), (2) and (3) $i=\frac{E}{2r}$, $x=0$ and $y=\frac{E}{2r}$. Hence the currents in different branches can be found out.

ABCD is a wheatstone's bridge. The resistances of the arms AB, BC, CD and DA are 100, 10, 25 and 200 ohms respectively. The points B and D are joined to a galvanometer of resistance 100 ohms. If a current of 0·1 amp enters the bridge at A and leaves it at C find the value of the current through the galvanometer. C.U. 1976

Let the distribution of currents be as shown in the diagram.

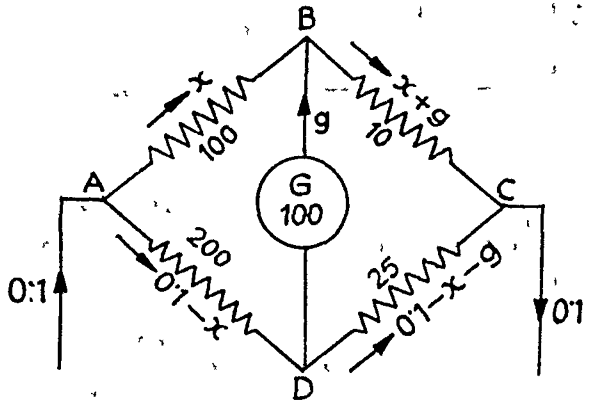

Fig. 121 (a)

Applying Kirchoff's second law, to the circuit ADB,

$$200(0{\cdot}1-x)+100g-100x=0$$

or $$3x-g=0{\cdot}2 \qquad \ldots(1)$$

Applying Kirchoff's second law to the circuit DBC,

$$100g+10(x+g)-25\,(0{\cdot}1-x-g)=0$$

or $$35x+135g=2{\cdot}5 \qquad \ldots(2)$$

Solving (1) and (2) $g=0{\cdot}00114$ amp.

Exercise VIII

1. Compare the resistances of two wires of the same material, one of which weighs 2·53 gms and is 3·6 metres long, while the other weighs 1·67 gms and is 4·2 metres long.

Ans. 1 : 2·06.

2. A uniform glass tube of internal diameter 0·1128 cm. contains mercury. If the resistance of one metre of this mercury be 0·9407 ohm find the sp. resistance of mercury.

Ans. $94{\cdot}01 \times 10^{-6}$ ohms per cm^3.

3. (*a*) Describe a dry cell.

(*b*) Describe a standard cell. Why is it so called?

4. Write short notes on Ohm's Law.

When the terminals of a cell are joined by a wire of resistance 7 ohms the current is 0·25 amp; when the wire is of 15 ohms resistance the current is halved. Find the E. M. F. and the internal resistance of the cell. Ans. 2 volts; 1 ohm.

5. Find the resistance of a cubic centimetre of copper (*a*) when drawn out into a wire of diameter 0·32 mm and (*b*) when hammered into a flat sheet of thickness 1·2 mm, the current flowing perpendicularly through the sheet from one face to the other. [Sp. resistance of copper is $1{\cdot}59 \times 10^{-6}$].

C. U. 1944. Ans. (*a*) 2·46 ohms (*b*) $2{\cdot}29 \times 10^{-8}$ ohm.

6. State Ohm's Law and describe how you would verify it experimentally.

A galvanometer of 50 ohms resistance shunted by a wire of resistance 5 ohms is in series with a cell of E. M. F 2 volts and an external resistance of 100 ohms. Find the current through the galvanometer. [Internal resistance of the cell is to be neglected]. Ans 0·00174 amp.

7. A galvanometer of resistance 100 ohms is in series with a battery and a resistance of 200 ohms. The galvanometer is shunted by a wire of 5 ohms and at the same time the external resistance is reduced to some value R so that the deflection of the galvanometer remains the same. Find R. Ans 9·52 ohm.

8. What is a shunt? Explain its use in an actual electric experiment. Explain the use of a universal shunt. C. U. 1949

The resistances of different steps of a universal shunt are 10, 90. 100, 300, 1500, 8000 ohms respectively. Find to what fractions the galvanometer current may be reduced when this universal shunt is used. Ans. $\frac{1}{5}$ $\frac{1}{20}$ $\frac{1}{50}$ $\frac{1}{100}$ $\frac{1}{1000}$.

9. What are the resistances of the different steps of a universal shunt which when used with a galvanometer, may reduce the galvanometer current to $\frac{1}{5}$, $\frac{1}{10}$, $\frac{1}{50}$, $\frac{1}{100}$ and $\frac{1}{500}$ of its maximum value? Ans 1, 4, 5, 40, 50, 400 ohms or any multiple of this series.

10. 80 exactly similar cells are to be arranged in mixed circuit to send a maximum current through an external resistance. If the internal resistance of each cell be one fifth of the external resistance find how the cells are arranged.

Ans. 20 cells in series in one row and 4 such rows in parallel.

11. Find the maximum current that can be sent through a wire of 4 ohms resistance with the help of 12 cells. Each cell is of E. M. F, 1·5 volts and of internal resistance 2 ohms.

Ans. 0·9 Amp

12. What is meant by the E. M. F. of a cell on open circuit? Why is it different from that on closed circuit?

A voltmeter of internal resistance 35 ohms is connected to the terminals of a battery of E. M. F 2 volts and of internal resistance 5 ohms. What voltage will the voltmeter indicate?

Ans. 1·75 volts.

13. The E. M. F. of a battery is 20 volts. If the terminals of the battery be joined by a wire of resistance 8 ohms the potential difference between the battery terminals falls to 16 volts. Find the internal resistance of the battery. Ans. 2 ohms.

14. The terminals of a battery are joined by a wire of 7 ohms resistance and the potential difference between the terminals is found to be 14 volts. When the wire is replaced by another of 4 ohms resistance the terminal potential difference drops to 12 volts. Find the E. M. F. and the internal resistance of the battery. Ans. 18 volts; 2 ohms.

15. Ten cells in series are used to send a current through a wire of 7·5 ohms resistance. A high resistance voltmeter connected to the terminals of the battery indicates 15 volts. When the battery is on open circuit the same voltmeter records 20 volts. Find the E. M. F and the internal resistance of each cell. Ans. 2 volts; 0·25 ohm.

16. State and explain Kirchoff's laws on the distribution of currents in a net work of conductors.

A battery of E. M. F. 12 volts and of internal resistance 6 ohms is connected in parallel to another of E. M. F. 8 volts and of internal resistance 4 ohms. The terminals of the composite battery so formed are joined by a wire of resistance 12 ohms. Calculate the current passing through the different parts of the circuit. Ans. $i_1 = \frac{2}{3}$ amp ; $i_2 = 0$: $i = \frac{2}{3}$ amp.

17. Four wires of resistances 2, 4, 6 and 3 ohms are joined end to end, so as to form a quadrilateral ABCD. Another wire of resistance 5 ohms joins the diagonal points B and D. If the terminals of a battery of E. M. F 4 volts be joined to A and C find the current through the 5 ohm resistance. Find also the equivalent resistance of the net work of conductors.

Ans. 0 amp : 3·6 ohm.

18. The four arms of a Wheatstone's net are of resistances 5, 2, 6, 4 ohms respectively. One terminal of the galvanometer (resistance 20 ohms) is joined to the junction of 5 and 2 and the other to the junction of 6 and 4. If the cell joined to the other junctions, be of E. M. F, 2 volts and of negligible internal resistance find the current through the galvanometer.

Ans. 0·026 amp.

C. U. Questions

1962, State and explain Kirchoff's Laws as applied to an electric network of conductors.

State briefly how these laws are applied in a Wheatstone's net work of conductors.

A battery of E. M. F. 12 volts and of internal resistance 6 ohms is connected in parallel to another of E. M. F. 8 volts and internal resistance 4 ohms. The terminals of the composite battery so formed are joined by a wire of resistance 12 ohms. Calculate the current through this last mentioned resistance.

Ans $\frac{2}{3}$ amp.

1963. State and explain Kirshoff's Laws as applied to an electrical network.

The positive poles of two cells of E. M. F's 2V and 1·5 V and each of internal resistance 1 ohm are connected by a uniform wire of resistance 10 ohms. The negative poles are also connected by a similar wire of resistance 6 ohms. If the middle points of these two wires are connected by a resistance of 2 ohms, calculate the current flowing through this wire and also the potential difference developed between these two points.

Ans. $\frac{7}{26}$ amp ; $\frac{7}{13}$ volt.

1965. State and explain kirchoff's Laws for the distribution of currents in a network of conductors.

[Problem worked out on Page 155].

1966. State Kirchoff's Laws for the distribution of currents in a network of conductors.

In a Wheatstone's net ABCD the resistances in ohms are AB=10, BC=15, AD=20, DC=25 and BD=10. If the P. D. between A and C is 20 volts find the current distribution in the network, neglecting the internal resistance of the battery.

Ans. Current in AB$=\frac{50}{61}$ amp, in AD$=\frac{26}{61}$ amp, in BD$=\frac{2}{61}$ amp. in BC$=\frac{48}{61}$ amp, in DC$=\frac{28}{61}$ amp.

1968. State Kirchoff's Laws for the distribution of currents in a network of conductors.

Obtain an expression for the galvanometer current in an unbalanced Wheatstone's Bridge in terms of the battery current.

1976. State and explain Kirchoff's Laws for distribution of currents in a network of conductors.

CHAPTER IX

HEATING EFFECT OF CURRENT

Art 73
Heat generated

If two points A and B *maintained* at different potentials,—say, by connecting them to the plates of a battery, be joined by a wire a current flows from higher potential to lower one. A current however means rate of flow of charge. If a current i flows for a time t it means that a charge Q ($=it$) has passed through the wire. If E be the P. D. between the points A and B, the work done in carrying the charge Q from A to B is $EQ=Eit$. Energy thus spent reappears as heat in the connecting wire. Thus if a current i flows for a time t

$$\left.\begin{aligned}\text{Heat generated} &= Eit\\ &= i^2Rt\\ &= \frac{E^2}{R}.t\end{aligned}\right\} \qquad \ldots \qquad \ldots \qquad (34)$$

Thus from the first equation, heat generated is proportional to any one of E, i and t when the other two remain constant. This is Joule's Law of generation of heat.

In working out numerical problems attention must be paid to the units used. The following relations should be remembered :—

Volt, Ohm and Ampere are practical units.

One Volt	=	10^8	C. G. S. units of E. M. F.
,, Ohm	=	10^9	,, ,, ,, Resistance.
,, Ampere	=	10^{-1}	,, ,, ,, Current.

If two of these relations be remembered the third may be found out by Ohm's Law.

If E. M. F., resistance and current be expressed in C. G. S. units and time in seconds the heat generated as determined by equation (34) is also expressed in C. G. S. units of energy, *i. e.* in ergs. Again, since one calorie $=4{\cdot}2\times10^7$ ergs*, in order to convert ergs into calories, we must divide by $4{\cdot}2\times10^7$.

N. B. One Joule $=10^7$ ergs. Hence one calorie $=4{\cdot}2$ Joules.

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* This is what is called J the mechanical equivalent of heat.

Thus if a current of i amperes flows through a resistance of R ohms for t secs

Heat generated $=(i\times10^{-1})^2\times(R\times10^9)\times t$

$$=i^2Rt\times10^7 \text{ ergs}$$

$$=\frac{i^2Rt\times10^7}{4.2\times10^7}$$

$$=\frac{i^2Rt}{4.2} \text{ calories} \quad \ldots \quad \ldots \quad (34a)$$

Thus if i and R be measured in practical units and t in secs, we may use the simplified formula (34a).

Problem

A wire of 8 ohms resistance is immersed in a calorimeter containing one litre of water at 20°C. If a current of 5 Amps passes through the wire for 10 minutes, find the final temperature.

$$\text{Heat generated}=\frac{5^2\times8\times10\times60}{4.2} \text{ calories}$$

Mass of water=1000 gms. If θ be the rise of temperature we have

$$1000\,\theta=\frac{5^2\times8\times10\times60}{4.2}$$

$$\therefore \quad \theta=\frac{5^2\times8\times10\times60}{4.2\times1000}=28.6 \text{ C}$$

$\therefore$ Final temperature$=20+28.6=48.6$ C

Art 74
Power

Power is the rate of expenditure of energy, *i. e.* power is equal to energy divided by time. Thus from the preceding article power consumed in a circuit is Ei or i^2R or $\frac{E^2}{R}$

The practical unit of power is Watt. Hence if E, i and R be expressed in practical units the power consumed$=$Ei, i^2R or $\frac{E^2}{R}$ Watts.

Thus Power = E. M. F. × Current.

or Watt = Volt × Amp.

In C. G. S. units one Watt

$=10^8\times10^{-1}=10^7$ ergs. per sec = one Joule per sec.

One Horse power (H. P.) $=550$ ft-lbs. per sec.
$=550g$ foot-poundals per sec.

Since one foot $=12\times 2\text{·}54$ cms.
and one lb $=453\text{·}6$ gms.

$\therefore$ One Horse Power $=550\times 981\times 12\times 2\text{·}54\times 453\text{·}6$
$=746\times 10^7$ ergs per sec
$=746$ Watts.

Again, energy consumed = power × time. If a power of one watt be consumed for one hour the energy consumed is one watt × one hour or one watt-hour. One Kilowatt-hour = 1000 Watt-hours.

This Kilowatt-hour (or K.W.H.) is the unit used in electric consumption. This is also called Board of Trade Unit or B. O. T. unit or simply B. T. U. Thus, if a bulb marked 220 volts 40 watts be lighted for 25 hours the energy consumed $=40\times 25=1000$ watt-hours $=1$ B. T. U. If the cost of one unit (one B. T. U.) of electricity be 4 as, the cost of lighting such a bulb (40 watts) for 25 hours is 4 as or 25 pice, *i. e.* the cost per hour is one pice.

A dynamo supplies current to 200 25-candle power lamps. If each lamp absorbs 1·2 watts per candle and if the difference of potential at its terminals be 200 volts, find the current from the dynamo. Find also the cost of lighting these lamps for 12 hours, the cost of one B. T. U. being 6 as.

Power of each lamp $=25\times 1\text{·}2=30$ Watts.

$\therefore$ Current through each lamp $=\dfrac{30}{200}=\dfrac{3}{20}$ amp

$\therefore$ Current from the dynamo = Current through 200 lamps.

$$=\frac{3}{20}\times 200=30 \text{ amps.}$$

Again, Power consumed by each lamp $=30$ watts.

$\therefore$ Power consumed by 200 lamps $=200\times 30$ watts.

$\therefore$ Energy consumed by these lamps in 12 hours
$=200\times 30\times 12$ watt-hours
$=72$ Kilowatt-hours.

$\therefore$ Required cost $=72\times 6$ as $=$ Rs. 27.

Art 75 The heating effect of the current has been utilised in the industry in numerous ways. The most important is perhaps the construction of electric bulbs. Edison was the first* to place incandescent lamps on the market. He succeeded in preparing carbon filaments by carbonising bamboo threads and using them in glass bulbs. As carbon readily combines with oxygen at higher temperature it was necessary to evacuate the bulbs as completely as possible. Although carbon melts at a very high temperature (about 4200°C) its running temperature cannot be raised beyond 1865°C, as otherwise it rapidly disintegrates and blackens the walls of the bulb. A further disadvantage is that with increasing temperature its resistance diminishes; as a result, it is very sensitive to small fluctuations of the E. M. F. applied. Nowadays carbon filaments have been almost entirely replaced by metal filaments. After numerous trials tungsten has been found to be the most suitable metal for the purpose. Its melting point is as high as 3390°C and it can stand a running temperature of 2080°C. Usually such bulbs are hard bulbs, *i. e.* the inside air is pumped out. In certain bulbs however—specially in high power bulbs—air is replaced by some inactive gas such as nitrogen or argon. These bulbs last longer and the temperature of the filament can also be raised to as high as 2625°C with safety.

There is hardly any definite relation between the candle power of the light emitted and the power consumed by a bulb. Usually for a 'hard' bulb the candle power is somewhat lower than the number of watts consumed. Thus, for a bulb requiring 30 watts, the candle power is in the neighbourhood of 25. For gas filled lamps manufacturers claim that the candle power is double the number of watts, *i. e.* for half a watt the candle power is one. Thus for a gas-filled lamp consuming 75 watts, the candle power is believed to be about

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* Hanoverian teacher Heinrich Goebel was the first to invent carbon filament lamps in 1155. He however used them for his personal use.

150. Such lamps are therefore sometimes called half watt lamps.

Art 76 We may here profitably discuss different methods by which electric energy may be converted into light energy. The lamps which have so far been described in the preceding article are known as glow lamps; they depend upon generation of heat when an electric current passes through a wire. There are however other methods by which electric energy may produce luminosity.

Arc lamps. The earliest method is perhaps due to what is known as the electric arc. If two carbon rods forming part of an electric circuit are brought into contact and *then* separated to a distance of about 6 mm a luminous arc is formed between them, the arc itself being the conducting path between the two rods. After a short time the ends of the carbon rods become white hot and begin to emit light. Light is emitted more profusely from these ends than from the arc itself. In fact the end of the positive carbon contributes about 85%, that of the negative carbon about 10% and the arc itself only about 5% of the total light. Slowly a crater is formed at the end of the positive carbon while the negative carbon becomes gradually pointed. The temperature of the positive end is something like 3500°C to 4000°C while that of the negative only about 2500°C. Both the carbon rods are gradually consumed. The positive carbon is however consumed twice as fast as the negative. The cross section of the positive carbon is therefore made twice as large as that of the negative. When alternating* current is used both rods become pointed and are consumed at the same rate. In that case the rods are made of the same cross-section area.

The potential difference between the two carbon rods necessary to maintain an arc is given by the following relation:—

$$E = \left\{ a + bl + \frac{c + dl}{i} \right\} \text{ Volts.}$$

* Vide Chapter XVIII.

where a, b, c and d are constants l the length of the arc in mm and i the current in ampere. It is found that in the case of continuous current a minimum potential difference of about 44 volts is necessary.

As the carbon rods are gradually consumed the distance between the ends of the rods increases and there must be some means for bringing them closer. Ultimately the carbon rods have also to be replaced. The inconvenience due to this has made the arc lamp almost obsolete in modern times. A modification of this known as *Flame Arc* lamp is however sometimes used. In such lamps small cavities made at the ends of the carbon rods are filled with certain metallic salts. When the arc strikes these salts are gradually vaporised and this produces a brilliantly luminous flame. The Flame Arc lamp consumes only about 0 3 watts per candle and is therefore much more efficient than even gas filled lamps.

Art 77
Discharge lamps

In modern times the ionisation* of gases is being extensively utilised in producing luminous sources. If an electric discharge is passed through a gas at low pressure the gas becomes ionised and light characteristic of the gas is produced. The mercury vapour lamps seen on festive occasions contains a little mercury which when vaporised by the electric discharge produces a brilliant greenish blue light. Neon on the other hand gives rise to the characteristic red glow. The familiar "Neon signs" consist of long tubes containing pure neon at a pressure of the order of a few mm of mercury. Similarly if a tube be filled with sodium vapour brilliant yellow light is produced by the electric discharge through the tube.

In a modified form the electrodes are themselves small incandescent filaments. These filaments are however used

Fluorescent lamps

not as sources of light but as sources of electrons †. When these filaments are made incandescent by an electric current electrons are emitted and the presence of these electrons

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* Vide Art 201 † Vide Chapter XX

makes the tube conducting. The inside of the tube is coated with fluorescent substances. And when the discharge passes these begin to emit profuse quantity of light. Such lamps known as fluorescent lamps are nowadays extensively used in houses and also for street lighting. There is of course some initial cost for installing these lamps but the low rate of consumption makes them very popular

An electric kettle or an electric stove is another example of the application of the same principle. The conducting material consisting of thin iron or platinum wire surrounds the kettle proper ; the whole is covered on the outside with a protecting material.

In electric furnaces also the same principle is utilised. Thin platinum or iridium wires are wound round the porcelain furnace. Very high temperature can be obtained by passing large currents through these wires.

Art 78
Efficiency

In electric kettles or electric furnaces substances are heated at the expense of electric energy If there is no loss amount of heat energy obtained is equal to the amount of electric energy spent. There is however always some loss of energy due to conduction, convection etc.; as a result the amount of heat energy is always less. The efficiency of the instrument is defined to be

$$= \frac{\text{amount of heat energy utilised}}{\text{amount of electric energy spent}}$$

This efficiency is always less than one.

N B. In order to find this ratio both must be expressed in the same unit, viz. ergs or calories.

An electric kettle taking 3 amps at 220 volts, brings one litre of water from 18°C to the boiling point in 11 minutes. Find its efficiency. $(J = 4{\cdot}18 \times 10^7)$ *C. U. 1937*

Heat generated by electric current

$$= \frac{220 \times 3 \times 11 \times 60}{4{\cdot}18} \qquad \text{(Vide Art 73)}$$

$$= 104210{\cdot}5 \text{ calories.}$$

Heat utilised by heating the water

$$= 1000 \times (100 - 18) = 82000 \text{ calories}$$

$$\therefore \text{ efficiency} = \frac{82000}{104210\cdot5} = 0\cdot79 \text{ or } 79\%$$

An arc lamp requires 60 volts and 15 amperes. The current is supplied at 220 volts. Find the value of the resistance to be included in the circuit. What is the cost of running the lamp for 10 hrs at 20 pice per BTU. *C. U. 1969*

Resistance of the arc lamp $= \frac{60}{15} = 4$ ohms

If R be the additional resistance required when the arc runs on 220 volts the total resistance is $(R+4)$ ohms. The current should be 15 amperes.

Hence $\frac{220}{R+4} = 15 \quad \therefore R = 10.7$ ohms.

Number of BTU consumed in 10 hrs. $= \frac{220 \times 15 \times 10}{1000}$

$\therefore$ Cost $= \frac{220 \times 15 \times 10}{1000} \times 20$ pice $=$ Rs. 6.60

Art 79 Determination of Joule's equivalent (J) by electric method.

Callender & Barnes' apparatus is the most suitable for the purpose. A wire R usually made of platinum is enclosed in a glass tube through which there is a steady flow of water. A steady current i is made to pass through this wire. Heat generated in the wire produces a change of temperature in

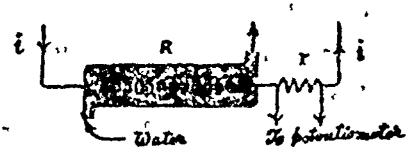

Fig. 122

the water as it passes through the tube. When the steady state has been reached the temperatures θ_1 and θ_2 of water at the entrance and at the exit, are carefully measured, preferably by a platinum resistance thermometer (Vide Art 110). The temperature of the apparatus being now steady

no heat is absorbed by any of its parts.

Water flowing through the apparatus in t secs is collected and weighed; let its mass be m gms. If E be the potential difference between the terminals of the wire R, the amount of heat generated in time t is $\frac{Eit}{J}$* calories. Hence if s be the average sp. heat of water between the temperatures θ_1 and θ_2 and if h be the amount of heat lost by radiation, we have

$$\frac{Eit}{J} = ms\,(\theta_2 - \theta_1) + h$$

The current i is measured by having a standard resistance r in series with R and by measuring the potential difference across r by means of a potentiometer (Vide Art 113). The potential difference E may be measured by an accurately calibrated voltmeter (Vide Art 93).

The amount of heat 'h' lost by radiation can be minimised by surrounding the glass tube by a vacuum jacket and then again by another constant temperature jacket. It may ultimately be eliminated by repeating the experiment with different values E′ and i' for the potential difference and the current, the rate of flow of water being however adjusted so that the temperatures θ_1 and θ_2 remain the same. Radiation loss which depends on temperatures is therefore kept constant. If m' be the new mass of water collected in time t we have a fresh equation

$$\frac{E'i't}{J} = m's\,(\theta_2 - \theta_1) + h$$

Subtracting this equation from the previous one,

$$\frac{(Ei - E'i')t}{J} = (m - m')\,s(\theta_2 - \theta_1)$$

Thus J may be determined.

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* In this expression E and i are expressed in C. G. S. units.

Exercise IX

1. A capacity 1 m.f is charged to 10,000 volts and suddenly discharged through a fine copper wire. If all the energy went to heating the wire, how many calories would be adiabatically liberated. C. U. 1943. Ans. 11·9 calories.

[1 m.f. $= 9 \times 10^5$ E. S. units of capacity ; 300 volts $=$ 1. E. S. unit of E. M. F.]

2. Two wires of resistances 20 ohms and 30 ohms respectively are connected in series to the terminals of battery. A third resistance r is to be used in parallel with 20 ohms resistance so that heat generated in 20 ohms resistance reduces to one-fourth of its former value. Find r. Ans 12 ohms.

3. What is watt ? How is it defined ?

A 10 ohm coil of wire is used to heat 1000 gms of water from 15°C to 100°C in 20 minutes How much current must be used ? What is the amount of power consumed ? If one B. T. U. costs 4 as, how much will you pay ?

Ans. 5·45 amp ; 297·5 watts ; 2·48 Pice

4. What is an arc lamp ? What are its advantages and disadvantages ? Describe other kinds of lamps that are in use in modern times.

5. A current of 3 amperes passes through a wire of resistance 0·5 ohm immersed in 100 gms of turpentine contained in a copper calorimeter of mass 95 gms. Calculate the rise in temperature produced in 5 minutes. [Sp. heat of copper $=$ 0·095 and Sp. heat of turpentine $=$ 0 42]. Ans 6 3°C

6. A current of 4 amperes passes through a wire of resistance 0·54 ohm immersed in water at 20°C. Ice is continuously added to water so that the temperature remains constant. If 18·5 gms of ice are required in 15 minutes find the value of J. [Latent heat of water $=$ 80 calories.]

Ans. $4{\cdot}20 \times 10^7$ ergs per calorie

7. A 50 volt lamp is to be run from a 220 volt supply. Explain how this can be done. If the current through the lamp be 0·54 amp. find how much power is taken from the supply and how much is actually consumed by the lamp.

Ans. 118·8 watts ; 27·0 watts.

8. In a house there are 15 20-candle power lamps each requiring 2·4 watts per candle. If these lamps are used on an average of 4 hours per day, calculate the cost of electrical energy consumed in a month of 30 days. One B.T.U. costs 5 as

Ans Rs 27.

9. In a local installation there are (1) 200 30-watt lamps (2) 300 15-watt lamps and (3) 10 1·5-H. P. electric fans. What is the horse power of the driving engine? What would be the cost of electrical energy consumed in a month of 30 days if the lamps and the fans are run on an average of 6 hours and 8 hours respectively in one day. [One B. T. U. costs 5 as.]

Ans. 29·1. H. P ; Rs 1429 14 as.

10. Determine the H. P. of the dynamo used to feed 40 220 volt 30 watt incandescent lamps. Determine also the total current supplied by the dynamo. [1 H. P. = 746 watts]

Ans 1·61 H. P. ; 5·45 amp.

11. How many 25 candle power incandescent lamps can be run by 1·2 H. P. dynamo, each lamp requiring 1·3 watts per candle? If the supply voltage be 220 what is the resistance of each lamp? Ans. 27 lamps. 1489 ohms.

12. Define electric power and efficiency and express the former in C. G. S. units.

A 10 H. P. engine is used to drive a dynamo supplying current to 200 30 watt lamps. What is the efficiency of the dynamo? Ans. 80·4%

13. A 1000 watt electrical kettle has an efficiency 85% How long does it take to raise one litre of water from 20°C to the boiling point? What is also the cost incurred if one unit is charged at 5 as? Ans. 6·59 mins : 0·55 as.

C.U. Question

1957. State Joule's Law for the rate of production of heat in a coil of wire carrying a current. What is a Joule and a watt?

Two resistances 20 and 40 ohms respectively can be connected to a source of E. M. F. Compare the rates of heat production in the two resistances when they are (a) connected

in series and (*b*) connected in parallel. Ans 1 : 2 and 2 : 1.

1964. State and explain Joule's law of generation of heat in an electric circuit.

Describe the experiment for measuring the mechanical equivalent of heat by electrical means.

A 660 watt electric heater is designed to operate from 220 volt main power source. What is the rate of generation of heat in calories/sec ? If the line voltage drops to 200 volts what power does the heater take ? (Change in heater resistance may be neglected) Ans. 157 1 cals/sec 545'5 watts.

1966 State Joule's law for production of heat in a coil of wire carrying a current. What is a Joule and a Watt ?

The heating element of an electric kettle has a resistance of 53 ohms. How long will it take to boil 1'5 kg of water at 15°C when connected to a 230 volt supply ? Neglect losses. (1 calorie = 4'18 Joules). Ans. 8 9 mins.

1968. Write notes on "Fluorescent Lamp".

1969. Deduce an expression for the heat developed in a conductor due to a flow of a steady current through it for a given time. Define Watt and Kilo-watt hour.

1973: Deduce the expression for the heat developed in a conductor due to the flow of a steady current through it for a given time. Define a Joule and a Watt.

The heating element of an electric heater has a resistance of 50 ohms. When it is immersed in 500 c.c. of water with a steady current of 2 amp. passing it for 10 minutes, the temperature rise is found to be 57.2 °C. Neglecting losses. calculate Joule's equivalent.

CHAPTER X

ACTION OF CURRENTS ON MAGNETS

Art 80 In the year 1820 Oersted of Copenhagen first demonstrated that a wire carrying a current produces a magnetic field in its neighbourhood.

Direction The *direction* of the field at any point is obtained by any of the following rules :—

(1) Ampere's rule : Suppose a man is swimming along the wire in the direction of the current with his face towards the point, then the direction of his left hand gives us the direction of the field at the point.

(2) Maxwell's Corkscrew rule : Imagine a cork screw driven along the wire in the direction of the current. The direction of rotation of the thumb gives us the direction of the field at any point.

Magnitude The *magnitude* of the field may be obtained from either of the following :—

(1) **Laplace's Law**

Let AB be a wire carrying a current i. At any point O of this wire consider an elementary length ds. Then the

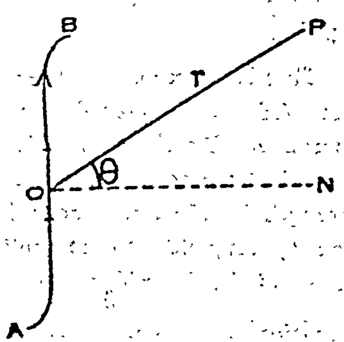

Fig. 123

field at any point P, due to the elementary current in ds, is given by

$$F = \frac{i\, ds \cos\theta}{r^2},$$

where OP $= r$ and θ is the angle between r and the normal to ds, the normal ON being drawn in the plane containing r and ds. By applying either of the two rules (for finding the direction) it may be seen that the direction of the field at P is perpendicular to the plane contained by ds and r ($=$ OP). The resultant field at P due to the whole of the current in AB may be found by integration.

(2) **Ampere's Theorem**

Any closed circuit carrying a current i is equivalent to a magnetic shell, the perimeter of which coincides everywhere with the wire, the strength of the shell being equal to the strength of the current viz. i.

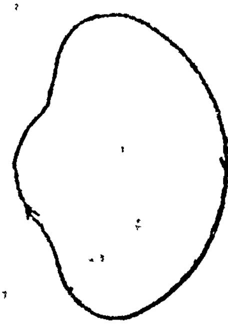

Fig. 124

The current being thus supposed to be replaced by the shell, the potential (magnetic) at any point due to the shell may be determined (Vide Art 10) ; and differentiating the potential, the intensity may be obtained.

(3) **Line Integral**

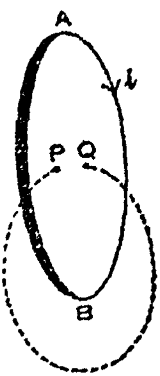

Fig. 125

This is an extension of Ampere's Theorem. Let AB be a closed circuit carrying a current i. By Ampere's Theorem this circuit may be replaced by a magnetic shell of strength i. If we now consider two points P and Q very close to the shell but on opposite sides of it, $V_P - V_Q = 4\pi i$ [Vide Art 10, equation (15)] *i. e.* the work done in carrying a unit North pole from P to Q $= 4\pi i$; it must be remembered however that the path along which the unit North pole is to be taken must nowhere cross the shell. Since the circuit carrying the

current is actually in existence and not the magnetic shell, the points P and Q may be made to approach each other until they coincide. In that case the path is a closed one linked with the circuit AB. Thus we have the law :—

Work done in carrying a unit North pole round a closed path once linked with the current i is equal to $4\pi i$.

If the path be linked n times with the circuit work done $= 4\pi n i$

Art 81 We shall consider a few examples illustrating the above laws.

1. Straight Current.

1st Method (a) BY LAPLACE'S LAW

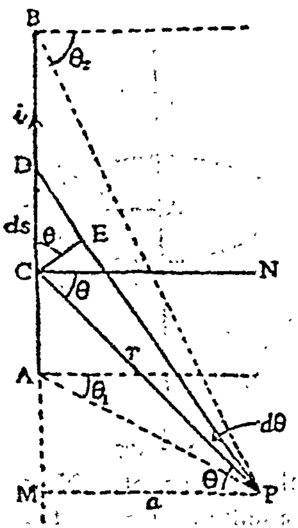

Fig. 128

Let AB be a straight wire carrying a current i. We are to find the magnetic field at any point P. Let CD be an elementary portion ds of the wire. Then the field at P due to the elementary current in ds

$$= \frac{i\, ds \cos\theta}{r^2}$$

where $PC^* = r$ and $\angle PCN = \theta$, CN being normal to CD. Join PC, PD. Drop PM perpendicular from P to AB (produced if necessary). Let it be equal to a. CE is an arc of a circle whose centre is P and radius equal to PC; CE may be regarded as normal both to PC and to PD.

Then $\angle MPC = \angle PCN = \theta$. $\therefore$ $\angle CPE = d\theta$

also $\angle DCE = \angle PCN = \theta$, for each is the complement of the $\angle NCE$.

$$\therefore \quad ds \cos\theta = CE = r d\theta \quad \text{and} \quad \frac{a}{r} = \frac{PM}{PC} = \cos\theta$$

* Since CD is infinitesimally small it does not matter whether we measure r from P to C or to D or to any other point in CD.

Hence $\quad \dfrac{i\, ds \cos\theta}{r^2} = \dfrac{i r d\theta}{r^2} = \dfrac{i\, d\theta}{r} = \dfrac{i \cos\theta\, d\theta}{a}$

$\therefore$ For the whole of the wire the intensity at P is

$$F = \int_{\theta_1}^{\theta_2} \frac{i}{a} \cos\theta\, d\theta = \frac{i}{a}\Big[\sin\theta\Big]_{\theta_1}^{\theta_2} = \frac{i}{a}(\sin\theta_2 - \sin\theta_1) \cdots (35)$$

If the wire be infinitely long $\theta_1 = -\dfrac{\pi^*}{2}$ and $\theta_2 = \dfrac{\pi}{2}$

$$\therefore \quad F = \frac{i}{a}\Big\{1-(-1)\Big\} = \frac{2i}{a} \qquad \cdots \qquad \cdots \qquad (35a)$$

(b) BY LINE INTEGRAL

2nd Method

Consider a point P at a distance a from an *infinitely long* straight wire carrying a current i. From P drop PA perpendicular to the wire. Then $AP = a$. With centre A and radius AP describe a circle in a plane perpendicular to the wire. From symmetry the intensity F at every point of this circle. is the same in magnitude and from Ampere's Swimming Rule or from Maxwell's Cork Screw

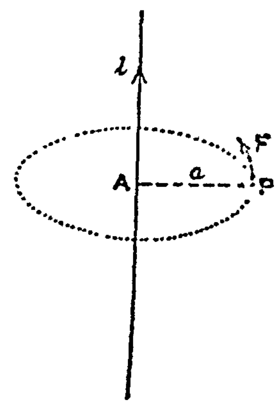

Fig. 127

Rule its direction is found to be tangential to the circle. Now intensity being equal to the force on a unit north pole, the work done in carrying a unit north pole round this circle

$$= \text{force} \times \text{distance} = 2\pi a F$$

But this circle is linked once with the current i.

$\therefore$ By the theorem of Line Integral

$$2\pi a F = 4\pi i \qquad \text{or} \qquad F = \frac{2i}{a}$$

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* When the wire extends to infinity in both directions, θ_1 lies on the other side of the normal and is therefore negative.

II. Circular current

Art 82 (1) INTENSITY AT THE CENTRE. BY LAPLACE'S LAW

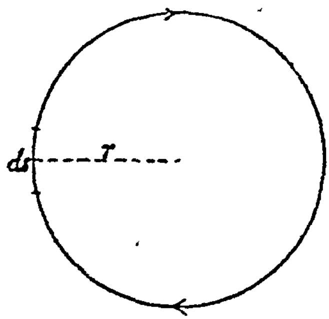

Fig. 128

In this case the radius r is normal to any elementary length ds of the circular wire. Hence the angle between r and the normal to ds, is zero. Thus from Laplace's Law field at the centre is given by

$$F=\Sigma\frac{i\,ds\cos\theta}{r^2}=\frac{i}{r^2}\Sigma\,ds$$

$$=\frac{i}{r^2}\times 2\pi r=\frac{2\pi i}{r}$$

If there be n turns of the wire

$$F=\frac{2\pi n i}{r} \qquad \ldots \qquad \ldots \qquad (36)$$

The direction of the field is normal to the plane of the coil and if the current flows clockwise as shown in Fig. 128 field is towards the plane of the paper.

(2) INTENSITY AT ANY POINT ON THE AXIS

(a) BY LAPLACE'S LAW.

1st Method

Let AB represent a circular current of radius r perpendicular to the plane of the paper. Let P be any point on the axis at a distance x from the centre O. Consider an elementary length ds of the wire at A. Obviously PA is perpendicular to the element. Hence by Laplace's law the intensity at P due to this elementary current $=\frac{i\,ds}{PA^2}=\frac{i\,ds}{r^2+x^2}$.

But this intensity acts in a direction perpendicular to the plane contained by ds and AP and hence perpendicular to AP. Resolving this along OP and perpendicular to OP, we notice that the component perpendicular to OP is equal and opposite to the corresponding component due to an elementary current

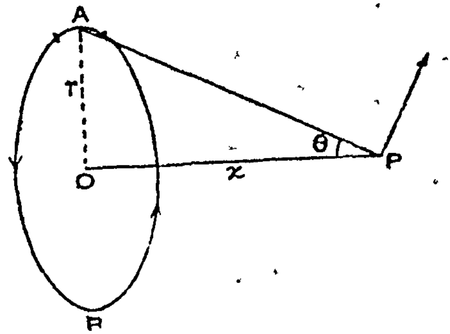

Fig. 129

at the opposite end of the diameter through A. Thus if we consider the field at P due to the entire circle the components at P perpendicular to OP cancel one another. Hence we need consider only the component along OP.

Due to the elementary current at A

$$\text{this component} = \frac{i\,ds}{r^2+x^2}\sin\theta = \frac{i\,ds}{r^2+x^2}\cdot\frac{r}{\sqrt{r^2+x^2}}$$

$$= \frac{ir\,ds}{(r^2+x^2)^{3/2}}$$

Hence for the whole of the circle the intensity at P is

$$F = \Sigma\frac{ir\,ds}{(r^2+x^2)^{3/2}} = \frac{ir}{(r^2+x^2)^{3/2}}\cdot\ \Sigma ds$$

$$= \frac{2\pi r^2 i}{(r^2+x^2)^{3/2}}$$

If there be n turns of the wire

$$F = \frac{2\pi n r^2 i}{(r^2+x^2)^{3/2}} \quad \dots \quad \dots \quad \dots \quad (37)$$

To an observer at P the current as shown in Fig. 129 appears *anti-clockwise*; the field at P due to this current is along the axis and is directed *away from the coil.*

(b) BY AMPERE'S THEOREM

2nd Method Let AB represent a circular coil perpendicular to the plane of the paper. Let P be any point on the axis at a distance x from the centre O of the

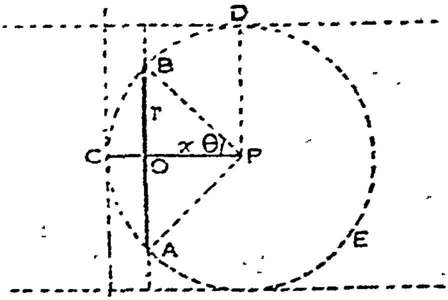

Fig. 130

coil. We replace the circular current by the corresponding magnetic shell. By Ampere's theorem the strength of this magnetic shell is equal to i the strength of the current. Hence

Potential at $P = i \cdot$ solid angle APB* [Vide (14) Art 10]

As deducted in (15a) Art 10 (a) this potential is given by

$$V = 2\pi i \left\{ 1 - \frac{x}{\sqrt{x^2+r^2}} \right\}$$

The potential may also be determined independently if we find out the solid angle by utilising the following theorem :

Area of a sphere intercepted between any two parallel planes = area of the circumscribing cylinder between the same two planes, the axis of the cylinder being perpendicular to the planes.

---

* AB is a circular coil and P is an external point on the axis of AB. If P be joined by straight lines to every point on the circle AB then these lines form a cone. The solid angle APB of this cone is the solid angle subtended by the coil at P.

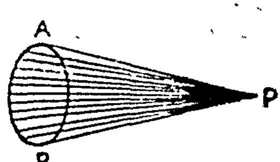

Fig. 131

We draw the sphere ACDE with centre P and radius PB. The area ACB of this sphere intercepted between the plane AB and the tangent plane at C = corresponding area of the circumscribing cylinder between the same two planes. $= 2\pi PD.OC$

Hence the potential at P

$$V = i \times \text{solid angle } APB$$

$$= i \times \frac{\text{area } ACB}{PB^2} = i \times \frac{2\pi PD.OC}{PB^2}$$

$$= 2\pi i \times \frac{OC}{PB} = 2\pi i \times \frac{PC - OP}{PB}$$

$$= 2\pi i \left\{ 1 - \frac{OP}{PB} \right\} = 2\pi i \left\{ 1 - \frac{x}{\sqrt{r^2 + x^2}} \right\}$$

$\therefore$ Intensity at P

$$F = -\frac{dV}{dx} = 2\pi i \frac{\sqrt{r^2 + x^2} - x\,\frac{1}{2}\,(r^2 + x^2)^{-\frac{1}{2}}\,2x}{r^2 + x^2}$$

$$= 2\pi i \frac{\sqrt{r^2 + x^2} - \dfrac{x^2}{\sqrt{r^2 + x^2}}}{r^2 + x^2} = 2\pi i \frac{r^2 + x^2 - x^2}{(r^2 + x^2)^{3/2}}$$

$$= \frac{2\pi r^2 i}{(r^2 + x^2)^{3/2}}$$

For n turns of the wire $\qquad F = \dfrac{2\pi n r^2 i}{(r^2 + x^2)^{3/2}} \qquad \ldots \quad (37)$

Art 83. III. Solenoidal Current.

1st Method.

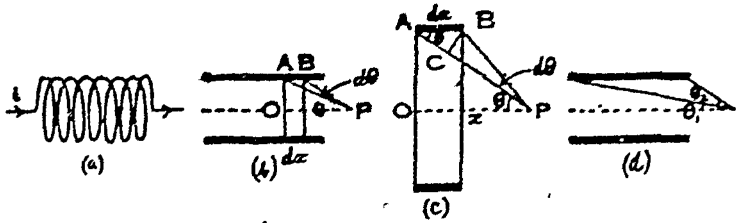

Fig. 182

The solenoid shown in Fig 182 (a) is represented as a continuous tube in Fig 182 (b).

Consider a solenoid of radius r carrying a current i. Let AB be an elementary length dx of this solenoid. If the number of turns per unit length be n this elementary length contains ndx turns. This elementary portion may therefore be regarded as a circular coil of ndx turns with centre O and radius equal to r. Let P be any point on the axis at a distance x from O.

Intensity at P due to the element AB

$$=\frac{2\pi nr^2 idx}{(r^2+x^2)^{3/2}}$$

From the magnified diagram Fig 132 (c),

$$dx \sin\theta = BC = BP\, d\theta = \sqrt{r^2+x^2}\, d\theta$$

$$\therefore \quad \frac{dx}{\sqrt{r^2+x^2}}=\frac{d\theta}{\sin\theta} \quad \text{and} \quad x=r\cot\theta.$$

$\therefore$ Intensity at P due to the element AB

$$=\frac{2\pi nr^2 i}{r^2+x^2}\cdot\frac{dx}{\sqrt{r^2+x^2}}=\frac{2\pi nr^2 i}{r^2+r^2\cot^2\theta}\cdot\frac{d\theta}{\sin\theta}$$

$$=\frac{2\pi nr^2 i}{r^2\operatorname{cosec}^2\theta}\cdot\frac{d\theta}{\sin\theta}=2\pi ni\sin\theta d\theta$$

If θ_1 and θ_2 be the limiting values of θ for the ends of the solenoid [Vide Fig 132 (d)] the resultant intensity at P

$$=\int_{\theta_1}^{\theta_2} 2\pi ni\sin\theta\, d\theta=-2\pi ni\Big[\cos\theta\Big]_{\theta_1}^{\theta_2}$$

$$=2\pi ni(\cos\theta_1-\cos\theta_2).$$

If the solenoid be infinitely long so that the point P is well within the solenoid, $\theta_1=0$ and $\theta_2=\pi$. Hence $\cos\theta_1=1$ and $\cos\theta_2=-1$.

$\therefore$ the intensity at any point within a long solenoid

$$=2\pi ni\,[1-(-1)]=4\pi ni \qquad \ldots \qquad (38)$$

N. B. (1) Here n represents the number of turns *per unit length* and not the total number of turns.

(2) This intensity is independent of the radius r.

(3) If we consider a point outside the solenoid, on the axis produced and if to an observer at this point, the current in the solenoid appears *clockwise*, then the intensity at the point is directed *towards the solenoid*, *i. e.* the solenoid carrying a current is equivalent to a magnet with the *South pole* nearer the observer to whom the current appears *clockwise*.

Consider an endless solenoid, *i. e.* a solenoid wound in the form of a ring of radius a. Consider a path along the axis of this solenoid. From symmetry the intensity F is the same at every point of this path and is everywhere directed along the path. Hence the work done in carrying a unit North pole round the path $= F.\ 2\pi a$. But if n be the number of turns per unit length of the solenoid the total number of turns linked with the path $= 2\pi an$.

2nd Method

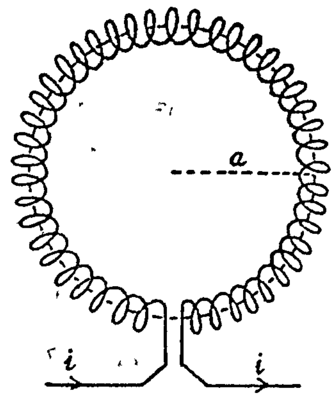

Fig. 133

$\therefore$ by the theorem of Line Integral

$$F.\ 2\pi a = 4\pi.\ 2\pi an.\ i$$
$$\text{or} \qquad F = 4\pi ni$$

But this result is independent of the radius a, *i. e.* it holds good for *all* values of a. If we now suppose that a is infinitely large, the ring solenoid becomes an infinitely long straight solenoid. Hence the intensity at any point within an infinitely long straight solenoid $= 4\pi ni$

Art 84 Tangent Galvanometer.

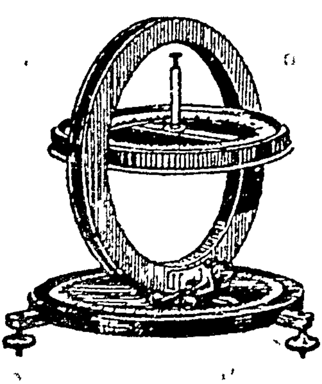

Fig. 184

Description. A tangent galvanometer essentially consists of a vertical circular coil of wire of a number of turns, at the centre of which a *short* magnetic needle is pivoted. A light but fairly long pointer usually made of aluminium. is rigidly attached at right angles to the needle. The pointer rotates on a horizontal graduated circle. The instrument rests on a suitable base provided with three levelling screws.

Adjustment. The instrument is first levelled ; the coil is then rotated until the plane of the coil comes in the magnetic meridian, *i.e.* the magnetic needle and the coil lie in one plane. This is done in order to make the field due to the current perpendicular to that due to the Earth, as otherwise the formula as deduced below breaks down.

Theory. If a current i is now passed through the coil the intensity at the centre due to this current is $F=\frac{2\pi ni}{r}$ and this is at right angles to the coil, where $n=$ no. of turns and $r=$ radius of the coil. [Vide (36) Page 179]

The intensity due to the Earth's horizontal component is directed along the magnetic meridian, *i. e.* is parallel to the coil.

The magnetic needle is therefore acted on by two fields, F due to the current and H due to the Earth at right angles to each other. If the resultant field make an angle θ with H we have

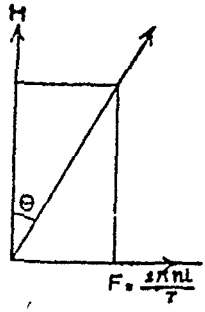

Fig. 185

$$\tan\theta=\frac{F}{H}=\frac{2\pi ni}{rH}$$

or $$i=\frac{rH}{2\pi n}\tan\theta=\frac{H}{G}\tan\theta \quad \ldots \quad \ldots \quad (39)$$

where $$G=\frac{2\pi n}{r}$$

$\therefore$ on passing the current the needle rotates through an angle θ given by the equation (39).

The quantity $\frac{H}{G}$ is called the reduction factor and G is called the constant of the galvanometer.

N.B. In the above theory we have tacitly assumed that intensity at the centre of the coil is the same as that at the poles of the magnetic needle. This is approximately true only when the size of the needle is extremely short in comparison to the radius of the coil. Hence the necessity of the shortness of the needle.

Writing k for $\frac{H}{G}$, $\quad i = k \tan\theta \quad \ldots \quad \ldots \quad$ (39a)

$\therefore \quad \log i = \log k + \log \tan\theta.$

Hence, differentiating,

$$\frac{di}{i} = \frac{\sec^2\theta d\theta}{\tan\theta} = \frac{d\theta}{\sin\theta\cos\theta} = \frac{2d\theta}{\sin 2\theta}$$

$\frac{di}{i}$ represents the proportional error in measuring the current i; this is minimum when $\sin 2\theta$ is maximum, *i. e.* when $2\theta = 90°$ or $\theta = 45°$

Art 85 Sine Galvanometer.

This instrument is essentially the same as the tangent galvanometer; the only difference is that in this case the vertical coil can also be rotated about a vertical axis and the amount of rotation can be measured by another graduated horizontal circle attached to the base.

To use it the instrument is levelled and the vertical coil is brought to magnetic meridian as in the case of a tangent galvanometer. When the current is passed the needle is deflected. This deflection however is not measured. Instead the vertical coil is rotated in the direction of the deflection of the needle. The deflection of the needle thereby increases but at a much slower rate; ultimately the coil overtakes the needle, *i. e.* the coil and the needle again lie in the same plane. The deflection ϕ of the coil from the initial position in the magnetic meridian is now measured. The needle is now acted on by two fields,—H along the magnetic meridian and F perpendicular to the coil. Since the needle is parallel to the coil the resultant of these two fields lies in the plane of the coil.

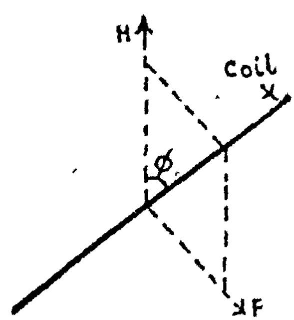

Fig. 138

Hence $$\sin\phi = \frac{F}{H} = \frac{2\pi n i}{rH}$$

or $$i = \frac{rH}{2\pi n}\sin\phi = k\sin\phi \qquad \ldots \qquad (39b)$$

Since the maximum value of $\sin\phi$ is one the maximum current that can be measured by a sine galvanometer is k. There is however no such restriction in the case of a tangent galvanometer.

Art 86
Sensitiveness

In both sine and tangent galvanometers the instrument is sensitive if k is small; for, in that case θ (or ϕ) is fairly large even if the current is small. But $k = \frac{rH}{2\pi n}$. To make k small we have to reduce r & H and increase n. There is a limit to which each of these can be adjusted. In the case of the radius r the limit is reached when the size of the needle becomes appreciable in comparison to r. n also cannot be increased indefinitely as otherwise r gradually begins to increase along with n; moreover the resistance of the coil is also unduly increased so that not much advantage is gained thereby. The external couple due to H can however be reduced to a very large extent by the use of an astatic pair of needles.

Astatic pair

If two magnetic needles of very nearly equal strengths m and m' be *rigidly* joined together so that they are parallel to each other but the

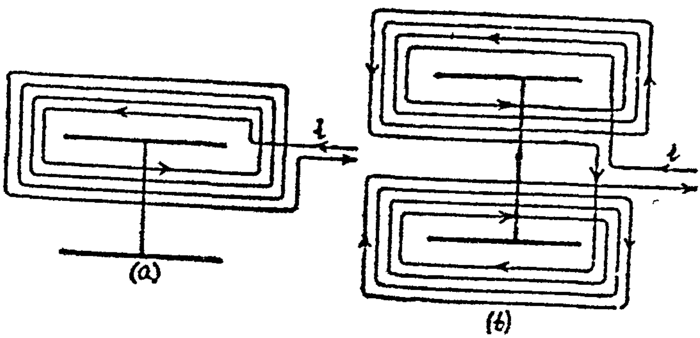

Fig. 137

north pole of one points the same way as the south pole of the other then the combination is said to be an astatic pair.

The horizontal component H of Earth's intensity acts on both the needles ; whereas if the coil be wound round one of the needles [as in Fig. 137 (*a*)] the intensity F due to the current acts only on one needle of pole strength m. Thus the resultant force due to H is $(m-m')$H but that due to F is mF If the resultant of these two forces makes an angle θ with H we have

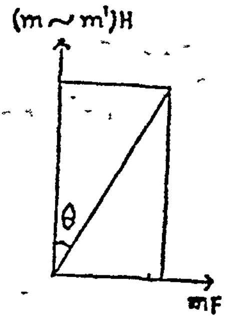

Fig. 138

$$\tan\theta=\frac{mF}{(m-m')H}=\frac{2\pi nmi}{(m-m')rH}$$

$$\text{or}\quad i=\frac{rH}{2\pi n}\,\frac{m-m'}{m}\tan\theta \qquad \ldots \quad (39c)$$

On passing the current the needle rotates through an angle θ given by (39*c*). The reduction factor is now $\frac{rH}{2\pi n}\frac{m-m'}{m}$ Clearly this can be reduced to a very large extent by making m nearly equal to m'

If the coil be wound round both the needles but in such a way that the current passes round them in opposite directions [as in Fig 137(*b*)] the force due to the current becomes $(m+m')$F ; the reduction factor is altered to $\frac{rH}{2\pi n}\,\frac{m\sim m'}{m+m'}$ Clearly this is much less than what it is in the first case.

A galvanometer in which an astatic pair of needles is used is known as an astatic galvanometer. Essentially it is a tangent galvanometer with its reduction factor very much reduced.

Art 87
Helmholtz's galvanometer

As we have seen the theory of tangent galvanometer so far discussed depends upon the assumption that the needle rotates in a uniform field of strength F. This is approximately true only when the size of the needle is small in comparison to the

radius of the coil. This necessitates the use of a short magnetic needle.

In Helmholtz's tangent galvanometer this defect is removed by using two parallel coils and having the needle midway between the two. The two coils are exactly similar but are so wound that the intensities at the mid-point C due to the two coils are in the same direction, *i. e.* the resultant intensity at C is twice that due to a single coil.

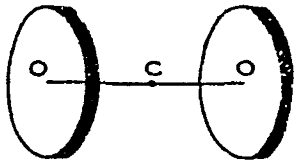

Fig. 189

Theory. Let the two coils be each of n *turns* and let the needle be placed at C midway between the two centres O and O′, at a distance x from either of them. The coils are wound in such a way that at C the field due to the two coils are in the same direction and the resultant field is therefore the sum of the two. Let r be the radius of either of the coils. Then the intensity at C due to one of the coils is given by

$$F = \frac{2\pi n r^2 i}{(r^2+x^2)^{3/2}} \qquad \text{[Vide (37), Page 180]}$$

As we pass from C towards either of the coils the field due to one increases and that due to the other decreases. Uniformity of the field in the space near about C is therefore secured if the rate of increase of the field due to one of the coils is equal to the rate of decrease of the field due to the other, *i. e.* if the rate of variation of the field is constant at C.

i. e. if $\dfrac{dF}{dx} = \text{const. at C}$, *i. e.* if $\dfrac{d^2F}{dx^2} = 0$ at C.

Now $F = \dfrac{2\pi n r^2 i}{(r^2+x^2)^{3/2}} = A(r^2+x^2)^{-\frac{3}{2}}$ writing A for $2\pi n r^2 i$

$$\therefore \quad \frac{dF}{dx} = A\left(-\tfrac{3}{2}\right)(r^2+x^2)^{-\frac{5}{2}}2x = -3Ax(r^2+x^2)^{-\frac{5}{2}}$$

$$\therefore \quad \frac{d^2F}{dx^2} = -3A\left\{(r^2+x^2)^{-\frac{5}{2}} + x(-\tfrac{5}{2})(r^2+x^2)^{-\frac{7}{2}}2x\right\}$$

$$= -3A\left\{(r^2+x^2)^{-\frac{5}{2}} - 5x^2(r^2+x^2)^{-\frac{7}{2}}\right\}$$

Since $\frac{d^2F}{dx^2} = 0, \quad \therefore \quad (r^2+x^2)^{-\frac{5}{2}} = 5x^2(r^2+x^2)^{-\frac{7}{2}}$

or $\quad r^2+x^2 = 5x^2$ multiplying both sides by $(r^2+x^2)^{\frac{7}{2}}$

or $\quad r^2 = 4x^2 \ \therefore \ x = \frac{r}{2}$ or $2x = r$

Hence in a Helmholtz's galvanometer the coils are separated by a distance equal to the radius of either of the coils and the needle is placed midway between the two.

The resultant field at C due to the two coils is given

by $$F = 2 \times \frac{2\pi n r^2 i}{(r^2+x^2)^{3/2}} = \frac{4\pi n r^2 i}{\left(r^2+\frac{r^2}{4}\right)^{\frac{3}{2}}}$$

$$= \frac{4\pi n r^2 i}{\left(\frac{5r^2}{4}\right)^{\frac{3}{2}}} = \frac{32\pi n r^2 i}{5r^3\sqrt{5}} = \frac{32\pi n i}{5r\sqrt{5}}$$

Hence if θ be the deflection of the needle at C we have

$$\tan\theta = \frac{F}{H} \text{ [As in Art 84 ; Fig. 135]}$$

$$= \frac{32\pi n i}{5rH\sqrt{5}}$$

$$\therefore \quad i = \frac{5rH\sqrt{5}}{32\pi n}\tan\theta = k\tan\theta$$

$$\text{where } k = \frac{5rH\sqrt{5}}{32\pi n}$$

This 'k' is called the reduction factor of the instrument.

This may be determined by passing a known current i through the instrument and observing the corresponding deflection θ.

N. B. Helmholtz's galvanometer is also essentially a tangent galvanometer.

A tangent galvanometer consists of two equal circular coils, each of one turn and of radius 9 cm placed at 24 cm apart on a common axis, the needle being on the axis midway between them. Find the galvanometer constant. *C. U. 1955*

$$\text{Field due to two coils} = 2 \times \frac{2\pi n r^2 i}{(r^2+x^2)^{3/2}}$$

$$= 2 \times \frac{2\pi \times 1 \times 9^2 i}{(9^2+12^2)^{3/2}} = \frac{12\pi i}{125}.$$

$$\frac{12\pi i}{125 \text{ H}} = \tan\theta \qquad \therefore i = \frac{125 \text{ H}}{12\pi}\tan\theta = \frac{\text{H}}{\text{G}}\tan\theta$$

$$\text{where G} = \frac{12\pi}{125}.$$

i.e. the galvanometer constant is $\frac{12\pi}{125}$

Exercise X.

1. State Laplace's Law for the field at a point due to a short element of current.

Obtain an expression for the field at the centre of a circular coil; hence deduce the formula for the tangent galvanometer.

2. Obtain an expression for the field at a point near a long straight wire carrying a current.

A current of 6 amps passes vertically upwards along a straight wire. Find the point where the field due to this current balances Earth's Horizontal Component. [H = 0·18].

Ans. At a point $6\frac{2}{3}$ cms west of the wire.

3. State the law expressing the equivalence of a circuit carrying a current and a magnetic shell. Hence or otherwise prove that the work done in carrying a unit North pole round a closed circuit once linked with a current i, is equal to $4\pi i$.

4. A current passes through a tangent galvanometer and

produces a deflection of $30°$. If the radius and the number of turns of the coil be 15 cms and 50 respectively find the strength of the current. [H = 0·18 C. G. S. unit.] Ans. 0·05 amp.

5. Explain the principle of the tangent galvanometer. Why is it necessary to use as small a suspended magnet as possible in the tangent galvanometer.

A battery, a resistance R and a tangent galvanometer are in series. If 500 turns of the coil of the galvanometer are used the deflection is $45°$. What would be the deflection if 50 turns of the same coil are used? [500 turns of the coil have a resistance 8R.] Ans. $26°34'$

[Hints :—If E be the E. M. F of the battery and r the radius of the coil,

$$\frac{E}{8R+R}=\frac{rH}{2\pi\times 500}\tan 45°$$

and

$$\frac{E}{\frac{8R}{10}+R}=\frac{rH}{2\pi\times 50}\tan\theta$$

Hence by division θ may be found out].

6. Explain how a tangent galvanometer may be used as a sine galvanometer.

When a certain current passes through a tangent galvanometer the deflection is $45°$. The plane of the coil is rotated until it makes an angle of $45°$ with the magnetic meridian. What would be the deflection if the same current now passes through the galvanometer? Ans. $67° 30'$

7. A current of 0·02 amp produces a deflection of $60°$ in a tangent galvanometer. If the radius of the coil be 20 cms, find the number of turns. What would be the deflection by half the current, if the instrument be used as a sine galvanometer? [H = 0·18 C. G. S. unit] Ans. 496 ; $59° 58'$

8. What is meant by the constant of a tangent galvanometer and what is its reduction factor? Explain why in determining the reduction factor of a tangent galvanometer, the deflection is made nearly $45°$.

If due to a certain current the deflection produced in a tangent galvanometer be $60°$ find what would be the deflection

when half the current passes through the instrument.

Ans. $40^\circ 54'$

9. What is an astatic pair of needles? Describe two methods of using an astatic pair in a tangent galvanometer. What is the advantage of using an astatic pair?

10. Deduce an expression for the magnetic field at a point on the axis of a circular current.

A current of 0·5 amp passes round a circular coil of diameter 20 cms. If the number of turns of wire in the coil be 50 find the intensity at a point on the axis at a distance of 10 cms from the centre. Ans. 0·56 C. G. S. unit

11. Explain the theory of Helmholtz's tangent galvanometer. In what respect is it superior to an ordinary tangent galvanometer?

12. In a Helmholtz's tangent galvanometer the coils are each of radius 10 cms and of 200 turns. If a current of 0·02 amp passes round the coils find the intensity at a point on the axis midway between the coils. What would be the deflection of a magnetic needle placed at this point? [H = 0·36 C. G. S. unit].

Ans. 0·36 C. G. S. unit ; 45°

13. Obtain an expression for the magnetic field at a point well within a solenoidal current.

A solenoid 20 cms long is of 800 turns. If a current of 0·25 amp. passes round the solenoid find the field at a point well within the solenoid. Ans. 12·57 C. G. S. unit.

14. How is the field at any point within a solenoidal current affected by the variation of (*a*) the diameter (*b*) the length and (*c*) the total number of turns of the solenoid?

A solenoid 60 cms long is wound with three layers of wire of 800 turns each. If a current of 2 amps. flows through the solenoid find the field at a point near the centre. Ans. 32π

[Hints : Here $n = \dfrac{800 \times 3}{60} = 40$].

C. U. Questions.

1963. Find the deflection of a magnetic needle when a current of 0·5 amp passes through a Helmholtz's tangent galvanometer in which each of the two coils has 100 turns of 20 cms mean radius. [H = 0·32 C. G. S unit]. Ans 81° 54′.

1963 Derive an expression for the magnetic field intensity at a point on the axis of a circular turn of wire carrying an electric current. Describe and explain the principle of operation of a Helmholtz double coil galvanometer and indicate its advantage over the single coil type.

1966. Find the intensity of the magnetic field at a point on the axis of a circular coil carrying a current. Describe an instrument where this principle is used for the measurement of current.

Find the field strength at the centre of a short coil 15 cms in diameter containing 10 turns and carrying a current of 10 amps. Ans. 20π C. G. S unit.

1969. Obtain an expression for the magnetic intensity at a point on the axis of a circular coil of wire carrying an electric current. Deduce therefrom an expression for the reduction factor of a Helmholtz's double coil galvanometer.

1971, 1974. Derive an expression for the magnetic field at any point along the axis of a circular coil carrying a steady curre[illegible] curr

[illegible]se the construction and principle of action of a tangent galva[illegible]meter. What is meant by the reduction factor of a tangent galvanometer.

1972. Describe a Helmholtz double coil galvanometer and explain its principle of action.

How is the reduction factor of the instrument determined in the laboratory?

1975. Find the value of the intensity of the magnetic field at any point along the axis of a circular coil carrying a steady current.

Describe a Helmholtz double-coil galvanometer and *explain* [illegible]

CHAPTER XI

ACTION OF MAGNETS ON CURRENTS
ACTION OF CURRENTS ON CURRENTS

Art 88. Laplace's law states that the magnetic field at P due to an element of the current at O is given by $F = \frac{ids \cos\theta}{r^2}$ where $OP = r$ and θ is the angle between r and the normal to ds. If therefore there be a pole of strength m at P the force on the pole due to the elementary current $= mF = \frac{mids \cos\theta}{r^2}$. Hence by Newton's

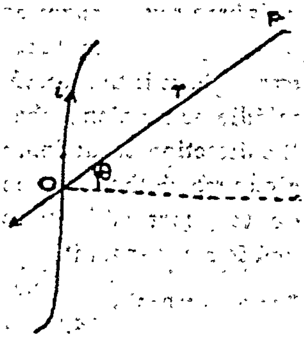

Fig. 140

third law the force on the elementary current at O, due to the pole m at P is $\frac{mids \cos\theta}{r^2}$. But $\frac{m}{r^2}$ is the field H at O due to the pole m, the line of force at O being along r. Thus the force on an elementary current is $Hids \cos\theta$, where H is the field in which the current is placed and θ is the angle between the direction of the field and the normal to ds, the normal being taken in the plane containing ds and the line of force at ds.

Straight current perpendicular to the magnetic field.

Let us consider a straight current i at right angles to a magnetic field of strength H. In the plane containing the current and the lines of force if a normal be drawn to the current, the angle between the normal and the direction of the field is zero. Thus in this case $\theta = 0°$ and $\cos\theta = 1$. $\therefore$ the force on the current $= Hil$ where l is the length of the wire carrying the current.

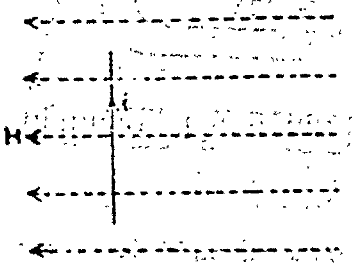

Fig. 141

N. B. If the current be along the field the force on the current is zero, because in this case $\theta=90°$ and $\cos\theta=0$.

The direction of the force is best given by Fleming's *Left* Hand Rule :—Stretch the thumb, the forefinger and the middle finger of your *left* hand so that each is perpendicular to the other two. Then if the forefinger be directed along the field, the middle finger along the current i, then the thumb represents the direction of motion, *i. e.* the thumb indicates the direction along which the force is acting. Thus in Fig. 141 the force on the current is perpendicular to the plane of the paper and is away from it.

Left hand rule

Barlow's wheel

The left hand rule is best illustrated experimentally by Barlow's wheel. A toothed wheel capable of rotating round the centre is so placed that as the wheel rotates different teeth dip into a cup of mercury one after another. A horse shoe magnet is placed in such a way that the two poles are on either side of the vertical tooth dipping into mercury. The terminals of a cell are connected to

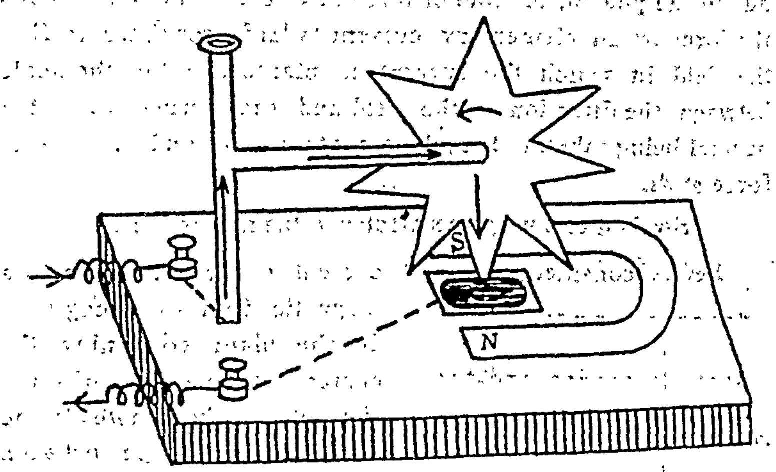

Fig. 142

the centre of the wheel and mercury as shown in the figure so that a current flows along the vertical tooth. Let us suppose that the North pole is on the front side and the current flow

downwards along the vertical tooth. By applying the Left Hand Rule it can be easily seen that the force on the vertical tooth is towards the right. The wheel therefore rotates in the direction of the arrow. As successive teeth come into contact with mercury the current flows through them and by the action of the magnetic field the wheel goes on rotating. If the direction of the current be reversed the wheel rotates in the opposite direction.

Art 89
Suspended coil type galvanometer

The above principle is uitilised in the construction of a suspended coil galvanometer. This instrument—also known as D'Arsonval galvanometer—essentially consists of a coil of wire suspended between the two poles of a horse shoe magnet. A spiral of wire at the bottom and the suspension fibre at the top are the two leads by which the current is made to pass through the coil. The suspension fibre is usually made of a thin strip of phosphor-bronze. Phosphorbronze is used because it is conducting, is not easily oxidisable and has a tensile strength approaching that of steel ; a strip is preferable to a wire because in that case the cooling surface is greater.

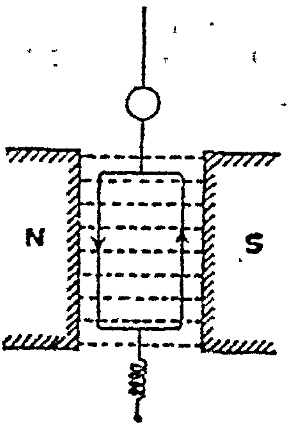

Fig. 143

Initially the plane of the coil is made parallel to the lines of force of the horse-shoe magnet. If the coil is rectangular—as is usually the case—the two horizontal wires lie along the magnetic field and therefore experience no force [Art 88]. On each of the two vertical wires, however, a force equal to Hil acts where H is the strength of the magnetic field, i the current and l the length of the coil. The currents in the two wires being oppositely directed the forces on them are also opposite in direction ; the coil therefore experiences a couple of moment

$= Hil \times b$ where b is the breadth of the coil.

$= AHi$, A being the area of the coil.

If there be n turns of the wire the total moment is $nAHi$.

By the action of this couple the coil rotates; the suspension fibre therefore becomes twisted. Equilibrium takes place when the couple due to twisting of the suspension fibre becomes equal to that due to the magnetic force. In the equilibrium position if θ be the angle through which the coil is rotated the moment of the couple due to twisting of the suspension fibre $= \mu\theta$, where μ is the coefficient of torsion; and since in this position the arm of the couple (due to magnetic force) is reduced to $b \cos \theta$, the moment of the couple $= nHil \times b \cos \theta = nAHi \cos \theta$

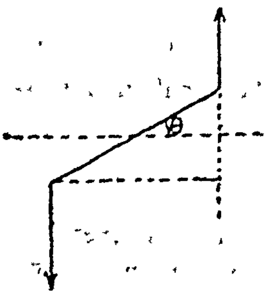

Fig. 144

$$\text{Hence} \qquad nAHi \cos \theta = \mu\theta$$

$$\text{or} \qquad i = \frac{\mu}{nAH \cos \theta}\theta$$

Usually θ is very small so that $\cos \theta$ may be taken to be equal to one.

$$\therefore \quad i = \frac{\mu}{nAH}.\theta$$

$$= k\theta \quad \text{where } k = \frac{\mu}{nAH} \qquad \cdots \qquad (40)$$

The current is thus proportional to the angle of rotation θ. To measure θ a small mirror is attached to the suspension fibre. A ray of light reflected by this mirror is incident on a scale. When the coil rotates the spot of light on the scale is displaced and θ is measured by this displacement. A galvanometer fitted with such a reflecting mirror is known as a mirror galvanometer.

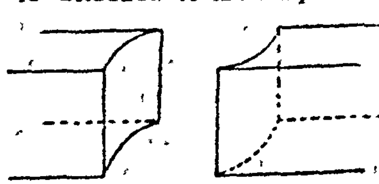

Fig. 145

The current necessary to produce a deflection of 1 *mm* on a scale placed at a distance of 1 metre from the galvanometer mirror, is known as the *figure of merit* of the galvanometer.

In modern galvanometers the pole pieces are almost invariably cylindrical in shape. [Vide Fig. 145] Further, a soft iron cylinder (of the same shape as the poles) is placed within the coil and is kept fixed in position, even when the coil rotates. This makes the field concentrated and radial. In this case, since the lines of force are radial, for all positions of the coil the couple is $nAHi$. Hence the assumption that θ is small and $\cos\theta$ is one, is not necessary.

Art 90 Ballistic and dead-beat Galvanometers A steady current when passed through a galvanometer produces a steady deflection and the amount of deflection gives us a measure of the strength of the current. If however the current is of very short duration (*e. g.* produced by the discharge of a condenser) there is no steady deflection but the coil receives a sort of impulsive blow and is jerked out of the initial position by the sudden passage of the current; but since the current ceases to exist long before the deflection becomes maximum, the coil tends to come back and after a few oscillations, is again brought to rest in its original position. A current existing for a short time means a quantity of charge passing through the galvanometer; the first maximum 'throw' or deflection of the coil gives us a measure of this quantity of charge. In such cases it is absolutely necessary that the damping should be made as small as possible. In suspended coil type instruments this damping is mainly electromagnetic in character and depends upon the nature of the framework round which the coil is wound. If the framework be made of a conducting material, as the coil rotates an induced* current is generated in the frame work and by Lenz's law† this always tends to stop the rotation of the coil. On the other hand if the frame be of a non-conducting material no such induced current exists and there is no such damping.

* Vide Chapter XVI † Vide Art 147

Galvanometers in which damping is reduced to a minimum are known as ballistic galvanometers. In these instruments therefore the coil is invariably wound over a non-conducting frame. When the current ceases the coil oscillates a number of times before coming to rest. These instruments are suitable for measuring quantities of electricity. If the damping is fairly large—as is produced by making the frame conducting*—the galvanometer is called dead-beat. In this case the coil may make one or two osciallions at the most but usually it makes no oscillation and gradually comes back to the initial position when the current is stopped. A dead-beat galvanometer is suitable for measuring a steady current. It may be noted that a suspended needle-type galvanometer may also be ballistic or dead-beat according as the damping is small or large.

Art 91 Theory of Ballistic galvanometers

We shall now proceed to discuss the theory of ballistic galvanometers. We shall first consider a **suspended magnet**** **type** galvanometer.

If at any instant i be the current in a circular coil the field at the centre is Gi where $G = \frac{2\pi n}{r}$ [Vide (36) Page 179]. If m be the pole strength of the short magnetic needle placed at the centre of the coil the force on each pole is Gim. Let us suppose

Fig. 146

that the current passes through the coil for a very short time T so that *the needle does not rotate appreciably before the current ceases to exist.* The force Gim on each pole is therefore of the nature of an impulsive force and its action is to give a sort of blow on each pole.

The force Gim acting on each pole for an infinitesimally short time dt produces an impulse $Gimdt$. Hence the total impulse in time T is $\int_0^T Gim\,dt$. But $\int_0^T idt$ is the total

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* If necessary, the damping may be further increased by other methods, such as attaching small vanes to the coil and further by immersing these in oil.

** A tangent galvanometer described in Art 84 is a suspended magnet type galvanometer.

charge Q which flows through the coil in the short time T. Thus the total impulse produced on each pole is

$$\int_0^T Gim\,dt = Gm\int_0^T i\,dt = GmQ$$

In linear motion Impulse = mass × velocity.
In rotatory motion Moment of the impulse = moment of inertia × angular velocity
Again, in linear motion Kinetic energy = $\frac{1}{2}$ × mass × (velocity)^2
In rotatory motion Kinetic energy = $\frac{1}{2}$ × moment of inertia × (angular velocity)^2

Let $2l$ be the length of the needle. The impulse on the two poles being in opposite directions the moment of the impulse is $GmQ \times 2l = GMQ$ where M is the magnetic moment of the needle. Due to this the needle begins to rotate with an angular velocity w.

Hence .. $GMQ = Iw$ ... ... (a)

where I is the moment of inertia of the needle.

When the needle starts rotating its kinetic energy is $\frac{1}{2}Iw^2$. As it rotates it is acted on by a controlling couple due to Earth's Horizontal component. Thus on each pole of the magnet AB a force mH acts. As the needle rotates the couple due to mH tends to bring the needle back to the original position CD. The needle stops momentarily at the extreme position when the work done against the couple is equal to $\frac{1}{2}Iw^2$

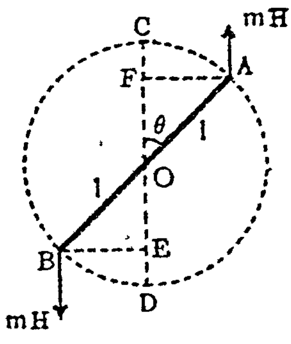

Fig. 147

In this connection the motion of the pendulum may be considered. When the bob passes through the central position it has some kinetic energy. Thereafter as the pendulum passes to the extreme position the pendulum bob rises through a certain height *i. e.* the bob does work against its weight. The bob comes to rest when the work done against the weight is equal to the kinetic energy in the central position.

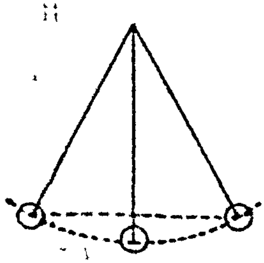

Fig. 148

Now when the needle comes to the extreme position AB work done against one of the forces mH (acting on B) is $mH \times DE = mH(OD - OE) = mH(l - l\cos\theta) = mlH(1 - \cos\theta)$

Hence total work done against both the forces

$$= 2mlH(1 - \cos\theta) = MH(1 - \cos\theta)$$

$$\therefore \tfrac{1}{2} I w^2 = MH(1 - \cos\theta) = 2MH\sin^2\frac{\theta}{2}$$

$$\text{or} \quad I w^2 = 4MH\sin^2\frac{\theta}{2} \quad \ldots \quad (b)$$

Eliminating w between (a) and (b)

$$Q^2 = \frac{I^2 w^2}{G^2 M^2} = \frac{I^2}{G^2 M^2} \times \frac{4MH}{I}\sin^2\frac{\theta}{2}$$

$$= \frac{I}{M} \cdot \frac{4H}{G^2}\sin^2\frac{\theta}{2} \quad \ldots \quad \ldots \quad (c)$$

Further if damping be small the needle oscillates a number of times before coming to rest. The period of this oscillation is given by

$$T = 2\pi\sqrt{\frac{I}{MH}}$$

$$\text{or} \quad T^2 = \frac{4\pi^2 I}{MH} \quad \therefore \frac{I}{M} = \frac{HT^2}{4\pi^2}$$

Hence from (c) $$Q^2 = \frac{HT^2}{4\pi^2} \cdot \frac{4H}{G^2}\sin^2\frac{\theta}{2}$$

$$= \frac{H^2 T^2}{\pi^2 G^2}\sin^2\frac{\theta}{2}$$

$$\therefore Q = \frac{HT}{\pi G}\sin\frac{\theta}{2} \quad \ldots \quad \ldots \quad (41)$$

$$i.e. \quad Q = k\sin\frac{\theta}{2} \quad \ldots \quad \ldots \quad (41a)$$

$$\text{where } k = \frac{HT}{\pi G} \quad \ldots \quad (41b)$$

We shall now consider a suspended coil type galvanometer.

If at any instant i be the current through the coil the force on each vertical *wire* of the coil is Hil where H is the strength

of the magnet and l is the length of the wire. If the coil be of n turns the force on each vertical *arm* of the coil is $nHil$. Let us now suppose that the current passes through the coil for a very short time T so that the *coil does not rotate appreciably before the current ceases to exist.* Hence the force $nHil$ is of the nature of an impulsive force. The impulse produced by this force is $\int_0^T nHildt = nHl \int_0^T idt = nHlQ$ where Q is the charge which flows through the coil in time T. If b be the length of the horizontal arm of the coil the moment of the impulse is $nHlQ \times b = nAHQ$ where A is the area of the coil. Hence if ω be the angular velocity with which the coil begins to rotate, we have $nAHQ = I\omega$ ... (*d*)

where I is the moment of inertia of the coil.

When the coil starts rotating its kinetic energy is $\frac{1}{2}I\omega^2$. As the coil rotates the suspension fibre gets twisted. Work is therefore done against the restoring couple produced by twisting. Let μ be the coefficient of torsion (*i. e.* the restoring couple produced by a unit twist). At any instant if the angle of the twist is ϕ the restoring couple is $\mu\phi$. For an aditional twisting $d\phi$ of the wire work done is $\mu\phi d\phi$. Hence if θ be the maximum deflection the total work done is $\int_0^\theta \mu\phi d\phi = \frac{1}{2}\mu\theta^2$.

Hence $\quad \frac{1}{2}I\omega^2 = \frac{1}{2}\mu\theta^2$ or $I\omega^2 = \mu\theta^2$ ... (*e*)

Eliminating ω between (*d*) and (*e*).

$$Q^2 = \frac{I^2\omega^2}{n^2A^2H^2} = \frac{I^2}{n^2A^2H^2} \cdot \frac{\mu\theta^2}{I}$$

$$= \frac{\mu I\theta^2}{n^2A^2H^2} \quad \dots \quad (f)$$

Again, if damping be small the coil oscillates a number of times before it finally comes to rest. The period of this oscillation is given by

$$T = 2\pi\sqrt{\frac{I}{\mu}} \quad \text{or} \quad T^2 = \frac{4\pi^2 I}{\mu} \quad \dots \quad (g)$$

Dividing (*f*) by (*g*), thus eliminating I, we have

$$\frac{Q^2}{T^2}=\frac{\mu^2\theta^2}{4\pi^2n^2A^2H^2}$$

$$\text{or}\qquad Q=\frac{\mu T}{2\pi nAH}\theta \qquad \ldots \qquad (42)$$

$$i.\,e.\qquad Q=k\theta \qquad \ldots \qquad (42a)$$

where k is a constant equal to $\frac{\mu T}{2\pi nAH}$ ... (42*b*)

It should be clearly understood that H in equation (41) and H in equation (42) are not the same magnetic field. In (41) H is Earth's horizontal component and in (42) H is the field due to the magnet between whose pole pieces the coil is suspended.

Art. 92
Damping

In the previous article we have seen that in the case of a ballistic galvanometer damping must necessarily be quite small. But although small damping can never be entirely absent. As a result when the coil*, (or the needle) oscillates the successive amplitudes become gradually smaller and smaller and after a few oscillations the coil (or needle) ultimately comes to rest. It is obvious that during the *first* swing also to the extreme position damping must play its part, *i. e.* in absence of damping the first swing would become a little larger. Or in other words the deflection θ observed as the first swing is slightly smaller than what it would be if there were no damping. To get the corrected value θ_0 of the first deflection we proceed thus:

Let θ, θ_1, θ_2, θ_3, ... be the successive deflections§ as observed on both sides of the central position. It is noticed† that the ratio of any one deflection to the next one is a constant. Thus

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* In the case of a suspended coil type galvanometer it is the coil which rotates. In the case of a suspended magnet type galvanometer however it is the magnetic needle which rotates.

§ In the case of a suspended coil type galvanometer these deflections are observed on the scale on which the beam of light is incident after being reflected by the galvanometer mirror.

† This may also be proved theoretically.

$$\frac{\theta}{\theta_1}=\frac{\theta_1}{\theta_2}=\frac{\theta_2}{\theta_3}= \quad \dots \quad \dots \quad = d \text{ (a constant)}$$

Let $d=e^{\lambda}$. Then $\log_e d=\lambda$. The constant d is called decrement and $\log_e d$ or λ is called the log decrement.

Thus for a half oscillation

$$\frac{\theta}{\theta_1}=\frac{\theta_1}{\theta_2}=\frac{\theta_2}{\theta_3}= \quad \dots \quad = e^{\lambda}$$

Hence for a complete oscillation

$$\frac{\theta}{\theta_2}=\frac{\theta_2}{\theta_4}=\frac{\theta_4}{\theta_6}= \quad \dots \quad = e^{2\lambda}$$

and so on

$\therefore$ If θ_0 be the correct value of the *first* deflection when there is no damping $\frac{\theta_0}{\theta}$ is therefore the ratio for a quarter oscillation

Hence $$\frac{\theta_0}{\theta}=e^{\frac{\lambda}{2}}$$

But $$e^{\frac{\lambda}{2}}=1+\frac{\lambda}{2}+\frac{\lambda^2}{4\underline{|2}}+\frac{\lambda^3}{8\underline{|3}}+ \quad \dots \quad \dots$$

Since λ is usually a small quantity its square and higher powers may be neglected.

Thus $$\frac{\theta_0}{\theta}=e^{\frac{\lambda}{2}}=1+\frac{\lambda}{2} \quad \text{or} \quad \theta_0=\theta\left(1+\frac{\lambda}{2}\right)$$

Hence θ in (41) and in (42) should be replaced by $\theta\left(1+\frac{\lambda}{2}\right)$. Thus for a suspended needle type galvanometer

$$Q=\frac{HT}{\mu G}\,\text{Sin}\,\frac{\theta}{2}\left(1+\frac{\lambda}{2}\right) \quad \dots \quad (41c)$$

And for a suspended coil type galvanometer

$$Q = \frac{\mu T}{2\pi n A H}\theta\left(1+\frac{\lambda}{2}\right) \quad \cdots \quad (42c)$$

Art 93 Ammeters and Voltmeters

Ammeters and Voltmeters are essentially galvanometers. Ammeters measure currents and are therefore used *in series* in an electric circuit, whereas voltmeters measure E. M. F.'s or potential differences and are placed *in parallel* between two points whose difference of potential is to be measured. Ammeters are necessarily *low resistance* galvanometers; for when used in series they must not alter the strength of the current in the circuit. Voltmeters, on the other hand, are *high resistance* galvanometers; when placed in parallel they absorb very little current and do not practically interfere with the main current.

In different types of these instruments the electric current passes through a straight wire or through a coil of wire; arrangements are made whereby as a result of this current a pointer moves over a graduated scale. The way how this is done differentiates one type of instrument from another.

Ammeters and voltmeters are accordingly classified as follows;—(*a*) Hot wire (*b*) Moving coil (*c*) Moving iron.

Art 94 Hot wire instrument

The instrument is diagrammatically shown in Fig. 149. The current to be measured passes through the wire A stretched between two suitable blocks. The wire is usually made of platinum - silver so that it may have a high melting point. A phosphorbronze wire F is attached at one end to some point near the middle of the wire A, the other end being attached to an insulated block C. The wires A and F are kept taut by a silk fibre B pulling F to one side; the silk fibre passes round a pulley W carrying the pointer P and is itself pulled by the spring S. As the wire A is heated by the passage of the current it expands and sags; consequently the

wire F also tends to sag but is kept taut by the pull of the silk fibre due to the action of the spring S. The pulley W consequently rotates and the pointer P moves over a scale.

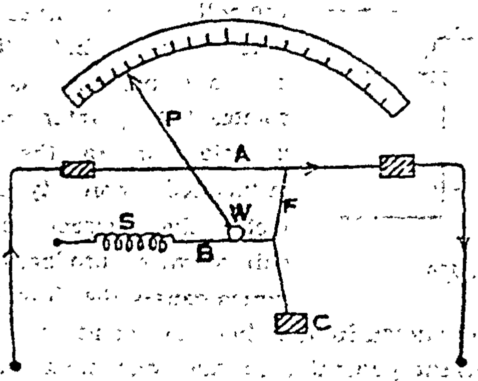

Fig. 149

By passing known currents through the wire or by using known potential differences between the terminals of the wire, the scale may be calibrated in amperes or volts as desired.

Since the heating of the wire A and hence its expansion is proportional to the square of the current the instrument may be used for both A. C. and D. C. currents; the scale however is non-uniform, being more crowded near the zero reading and more opened out for larger currents.

The principal defect of this instrument is the wandering of the zero reading caused by the fluctuation of the room temperature as a result of which the wire expands differently at different times and the pointer moves even if no current passes through the instrument.

Art 95 Moving coil instrument

This instrument is very much analogous in construction to a suspended coil galvanometer. An insulated copper wire is wound round a copper or aluminium rectangular frame. The frame is placed between two polepieces of a permanent horse-shoe magnet and is so pivoted that it can rotate between the

pole-pieces. The pole pieces are made cylindrical in shape and a soft iron cylinder (not shown in the diagram) is kept fixed within the coil, so that lines of force are everywhere radial. The frame carries a pointer: as the frame rotates the pointer moves over a scale. As in the case of a suspended coil type galvanometer the current through the coil of wire produces a torque which causes the frame to rotate. A spring (not shown in the diagram) controls this rotation. Induced current generated in the conducting frame makes the instrument sufficiently dead-beat.* Due to the fixed cylinder of soft iron placed within the frame lines of force are more concentrated.

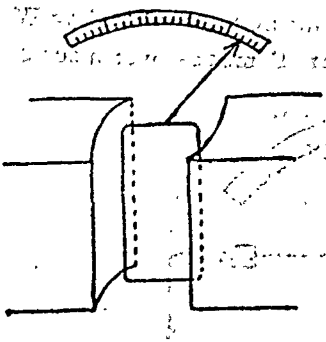

Fig. 150

The torque produced by the current is directly proportional to the current; the instrument can therefore be used only for D. C. current. The scale is uniform throughout the range and can be calibrated in amperes or volts as desired.

The instrument is very little affected by external magnetic effect; the permanent horse-shoe magnet if specially prepared retains its magnetism for a sufficiently long time. Moving coil instruments are therefore the most accurate and most satisfactory for all D. C. measurements.

Art 96 Moving iron instrument

In these instruments the electric current passing through a fixed coil magnetises one or more soft iron pieces. Due to the force existing between the current and a magnetised soft iron or between two such soft iron pieces, one of the iron pieces rotates; a pointer attached to this soft iron consequently rotates over a graduated scale. Two types of such instruments are in use according as the force is of attraction or of repulsion.

* Vide Art 90

We first describe an *attracted iron* type instrument. The coil C carrying the current is fixed. The soft iron piece M is excentrically pivoted and being magnetised by the current is attracted within the coil. A pointer P attached to this soft iron M moves over a graduated scale. The weight of the soft iron generally produces the necessary controlling couple; in some cases a spring is also provided for this purpose.

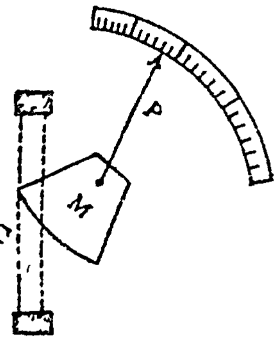

Fig. 151

If the current be not very large the magnetisation of the soft iron is not near the saturation* point and is in that case very nearly proportional to the current. The force of attraction and hence the deflection of the pointer is proportional to the product of the current in the coil and the strength of magnetism; it is therefore proportional to the square of the current. The scale is therefore non-uniform, being more opened out for larger current.

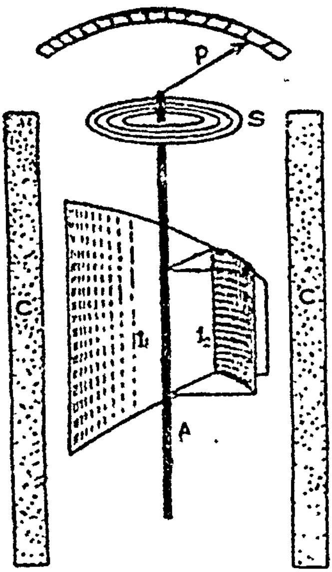

Fig. 152

In the *repulsion* type the coil C is cylindrical in shape. Parallel to the inner surface of the cylinder a curved soft iron I_1 is fixed. This is so shaped that it is much broader at one end than at the other. Along the axis of the cylinder a spindle A is pivoted, carrying another soft iron I_2 much

* Vide Art 139

smaller in area. This iron is also curved so as to be parallel to the other iron I_1. Due to the current in the coil C both the iron pieces are magnetised longitudinally in the same direction and therefore repel each other. The magnetic effect in I_1 being much greater at the broader end the force of repulsion is greater in that region; the rotating iron I_2 rotates so that it tends to face the shorter end. A pointer P attached to the spindle A rotates over a graduated scale. A spring at the top of the spindle provides the necessary controlling couple.

The force of repulsion is proportional to the product of the strengths of magnetism in the two iron pieces and since each of the latter varies directly as the current; the force of repulsion and hence the deflection of the pointer is proportional to the square of the current. The scale is thus non-uniform as before.

Although these moving iron instruments are not so accurate as moving coil instruments they are more robust in construction and are simple in design. They are however susceptible to exernal magnetic fields; this can be minimised by enclosing the instrument in an iron case. They are also liable to hysteresis* errors so that they read low when currents are increasing and high when currents are decreasing. In modern instruments a nickel-iron alloy having negligible hysteresis is used instead of soft iron.

Art 97 Ammeters can be converted into voltmeters and voltmeters into ammeters. The range of either of these instruments may also be increased by using a shunt or by inserting a resistance as the case may be. We shall explain these cases separately with the help of problems.

I. An ammeter to be converted into a voltmeter.

Explain how an ammeter of 0·5 Amp range can be converted into a voltmeter reading up to 50 volts. The resistance of the ammeter is 2 ohms.

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* Vide Art 139.

A voltmeter being a high resistance galvanometer, a resistance R is to be inserted in series with the ammeter

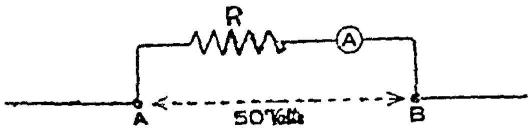

Fig. 153

Full scale deflection of the ammeter is produced when a current of 0·5 amp passes through the instrument. Since the converted voltmeter is to read up to 50 volts, it is obvious that when a P. D. of 50 volts is applied between the new terminals A and B, a current of 0·5 amp must pass through the instrument. The total resistance between A and B being R + 2, we have

$$50 = 0{\cdot}5\,(R+2) \quad \text{or} \quad R+2 = 100 \quad \therefore\ R = 98 \text{ ohm.}$$

II A voltmeter to be converted into an ammeter.

Explain how a voltmeter reading up to 150 volts can be converted into an ammeter of 8 Amps range. The resistance of the voltmeter is 300 ohms.

A voltmeter being a high resistance galvanometer a shunt of resistance R must be used in parallel with the instrument. Since the voltmeter reads up to 150 volts and since the resistance of the voltmeter is 300 ohms, maximum current that can be passed through the instrument is $\frac{150}{300}$ or 0·5 amp. And since the instrument is to be used as an ammeter of 8 amp range the current through the shunt R is $8 - 0{\cdot}5 = 7{\cdot}5$ amps. Hence if A and B be the terminals of the instrument,

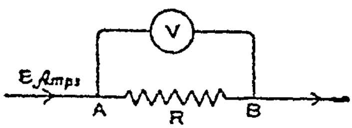

Fig. 154

$V_A - V_B = 7{\cdot}5\,R$ considering the shunt circuit.

Also, $V_A - V_B = 0{\cdot}5 \times 300$ „ voltmeter „

Hence $7{\cdot}5\ R = 0{\cdot}5 \times 300$

$$\therefore \qquad R = \frac{0{\cdot}5 \times 300}{7{\cdot}5} = 20 \text{ ohms.}$$

III. The range of an ammeter to be increased.

Explain how a milli-ammeter of 50 milli-amp range can be made to read up to 5 amps. The resistance of the milli-ammeter is 5 ohms.

Here a shunt R is to be used in parallel with the instrument. The maximum current to be used being 5 amps and

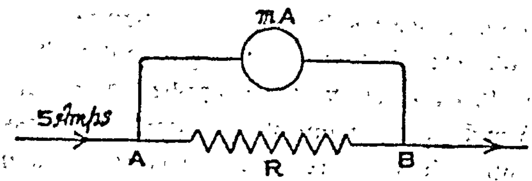

Fig. 155

the maximum current *through the instrument* being 50 milli-amps or 0·05 amp, the current through the shunt is $5 - 0{\cdot}05 = 4{\cdot}95$ amps. Hence if A and B be the terminals of the instrument,

$V_A - V_B = 0{\cdot}05 \times 5$ considering the milli-ammeter circuit.
also $V_A - V_B = 4{\cdot}95\ R$ „ „ shunt „

Hence $4{\cdot}95\ R = 0{\cdot}05 \times 5$

$$\therefore \qquad R = \frac{0{\cdot}05 \times 5}{4{\cdot}95} = 0{\cdot}0505 \text{ ohm.}$$

IV. The range of a voltmeter to be increased.

Explain how a voltmeter of 5 volts range can be made to read up to 500 volts. The resistance of the voltmeter is 20 ohms.

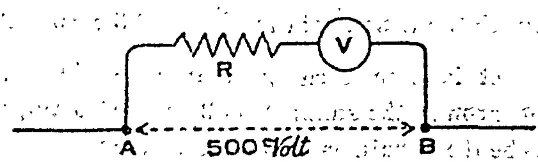

Fig. 156

Here a resistance R is to be used in series with the volt-

meter. Since the resistance of the instrument is 20 ohms and since the original range of the instrument is 5 volts maximum current that can be passed through the instrument is $\frac{5}{20}=\frac{1}{4}$ amp. Hence when R is in series with the voltmeter the same current $\frac{1}{4}$ amp must pass through the instrument when a P. D. of 500 volts is applied between the new terminals A and B. Thus

$$500=(R+20)\times\tfrac{1}{4}$$

$$\text{or} \quad R+20=2000 \quad \therefore \quad R=1980 \text{ ohms.}$$

Art 98. Action of current on current.

Consider two like parallel currents* i_1 and i_2 separated by a distance r. The field due to i_1 at a distance r is $\frac{2i_1}{r}$ [Vide (35a) Page 178]. By Maxwell's corkscrew rule [Art 80] the direction of the field may be easily found; in the case shown in Fig. 157 it is perpendicular to the plane of the paper and is towards the paper.

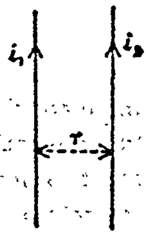

Fig. 157

The current i_2 is placed in this field. Hence the force on the current i_2 (per unit length of the wire) is $\frac{2i_1i_2}{r}$ and by Fleming's Left Hand Rule this is directed towards the current i_1, *i. e.* the currents i_1 and i_2 attract each other. If the currents be unlike they mutually repel each other. Thus the rule is "Like currents attract, unlike currents repel".

In the case of oblique currents, the rule is :—

If both currents flow towards or away from the apparent or real point of intersection they attract each other; if one of the currents flows towards and the other away from the point of intersection, they repel each other.

The magnitude of the force between two currents is proportional to the product $i_1 i_2$; if the same current i passes

* Both wires carrying currents, are supposed to be sufficiently long.

through both the wires the force is proportional to the square of the current i.

Art 99 Kelvin's Ampere Balance.

The principle explained in the last article has been utilised in Kelvin's Ampere Balance. In the instrument

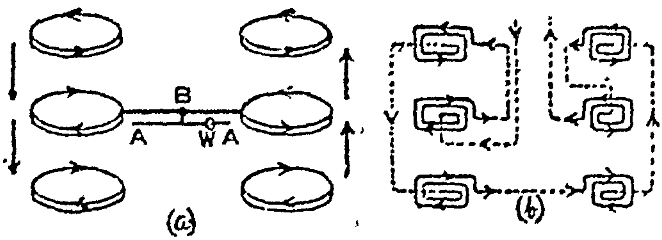

Fig. 158

there are six exactly similar coils placed in two columns—three in each. [Vide Fig. 158 (*a*)]. The two uppermost and the two lowermost coils are fixed and the two middle coils are held by a lever which can rotate about the fulcrum B. A horizontal arm AA attached to this lever carries a small moveable weight W. All the six coils are electrically connected to one another so that the same current i passes through all of them [Vide Fig. *158 (b)*]. *It will be* seen that currents in different coils are so directed that on the left hand side the middle coil is repelled by the upper and attracted towards the lower coil ; it therefore tends to come down. In a similar way on the right hand side the middle coil tends to go up. The lever carrying the middle coils therefore tends to rotate about B in the anti-clockwise direction. This may be prevented by suitably adjusting the position of the moveable weight W. Known currents are first passed through the instument and corresponding positions of W along the arm AA are noted. Afterwards any unknown current may be determined by noting the corresponding position of W.

In this instrument the force between two coils is proportional to the square of the current and is therefore

ndependent of the direction of the current ; hence alternating currents may also be measured by this apparatus.

Art 100 Wattmeter.

If a current passes through a circuit or an instrument some power is usually absorbed. A system in which power is absorbed is technically known as a *load*. A wattmeter measures the power absorbed by a load. It essentially consists of two coils of wire—a low resistance coil used in series with the load and a high resistance coil placed in parallel with the load. The former is known as the current coil and the latter as the pressure coil. The current coil is usually fixed and produces a field proportional to the current. The pressure coil—which is moveable—is placed in this field. Hence for any given positions of the two coils the couple at any instant acting on the moveable coil is proportional to the product of the P. D. and the current at that instant, *i. e* proportional to the instantaneous power. If the power fluctuates as in alternating current circuits, the mean couple acting on the moveable coil is proportional to the mean power.

Siemen's Electro-dynamometer may be used both as an ammeter and also as a wattmeter. Two vertical coils AA′ and BB′ are situated at right angles to each other. The coil AA′ is fixed ; but the coil BB′ (whose terminals are dipped into two mercury cups) can rotate about the common vertical axis. This rotation is controlled by the spring S attached to a torsion head at the top. If the coils are in series as in Fig. 159 (*a*), the instrument is used as an ammeter. It will be seen that when the current *i* passes through the two coils the arm B of the moveable coil is attracted by the arm A and repelled by the arm A′ of the fixed coil ; the arm B′ is similarly attracted by A and repelled

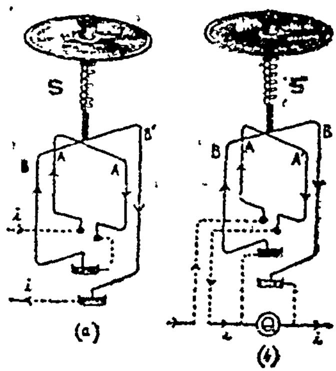

Fig. 159

by A. The coil BB′ accordingly rotates in the clockwise direction. It is again brought back to its initial position by rotating the torsion head at the top. The amount of torsion, *i. e.* the angle θ through which the torsion head is rotated is measured by the pointer P rotating over a graduated circular scale. The forces of attraction and of repulsion between the arms and therefore the couple tending to produce rotation is proportional to i^2. Hence when the coil is finally brought back to the initial position

$$i^2 \propto \theta \qquad \text{or} \qquad i = k\sqrt{\theta}.$$

The constant k may be determined by observing θ for a known current.

If the instrument is to be used as a wattmeter—say, for measuring the power absorbed by an electric bulb Q the low *resistance coil AA′ is connected in series and the high* resistance coil BB′ in parallel with the bulb Q [Vide Fig. 159 (*b*)]. If i be the current through the bulb and E the P. D. between the terminals of the bulb the current through AA′ is i and that through BB′ is $\frac{E}{R}$ where R is the resistance of BB′. Hence the couple tending to produce rotation of the coil BB′ is proportional to $\frac{Ei}{R}$. As before if θ be the torsion necessary to bring back the coil to the initial position $\frac{Ei}{R} \propto \theta$ Thus power $(=Ei) \propto \theta$ $\quad \therefore$ Power$=k\theta$

The constant k may be determined by observing θ when a known power is used.

Art 101
Energy mote

The instrument described above is known as an indicating wattmeter, *i. e.* it indicates the watt or the *rate* at which energy is supplied. In house supply meters the instrument must not only indicate the power or the rate of supply of energy but must also take into account the length of time for which energy is supplied. Such meters are therefore known as energy meters. These energy meters are again of two kinds ; where the

supply voltage is maintained constant—as is usually the case in the town supply—it is enough if only the current is measured. The current in amperes when multiplied by the number of hours for which the energy is supplied to a circuit gives us the number of ampere-hours. The number of watt-hours supplied to such a circuit is obtained by multiplying these ampere-hours by the constant voltage applied. Thus the amount of energy is simply the current in amperes multiplied by a constant varying with time. Such an instrument is called an ampere-hour meter. Where the variation of the supply voltage is also taken into account the instrument is known as an watt-hour meter.

Meters of the "motor" type are most generally used in actual practice. These meters may be used with direct current as well as with alternating current. In an indicating instrument the moving system comes to rest after rotating through a fraction of a revolution; but in this class of instruments the current passes through an electric motor and the moving system rotates continuously. The speed of revolution is obviously proportional to the current passing through the motor. It follows therefore that the number of revolutions made by the revolving system in any given time is proportional to the quantity of electricity supplied in that time and hence—if the supply voltage is constant—to the energy supplied. The number of revolutions is recorded by a counting mechanism consisting of a train of wheels to which the spindle of the rotating system is geared. The registering dials are calibrated in Kilowatt hours so that the energy supplied can be directly read off. Usually a copper or an aluminium disc rotates along with the motor. Current induced in this disc by a permanent magnet controls the speed of revolution, the system attaining a steady speed when the retarding torque by these induced currents, balances the driving torque produced by the supply current.

One of the commonest forms of house supply meters is Ferranti's mercury meter. A thin amalgamated copper disc C is mounted at its centre on jewelled cup bearings inside a

shallow circular box B of some non-conducting material. Two magnets—one for driving purpose and the other for braking, are fixed with their poles, one above and the other below the

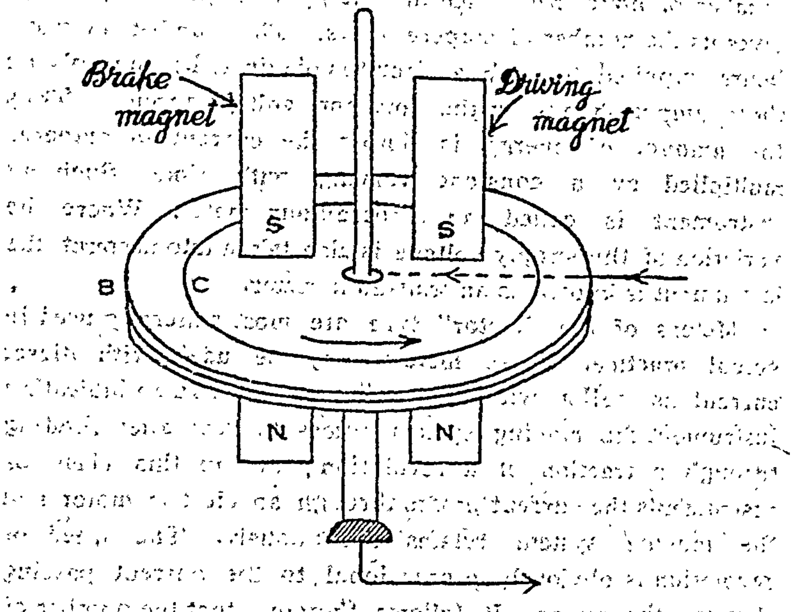

Fig. 160

circular box. The space inside the box is filled with mercury. The current is led into the box through mercury at some point on the right; it then flows radially to the centre through the copper disc and and afterwards passes out along the spindle on which the disc is mounted. Due to the magnet on the right a torque is produced on the radial current as a result of which the disc rotates in the direction as indicated by the arrow. By the rotation the disc cuts through the field of the magnet on the left; induced current thus generated controls the speed of rotation of the disc. The upper part of the spindle (rotating with the disc) has a worm cut in it whereby the gear wheels of the recording dials are engaged. This instrument is obviously a D. C. ampere-hour meter; when working on a fixed voltage it indicates Kilowatt-hours on the dials.

Exercise XI

1. A mirror galvanometer of resistance 200 ohms is shunted by a wire of 2 ohms resistance : it is then connected through a resistance of 1000 ohms to the terminals of a cell of negligible resistance and of E. M. F. 2 volts. If the deflection be 20 cms on a scale at a distance of 120 cms from the galvanometer find the figure of merit of the galvanometer.

Ans. 11.86×10^{-8} amp per mm deflection.

2. A galvanometer of resistance G is connected to the terminals of a battery of internal resistance R. On shunting the galvanometer with a resistance S it is found that the galvanometer current is halved, while the battery current is doubled. Prove that $G = 2R = 3S$.

3. You are given a milli-voltmeter of range 1 to 30 millivolts and of internal resistance 25 ohms. Show by diagrams how you will use the instrument to measure (*a*) potentials between 1 to 30 volts and (*b*) currents between 0·1 and 3 amperes. Ans. (*a*) 24975 ohms in series (*b*) 0·01 ohm in parallel.

4. A moving coil galvanometer has a resistance of 10 ohms and gives a full scale deflection when the current through it is one milli-ampere. What will you do to convert it into an ammeter reading up to ten ameres?

Ans. 0·001 ohm in parallel

5. A certain ammeter has a resistance of one ohm and the full scale deflection is obtained when a current of 0·05 ampere flows through it. Find what shunt must be connected with it in order that the ammeter may read up to 10 amperes.

Ans. 0·005 ohms.

6. A portable galvanometer whose needle deflects 5 scale divisions per milli-ampere, is to be used as an ammeter. Its resistance is 238 ohms. What should be the resistance of the shunt in order that the needle may deflect 10 divisions per ampere ? C. U. 1949. Ans. 0·477 ohm.

7. Two resistances 150 and 250 ohms are in series and are connected to a source of E. M. F. 200 volts. A high resistance voltmeter is connected in turn across each of the resistances.

What are the readings? How are the readings altered if the voltmeter be of resistance 200 ohms only?

Ans. 75 volts; 125 volts; 51·07 volts; 85·11 volts

8. Full scale deflection is produced when a current of 0·1 amp is passed through a certain galvanometer of resistance 25 ohms. How can the galvanometer be used as (*a*) an ammeter reading up to 10 amps (*b*) a voltmeter measuring potentials up to 10 volts.

Ans. (*a*) 0·253 ohm in parallel (*b*) 75 ohms in series.

9. A milli-ammeter of resistance 20 ohms reads up to 0·05 amp. Explain how the instrument can be used to measure (*a*) currents up to 5 amps (*b*) voltages up to 50 volts.

Ans. (*a*) 0·202 ohm in parallel (*b*) 980 ohms in series.

10. The scale of a voltmeter (resistance 50 ohms) is graduated from 0 to 5 volts. Show how the voltmeter can be adapted for use as (*a*) a voltmeter of range 500 volts (*b*) an ammeter reading up to 5 amps.

Ans. (*a*) 4950 ohms in series (*b*) 1·02 ohm in parallel.

C. U. Questions

1964. Describe a moving coil galvanometer and show how a current is measured with it.

A voltmeter reads 50 volts when connected across an unknown potential difference. A 5000 ohm resistor is then inserted in series with the meter across the same potential difference and the reading drops down to 33·3 volts. *Find* the resistance of the voltmeter and the value of the current through the voltmeter when it reads 50 volts.

Ans. 10,000 ohms; 0·005 amp.

1965. What is the difference between an ammeter and a voltmeter? Discuss their constructional details with the help of diagrams.

Explain how a voltmeter of 10 volts range can be made to read up to 500 volts, the resistance of the voltmeter being 500 ohms.

Ans. 24,500 ohms in series.

1965 Describe the construction of a moving coil galvanometer and show how a current can be measured by it.

The moving coil of a galvanometer has 60 turns, a width of 2 cms and a length of 3 cms. It hangs in a uniform radial magnetic field of 500 C. G. S. units. Find the current in milli-amps when the controlling couple due to twist in the suspension is 18 dyne-cms. Ans. 1 milli-amp

1965 Describe the construction of a Ballistic galvanometer of suspended coil type.

Establish an expression for the quantity of charge flowing through the coil.

Explain the measurement of the capacity of a condenser with it.

1966 Describe a moving coil pivoted type of ammeter with a neat diagram. How does it differ from a voltmeter ?

An ammeter of resistance 15 ohms gives a full scale deflection when 5 amps of current passes through it. Calculate the value of the shunt necessary to convert it into a meter reading up to 10 amps. Ans. 15 ohms.

1966, 1971, 1974. Write notes on "Barlow's wheel."

1967. Find the force per unit length of a current carrying conductor placed in a uniform magnetic field. Hence find the couple acting on a rectangular coil placed in a uniform magnetic field. Describe the construction and mode of action of a suspended coil galvanometer.

1967, 1969. Write notes on "Ballistic galvanometer."

1968. Write notes on "Moving coil voltmeter".

1970. Explain with diagram the construction and action of a suspended coil D'Arsonval type of galvanometer. What are the points of difference between a ballistie and a dead-beat type of galvanometer ? Write down the working formula of a ballistic galvanometer explaining the symbols used.

1971. Explain the construction and working principle of a Wattmeter

1973. Explain the construction and working principle of a meter for recording the consumption of electrical energy.

1973, 1976. Explain the construction and action of a suspended coil type galvanometer. What are the factors on which its sensitiveness depends? Explain how this instrument may be converted into the following two instruments: (*a*) Ammeter and (*b*) Voltmeter.

CHAPTER XII

ELECTRIC MEASUREMENT

Measurement of Resistance

Art 102 Measurement of a resistance depends on the principle of Wheatstone's net. The complete theory of Wheatstone's net based on Kirchoff's Laws, has already been given in Art 72. A more elementary theory is given below.

Four resistances P, Q, S, R are connected in the form of a quadrilateral ABCD; the galvanometer and the battery are placed along the two diagonals. The resistances are so adjusted that no current flows through the galvanometer. Then the current i_1 through AB is the same as that through BC and the current i_2 through AD is the same as that through DC.

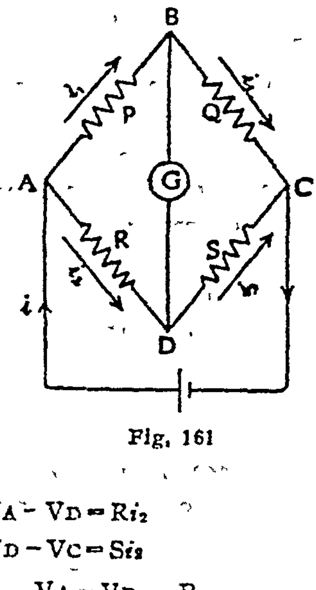

Fig. 161

Then potential difference

$$V_A - V_B = Pi_1 \qquad \text{and} \qquad V_A - V_D = Ri_2$$
$$V_B - V_C = Qi_1 \qquad \qquad V_D - V_C = Si_2$$

$\therefore$ by division $\dfrac{V_A - V_B}{V_B - V_C} = \dfrac{P}{Q}$ and $\dfrac{V_A - V_D}{V_D - V_C} = \dfrac{R}{S}$

And since there is no current through the galvanometer, $V_B = V_D$

$$\therefore \qquad \frac{P}{Q} = \frac{R}{S} \qquad \ldots \qquad \ldots \qquad (43)$$

If three of these four resistances be known the fourth can be easily determined.

Art 103 The Metre Bridge (or Wheatstone's Bridge) is an application of the above principle. It essentially consists of a uniform wire (usually made of german silver or constantan) one metre long, attached to two copper strips at the two ends. Another copper strip is placed on the same wooden board, so that there are two gaps among the three copper strips. A jockey slides along the metre wire and a knife edge attached to the jockey may be made to contact the wire at any desired point.

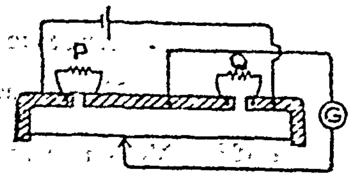

Fig. 162

The wire is stretched along a metre scale so that the point of contact may be determined readily. Two resistances P and Q, one known and the other unknown are placed in the two gaps; the galvanometer and the battery are connected as shown in the figure. The position of the jockey is adjusted until on its coming in contact with the wire no current flows through the galvanometer. The two resistances P, Q and the two portions of the metre wire on the two sides of the jockey, form the four arms* of the Wheatstone's net. Hence if l be the null point reading and if σ be the resistance per unit length of the wire we have

$$\frac{P}{Q}=\frac{l\sigma}{(100-l)\sigma}=\frac{l}{100-l} \quad \ldots \quad \ldots \quad (43a)$$

Hence, of the two resistances P and Q, one being known the other can be found out.

Art 104 There are several sources of error which must be avoided in carrying out the above experiment :—

(1) There are usually themo currents† in the circuit. These

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* Resistances of copper strips and of connecting wires are neglected.

† If different junctions are at different temperatures small currents known as thermo-currents flow through different branches of the circuit. A fuller account of these currents will be given in Chap XIV.

tend to deflect the null point towards one side and thus create an error. This is avoided by using a commutator in the battery circuit. By the commutator the battery current is reversed; but thermo-currents persisting in the same direction as before, the null point is now shifted to the other side of the correct reading. Null point readings are therefore taken both when the current is direct and also when it is reversed by the commutator. The mean of these two readings gives us the correct reading.

Thermo-current

(2) There is usually an arrow mark on the jockey. The position of this arrow mark against the scale is taken as the position of the null point. It may be that the actual null point *i. e.* the point at which the knife edge of the jockey touches the wire, may be slightly different from that indicated by the arrow mark. To avoid this error P and Q are interchanged and a second set of readings is taken. The unknown resistance is calculated from each of the two sets and the mean of the two results is the correct value of the unknown resistance.

Non-coincidence of arrowmark with actual null point

(3) There is usually some resistance at the two ends of the bridge wire. This is mainly due to soldering by which the wire is joined to the copper strips. These are known as end-errors. The bridge wire is also seldom *exactly* 100 cms; it is either slightly longer or slightly shorter. This also produces what are called end-errors. These end-errors are actually calculated in terms of resistances of so many centimeters of the bridge wire. For this purpose two *known* resistances P and Q are used in the two gaps. If the errors at the two ends are equal to the resistances of α cms and β cms of the bridge wire, we have

End-correction

(1) When P and Q are in left and right gaps respectively and l_1 is the null point,

$$\frac{P}{Q}=\frac{l_1+\alpha}{100-l_1+\beta} \qquad \dots \qquad \dots \qquad (a)$$

and (2) when P and Q are interchanged and l_2 is the null point,

$$\frac{Q}{P}=\frac{l_2+\alpha}{100-l_2+\beta} \quad \cdots \quad \cdots \quad (b)$$

The quantities involved in equations (a) and (b) are all known excepting α and β; α and β can therefore be calculated. These values can afterwards be utilised in determining any unknown resistance.

Non-uniformity of the wire

(4) The bridge wire may not be uniform in cross section throughout its length so that the ratio of the resistances of the two portions of the wire on both sides of the null point, is not equal to the ratio of the corresponding lengths. This can be remedied only by calibrating the wire, *i. e.* by dividing the whole wire into a large number of elementary lengths and by measuring the resistances of each of these elementary lengths; in this way the resistances of the two portions of the wire on the two sides of the jockey can be determined and their ratio can be found out.

Art 105 The **Post Office Box** is a more compact form of Wheatstone's Bridge. It essentially consists of a number of coils arranged so as to form the three arms of a Wheatstone's net. The unknown resistance R forms the fourth arm.

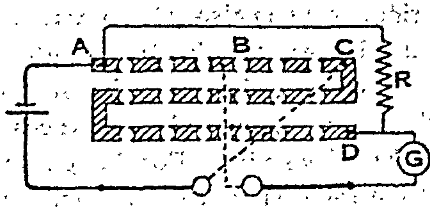

Fig. 163

There are two keys one of which is placed in the battery circuit and the other in the galvanometer circuit. The first two arms AB and BC are known as ratio arms; each of them consists of three resistance coils of 10, 100 and 1000 ohms.

The third arm CD consists of a number of resistances*, so that any whole number of ohms from 1 up to 10,000 is available. In using the instrument at first the ratio 10 : 10 is used in the ratio arms; it is then changed to 10 : 100 and finally to 10 : 1000. The resistance in the third arm is also suitably adjusted until the galvanometer shows no deflection. The unknown resistance can then be easily calculated by (43).

Art 106 Measurement of low resistance.

The classification of resistances into three groups—high, low and ordinary, is rather arbitrary. There is no definite boundary line between any two of these different groups. Generally speaking, resistances under one ohm are called low, those between 1 ohm and 1,000 ohms are termed ordinary and those above 1,000 ohms are considered as high.

In the usual method of measuring resistances by a Wheatstone's Bridge or by a Post Office Box, we neglect the resistances of copper strips and of connecting wires. So long as the resistance to be measured is fairly large (*i. e.* over one ohm) this is justified. But this is hardly proper when we come to the measurement of low resistances. The ordinary method therefore fails in this case.

The simplest arrangement by which a low resistance can be measured with sufficient accuracy, is by **Mathiessen** and Hopkinson's method.

Two low resistances r_1 and r_2—one known and the other unknown—are placed in the two gaps of a Wheatstone's Bridge. The two terminals of a battery are connected to the two end copper strips (as in ordinary method). A commutatot to reverse the current and a resistance to diminish the strength of the current are usually inserted in the battery circuit. One terminal of the galvanometer is as usual connected to the jockey but the other terminal is connected successively to the four terminals A, B, C, D of the two low resistances. This may conveniently be done with the help of a four way key as shown in the figure.

* All resistances in the Post Office Box are wound non-inductively Vide Art 149. Note (2)

The current from the battery on entering the bridge divides itself in two parts—one part i_1 flows through the low resistances and the other part i_2 flows through the bridge

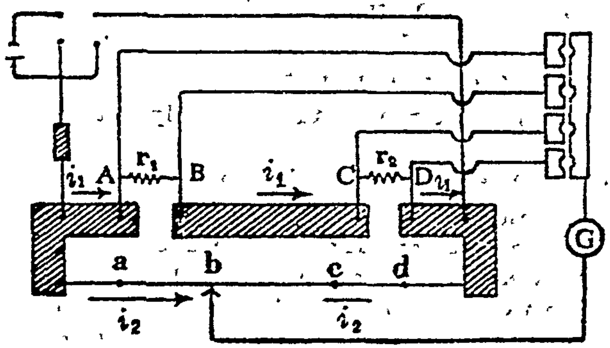

Fig. 164

wire. If a, b, c, d be the null points on the bridge wire when the galvanometer terminal is successively connected to A, B, C and D, we have

$$V_A = V_a \quad V_B = V_b \quad V_C = V_c \text{ and } V_D = V_d$$

or $\qquad V_A - V_B = V_a - V_b \quad \therefore \; i_1 r_1 = i_2 l_1 \sigma$

and $\qquad V_C - V_D = V_c - V_d \quad \therefore \; i_1 r_2 = i_2 l_2 \sigma$

where l_1 and l_2 are the lengths ab and cd and σ is the resistance per unit length of the bridge wire.

$$\therefore \text{ by division } \quad \frac{r_1}{r_2} = \frac{l_1}{l_2}$$

Art 107 **High resistance.**

It can be proved that Wheatstone's net arrangement is most sensitive when resistances in the four arms are of the same order. If two of them be unusually large the arrangement becomes insensitive. The ordinary method therefore fails in the case of measurement of a high resistance.

Method of substitution

We shall here consider the simplest method for measuring a high resistance.

Two high resistances R and X (one of them—say, X is unknown) are successively

joined in series with a battery and a galvanometer. This is conveniently done with the help of a two way key, as shown in Fig. 165. The current can be reversed by a Pohl's commutator. The shunt resistance S (used in parallel with the galvanometer G) is in each case adjusted so that the deflection is within the scale of the galvanometer. Let S_1 be the shunt resistance corresponding to R. Then the galvanometer and the shunt are together equivalent to a resistance $\frac{GS_1}{G+S_1}$. The total resistance in the battery circuit being thus equal to $R+\frac{GS_1}{G+S_1}$ the current from the battery

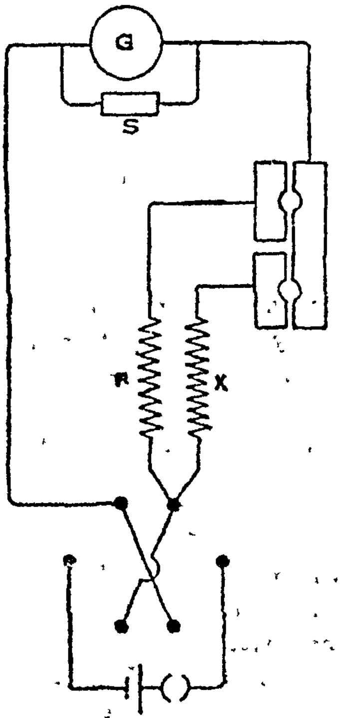

Fig. 165

$$=\frac{E}{R+\frac{GS_1}{G+S_1}}$$

∴ the current through the galvanometer

$$=\frac{E}{R+\frac{GS_1}{G+S_1}}\times\frac{S_1}{G+S_1}=\frac{ES_1}{R(G+S_1)+GS_1}$$

If this current produces a deflection θ_1

we have $$\frac{ES_1}{R(G+S_1)+GS_1}=k\theta_1 \quad \ldots \quad \ldots \quad (a)$$

Similarly if S_2 be the shunt resistance corresponding to X and if θ_2 be the corresponding deflection

$$\frac{ES_2}{X(G+S_2)+GS_2}=k\theta_2 \quad \dots \quad \dots \quad (b)$$

$\therefore$ by division, $$\frac{S_1}{S_2}\cdot\frac{X(G+S_2)+GS_2}{R(G+S_1)+GS_1}=\frac{\theta_1}{\theta_2} \quad \dots \quad (c)$$

Hence the unknown resistance X can be determined.

N. B. S_1 and S_2 should be so adjusted that θ_1 and θ_2 are nearly equal; otherwise strictly speaking, the constant k will not have the same value in both cases.

Art 108 Galvanometer Resistance,

The resistance of a galvanometer can best be obtained by clamping the galvanometer coil and by measuring its resistance by any ordinary method. But this requires the use of another galvanometer.

We shall discuss here two methods in which the use of the second galvanometer is avoided.

1st Method HALF-DEFLECTION METHOD

Connections are made as shown in the figure. P and Q are two resistance boxes; r is a low resistance. The current can be reversed by a Pohl's commutator. Keeping P equal to zero Q is at first adjusted until the deflection θ is within the scale. Resistances r and Q complete the battery circuit. Neglecting r in comparison to Q the current from the battery $=\frac{E}{Q}$

$\therefore$ Potential difference between the terminals of $r=\frac{Er}{Q}$.

Hence the current through the galvanometer

$$=\frac{Er}{QG}=k\theta \quad \dots \quad \dots \quad (a)$$

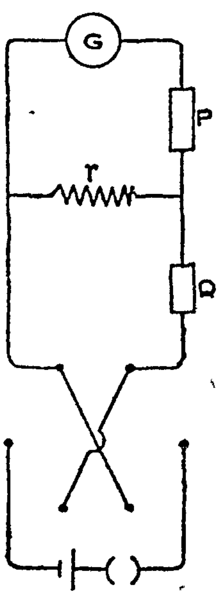

Fig. 166

Next without changing Q any more

P is adjusted until the deflection becomes half of what it was before.

$\therefore$ the current through the galvanometer

$$=\frac{Er}{Q(P+G)}=k\frac{\theta}{2} \quad \ldots \quad (b)$$

Hence, by division

$$\frac{P+G}{G}=2 \quad \text{or} \quad P+G=2G \quad \therefore \quad P=G.$$

Thus the resistance in P box is the same as the galvanometer resistance.

2nd Method THOMSON'S METHOD

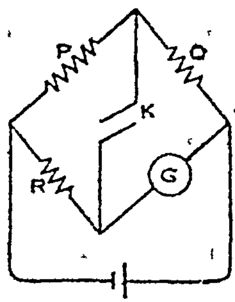

Fig. 167

In this method the galvanometer is placed in the fourth arm of a Wheatstone's net arrangement. A steady current passes through the galvanometer so that there is always a deflection. Resistances P, Q, R are adjusted until on pressing the key K (placed along one of the diagonals) there is *no change* in the deflection. In that case the terminals of the key are at the same potential and the usual relation $\frac{P}{Q}=\frac{R}{G}$ holds good.

Art 109 Battery Resistance

MANCE'S METHOD

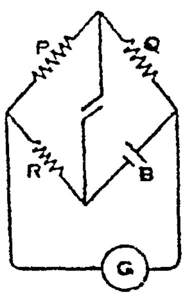

Fig. 168

The battery is placed in one arm of a Wheatstone's net arrangement. A galvanometer and a key are placed in the two diagonal arms. As in the case of Thomson's method (of measuring the galvanometer resistance), a steady current passes through the galvanometer so that there is always a steady deflection. The resistances

P, Q, R in the three arms are adjusted until on pressing the key there is *no change* in the deflection of the galvanometer. In this case if B be the internal resistance of the battery

$$\frac{P}{Q}=\frac{R}{B}$$

N. B. In all methods hitherto described what is actually determined is the ratio of the unknown resistance to a known one ; in order to determine an unknown resistance, we therefore always require a known resistance. The absolute determination of a resistance, *i, e.* determination of an unknown resistance independent of any other known resistance, is rather difficult ; this has been the subject matter of research work by Rayleigh, Lorentz and others. A full description of their experiments is beyond the scope of this book.

Art 110 Effect of temperature on resistance

The resistance of all metallic wires increases with temperature. To a first approximation the relation may be expressed as $R_t = R_0 \ (1+at)$ where R_0 is the resistance of the wire at 0°C, R_t the resistance at t°C and a is a constant for the material of the wire. This constant is known as the temperature coefficient of the material of the wire. This provides us with an efficient method of determining any unknown temperature. For, if we measure R_0 and R_t any unknown temperature t can then be determined from the above relation, provided a is known. Platinum is the material usually chosen for this purpose. In order to have a high degree of accuracy it is necessary to modify the ordinary arrangement for measuring a resistance ; for in the ordinary method the resistances of the connecting leads* cannot be eliminated and an error is therefore introduced.

Platinum Thermometer

The platinum wire is wound non-inductively § on a mica frame and is enclosed in a grass or silica tube. Two copper rods connected outside to two fairly long copper wires are attached to the two terminals of the platinum wire ; these

* The wires which *lead* the current from and to the platinum wire are known as *connecting leads*, *i.e.* they are the wires which connect the platinum wire to the measuring apparatus.

§ Vide Art 149 Note (2).

connecting leads are known as platinum leads. Two other exactly similar copper rods connected outside to equally long copper wires, are also inserted within the tube but the free ends of these copper rods are soldered together; these are known as compensating leads. This entire combination is what we call a platinum resistance thermometer. Obviously if

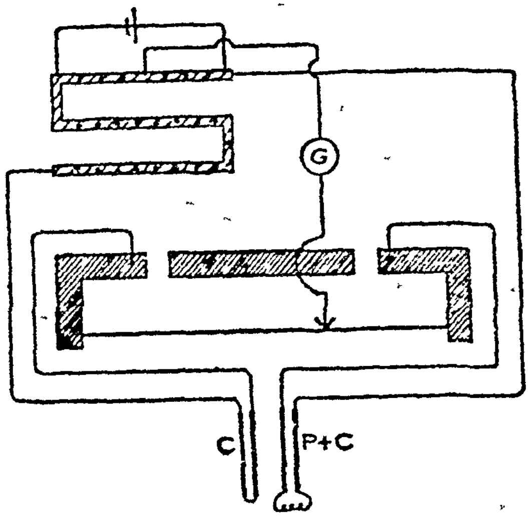

Fig. 169

C be the resistance of the compensating leads the resistance of the platinum leads together with the platinum wire, is P+C where P is the resistance of the platinum wire itself. Connections are made with a Post Office Box and a Wheatstone's bridge as shown in Fig. 169. This arrangement is known as Callendar and Griffith's Bridge. Resistances each equal to 10 ohms are inserted in the ratio arms of the Post Office Box. If R be the resistance in the third arm and if l be the null point reading on the bridge wire we have $R+C+l\sigma=P+C+(100-l)\sigma$, where σ is the resistance per unit length of the bridge wire.

Callendar & Griffith's Bridge

$\therefore$ $P=R-(100-2l)\sigma$. If σ be known P may be determined.

The relation $P_t = P_0(1+at)$ is however only approximate; hence the temperature obtained from this relation is not quite correct. We call it platinum temperature and designate it by t_p. Thus we write $P_t = P_0(1+at_p)$ The platinum thermometer is successively placed in melting ice and in steam under atmospheric pressure. The corresponding resistances P_0 at 0°C and P_{100} at 100°C are measured. If P_t be the resistance when the thermometer is placed in contact with any unknown temperature t, we have

$$P_{100} = P_0(1+100a) \quad \text{or} \quad P_{100} - P_0 = P_0 . 100a$$

and $$P_t = P_0(1+at_p) \quad \text{or} \quad P_t - P_0 = P_0 . at_p$$

$\therefore$ by division $$\frac{t_p}{100} = \frac{P_t - P_0}{P_{100} - P_0}$$

Thus the platinum temperature t_p may be obtained. The relation between platinum temperature t_p and the correct temperature t is given by $t - t_p = \delta \dfrac{t}{100}\left(\dfrac{t}{100} - 1\right)$ where δ is a constant* equal to 1·5 for pure platinum. Hence the correct temperature t may be calculated.

A platinum thermometer has a wide range; it can be satisfactorily used for measuring a temperature within the range of about −200°C to 1200°C.

In research work it is necessary that standard resistances are constructed of substances which do not change their resistance with temperature. After many trials an alloy 'manganin' (84% Cu, 4% Ni. 12% Mn) has been prepared, whose resistance may be taken as constant over a wide range of temperature.

It may be noted that unlike metals resistances of carbon filaments and also of electrolytic substances decrease with the rise of temperature.

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* This constant itself may be determined by measuring the resistance when the platinum thermometer is in contact with another fixed known temperature—usually boiling point of sulphur.

Art 111 Effect of Magnetic field on resistance.

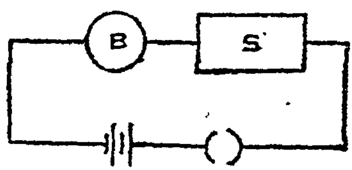
Bismuth is the only metal whose resistance is affected by a magnetic field. Its resistance increases with the strength of the magnetic field in which it is placed.

Bismuth spiral A fairly long bismuth wire is wound in the form of a spiral so that the total area covered by the wire is small. The bismuth spiral is first placed successively in a number of magnetic fields of known strengths and the corresponding resistances of the spiral are measured; a calibration curve showing the relation between the magnetic field and the resistance is thus obtained. Any unknown magnetic field can then be determined by measuring the corresponding resistance of the spiral placed in the magnetic field.

Art 112 Effect of light on resistance.

Selenium and a few crystals have got this property that in darkness their resistance is enormous but as soon as light is incident on them their resistance diminishes considerably, depending upon the intensity of the incident light. This property is most prominent in the case of selenium and has given rise to what we call a selenium 'cell'. It consists of two long metallic wires coiled and wound in such a way that they do not touch each other. Selenium — its gray modification variety—is fused between them. Thus if two ends of the wires be connected to the two terminals of a battery the circuit is completed through selenium. So long as this selenium 'cell' is covered up and is in darkness

Selenium cell



B—Bell
S—Selenium

Fig. 170

ciable current flows round the circuit due to resistance of selenium. But as soon as is opened and the cell is exposed to

selenium diminishes and if an electric bell be included in the circuit the bell begins to ring.

In the earlier days of "talkies" selenium cells were utilised for reproduction of sound from the film but now-a-days they have been entirely replaced by photo-electric cells. (Vide Art 247)

Art 113 Measurement of E. M. F.

The E. M. F. of a cell may be readily determined by means of a suitable voltmeter (Vide Art 93). The E. M. F's of two different cells may be compared by either of the following methods.

1st Method SUM AND DIFFERENCE METHOD

Two cells (whose E. M. F's E_1 and E_2 are to be compared) are joined in series with a resistance R and a tangent galvanometer. If B_1 and B_2 be the internal resistances of the two cells and if r be the galvanometer resistance

$$\frac{E_1+E_2}{R+B_1+B_2+r}=k\tan\theta_1 \quad \dots \quad (a)$$

k being the constant of the tangent galvanometer and θ_1 being the deflection produced.

If the terminals of one of the cells be now reversed, *i. e.* if the cells be now in opposition

$$\frac{E_1-E_2}{R+B_1+B_2+r}=k\tan\theta_2 \quad \dots \quad (b)$$

θ_2 being the new deflection.

$\therefore$ by dividing (a) by (b),

$$\frac{E_1+E_2}{E_1-E_2}=\frac{\tan\theta_1}{\tan\theta_2} \quad \therefore \quad \frac{E_1}{E_2}=\frac{\tan\theta_1+\tan\theta_2}{\tan\theta_1-\tan\theta_2}$$

2nd Method POTENTIOMETER METHOD

The principle of the method can be understood from Fig 171. AB is a long uniform wire to the ends of which the terminals of a cell E are joined. A resistance R and a key K are usually included in the circuit. If the current flows

in the direction A to B there is a fall of potential along the wire from A to B. If there is a parallel circuit AGC_1 (containing a galvanometer G) joining A to some point C_1 of the wire a current tends to flow in the direction AGC_1 due to the P. D. between A and C_1. If this parallel circuit also includes another cell of E. M. F. E_1 and if this cell tends to send the current in the *opposite** direction, two

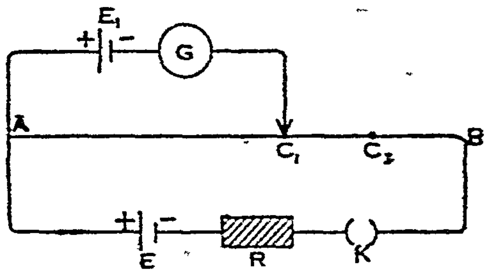


Fig. 171

currents tend to flow in this circuit in opposite directions,—one current due to P. D. between A and C_1 and the other current due to the E. M. F. E_1. If therefore the P. D. between A and C_1 be equal to E_1 there will actually be no current and hence no deflection of the galvanometer. Conversely, if by trial a point C_1 be found on the wire AB such that there is no current through the galvanometer the E. M. F. E_1 of the cell must be equal to the P. D. between A and C_1. If the cell E_1 be replaced by another cell of E. M. F. E_2 and if the corresponding null point on the wire AB be C_2, the two E. M. F's E_1 and E_2 may be compared thus :—

$$\frac{E_1}{E_2}=\frac{\text{P. D. between A and } C_1}{\text{P. D. between A and } C_2}$$

$=\dfrac{l_1\sigma i}{l_2\sigma i}$ where l_1 and l_2 are the lengths AC_1 and

* This requires that positive poles of E and E_1 are connected to A. If the negative poles of both E and E_1 be connected to A the result will be identical.

AC_2, σ is the resistance per unit length of the wire AB and i is the current along AB. Hence $\frac{E_1}{E_2}=\frac{l_1}{l_2}$.

The wire AB is called the potentiometer wire. It may be seen that if σ and i be separately determined the actual values of E_1 and E_2 can be easily obtained; for $E_1=l_1\sigma i$ and $E_2=l_2\sigma i$. The current i is determined by having a milliammeter in series with the potentiometer: σ is obtained by measuring the total resistance of the potentiometer wire,—say by a Post Office Box and by dividing this resistance by the total length.

N. B. (1) The E. M. F. E must be *greater* than E_1 or E_2; otherwise the null point C_1 or C_2 will not lie within AB.

(2) *Similar* poles of E and of E_1 or E_2 must be joined to A; otherwise there will not be any null point at all.

(3) The resistance R should be adjusted so that null points are brought near the end B of the potentiometer wire.

Art 114 Measurement of current.

A current may be measured by either of the following methods:—

1. By inserting a suitable ammeter in the circuit (Art 93).
2. By using a voltameter*. (Vide next chapter)
3. By potentiometer.

POTENTIOMETER METHOD.

This is essentially a method of comparison of two E.M.F.'s (Vide Art 113). A small but accurately known resistance r is inserted in the circuit in which the current i is to be determined. The potential difference ir between the terminals of the resistance r is compared with the known E. M. F. E of a cell by means of a potentiometer. The actual connection is

* Students must distinguish between voltmeters and voltameters.

shown in Fig. 172. If l_1 and l_2 be the null point lengths on the potentiometer wire corresponding to ir and E respectively we have

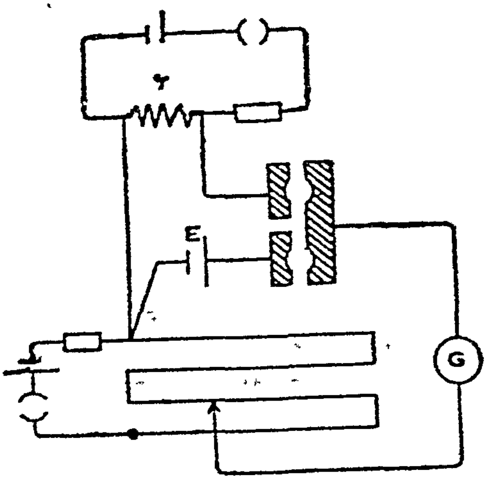


Fig. 172

$$\frac{ir}{E} = \frac{l_1}{l_2}.$$

Hence knowing other quantities, i can be determined.

N.B. Very large currents can be accurately measured by this method, *e. g.* if the current be 1000 amperes, a low resistance of the order of 0·001 ohm should be chosen for r. The P. D between its terminals would therefore be of the order of one volt. This may easily be compared with the E. M. F. of a standard cell.

Art 115 **Measurement of capacity** The capacity of a condenser may be measured by a ballistic galvanometer if a cell of known E.M.F be readily available.

A condenser of capacity C, a cell of known E. M. F. E and a ballistic galvanometer G are connected as shown in Fig 173. Keeping the key K_2 open as the key K_1 is closed the condenser acquires a charge CE. If now the key K_1 is opened and immediately thereafter* the key K_2 is closed

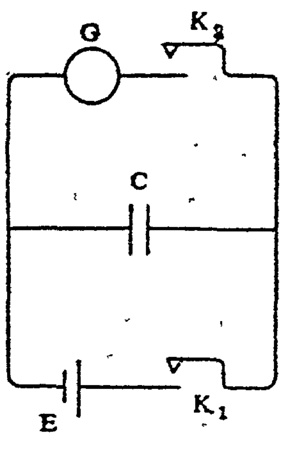


Fig. 173

* A special type of key containing both K_1 and K_2 is used. It is so arranged that by a single operation K_1 is opened and immediately thereafter K_2 is closed.

then the condenser is discharged through the ballistic galvanometer and the *throw* θ is observed. Hence $CE = k\theta$ where k is the galvanometer constant. If the galvanometer be previously calibrated, *i. e.* if the constant k be previously determined the capacity may be found out if E be known.

[Vide also Arts 58 and 155]

Exercise XII

1. 225 cms of a potentiometer wire are required to balance a Leolanche cell. If the cell be however shunted by a wire of resistance 5 ohms only 200 cms of the potentiometer wire are required for balancing. Find the internal resistance of the Leelanche cell. Ans. 0·625 ohm.

2. A 4-volt cell of internal resistance one ohm is used to send a current through a potentiometer 2 metres long and 5 ohms in resistance. Find what length of the wire would be required to balance a cell of E. M. F. 1·5 volts. What would be this length if a resistance of 3 ohms is placed (*a*) in series (*b*) in parallel with the potentiometer.

Ans. 90 cms ; (*a*) 135 cms (*b*) 115 cms.

3. A current of 72 milli-amperes passes through a potentiometer wire of resistance 25 ohms and of length 10 metres. If 600 cms are required to balance a cell find the E. M. F. of the cell.

[Hints :—Resistance of 600 cms $= 600 \times \frac{25}{1000} = 15$ ohms.

$\therefore$ E. M. F. $= 72 \times 10^{-3} \times 15 = 1{\cdot}08$ volt]

4. How does the change of temperature affect the resistance of a conductor ?

Describe an experimental arrangement illustrating how this property of conductors is utilised in the measurement of high temperatures. C. U. 1933

5. If the specific resistance of platinum at 0°C is $8{\cdot}96 \times 10^{-6}$ ohms and its temperature coefficient 32×10^{-4}, find the length of the wire of diameter 0·0274 cm., which has a resistance of 4 ohms at 50°C. C. U. 1938 Ans. 227 cms.

C U. Question

1950. Write short notes on (*a*) Platinum Resistance thermometer (*b*) Selenium cell.

1951. Why is a Wheatstone's Bridge unsuitable for the measurement of very low resistances? What method would you adopt for the accurate measurement of such resistances? Give a neat electrical diagram of the method and deduce the underlying theory.

1958. Write notes on Selenium cell.

1966. Describe the Wheatstone's bridge method of measuring resistance.

In a slide wire bridge the wire is 100 cms long and the standard resistance R_s is 100 ohms; a balance is obtained at a point 47·15 cms from the end to which the unknown resistance R_x is connected. On inter-changing R_s and R_x the balance point occurs 52·75 cms from the same end. Find the value of R_x.

Ans. 89·41 ohms.

1967. Describe the laboratory type potentiometer and explain the principle of its working with a suitable diagram. How can two E. M. F's be compared with it?

The current in a slide wire potentiometer is adjusted with a rheostat so that the galvanometer reads zero when a 1·08 volt standard cell connected to a slider, intercepts 300 cms of the calibrated resistance wire. An unknown dry cell substituted for the standard cell produces a null balance at 450 cms. What is the dry cell E. M. F? Ans. 1·62 volt.

1968 Describe the construction of a Platinum resistance thermometer and explain how it may be used to determine the boiling point of a liquid.

1969. Write notes on "Measurement of current by potentiometer"

1975. Write short notes on "Potentiometer and its use."

CHAPTER XIII

ELECTROLYSIS

Art 116 We have hitherto considered "Heating Effect" and "Magnetic Effect" of an electric current. The passage of an electric current through a substance generates heat within the substance and creates a magnetic field in the neighbouring space but usually produces no change in the composition of the substance itself. There is however a class of conducting substances—usually fused salts or solutions of salts in some liquids—which exhibit a rather curious phenomenon when a current passes through them. The molecules of these substances consist of two parts—a basic radical (or a positive*ion) and an acid radical (or a negative*ion). It is found that on passing a current through these substances these two constituent parts or ions are liberated near the two electrodes—positive ions

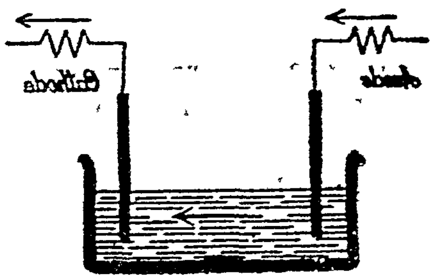


Fig. 174

near the negative electrode or cathode and negative ions near the positive electrode or anode. These ions are also known as anions and kations. This phenomenon is known as Electrolysis, such substances are called electrolytic substances

* Why these ions are called positive and negative will be explained later.

and the apparatus in which the decomposition of the substance takes place by the electric current, is called a voltameter. §

The electrode through which the current enters the solution, is the anode and the electrode through which the current goes out of the solution is the cathode. The current passes *through the solution* from anode to cathode. In the case of a cell it is just the opposite; the current passes *through the cell* from the cathode to the anode.

Thus if we electrolyse acidulated† water hydrogen and oxygen come out near the cathode and the anode respectively. In the case of dilute hydrochloric acid solution hydrogen and chlorine evolve near the two electrodes. Sometimes the process is slightly more complicated. Thus when a current is passed through NaCl solution the liberated Na ion reacts with water producing NaOH, $2Na+2H_2O=2NaOH+H_2$; hydrogen is thereby liberated at the cathode. At the other end chlorine of course comes out. The nature of the electrodes also, sometimes plays an important part. If we use copper electrodes in a $CuSO_4$ solution Cu gets deposited on the cathode; and at the other end SO_4 combines with the copper of the anode, forming $CuSO_4$ again. So *nothing* comes out near the anode or the cathode and the strength of $CuSO_4$ solution is maintained. If however the electrodes are made of platinum instead of copper SO_4 does not combine with platinum but reacts whith H_2O (of the solution) producing H_2SO_4, $2SO_4+2H_2O=2H_2SO_4+O_2$; Oxygen thus comes out at the anode. At the cathode Cu is of course deposited on platinum.

Art 117. The following definitions are very useful:—

The gram atom of an element is equal to its atomic weight expressed in grams. Thus the gram atom of silver is 108 gms.

The gram molecule of a compound is its molecular weight expressed in grams. Thus the gram molecule of $CuSO_4$ is $63+32+4\times16=159$ gms.

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§ Students must distinguish between a voltmeter and a voltameter.

† Pure water is a non-conductor of electricity. To make it conducting a little acid is added to it.

The gram equivalent of a compound is equal to the gram molecule divided by the valency of the basic radical. Thus the gram equivalent of $CuSO_4$ is $\frac{159}{2} = 79{\cdot}5$ gms.

Gram atoms of all elements contain the same number of atoms; gram molecules of all compounds contain the same number of molecules. This number (same in either case) is Avogadro's number and is usually denoted by the letter N.

The **chemical equivalent** of an element is its weight in grams, which combines with or displaces one gram of hydrogen. Numerically, chemical equivalent $= \frac{\text{gram atom}}{\text{valency}}$

	Atomic Wt	Valency	Chemical Equivalent
Thus for Hydrogen	1	1	1 gm
„ Sodium	23	1	23 gms
„ Copper (Cupric)	63	2	31·5 „
„ Iron (Ferric)	56	3	18·7 „
„ Silver	108	1	108 „

etc, etc, etc.

Art 118 Faraday studied the phenomenon of electrolysis extensively and formulated the following laws in 1833 :—

Faraday's Laws *1st Law. The amount of a substance liberated by a current is proportional to the quantity of electricity passing through the electrolyte.*

2nd Law. If voltameters containing different electrolytes be connected in series, so that the same quantity of electricity is passed through all of them, amounts of different substances liaberated at different electrodes are proportional to their respective chemical equivalents.

Since the quantity of electricity is equal to the product of the current and time ($Q = it$), the 1st Law may be split up into two laws :—

(*a*) *The amount of a substance liberated by a current is proportional to the current.*

(*b*) *The amount of a substance liberated by a current is proportional to the time for which the current passes through the electrolyte.*

∴ Mathematically,

Mass of ion liberated $W \propto it$

∴ $W = i z t$, where z is a constant.

This constant z is called the electro-chemical equivalent (E. C. E) of the substance. It is defined as the amount of the substance liberated by a unit quantity of electricity.

The 2nd law may also be enunciated thus :—

Chemical equivalents of different substances are liberated by the same quantity of electricity.

By careful experiments this quantity has been found to be equal to 96500 coulombs or 9650 C. G. S. units of electricity. This is also sometimes called a Faraday of electricity.

Thus 1 gm of H_2, 23 gms of Na, 31·5 gms of Cu, etc., etc. are liberated by 96500 coulombs.

Hence from the definition of E. C. E.

$$\text{E. C. E of } H_2 = \frac{1}{96500}, \text{ E. C. E. of Na} = \frac{23}{96500} \text{ etc., } i.\ e.$$

$$\text{E. C. E. of a substance} = \frac{\text{Chem. Equiv. of the substance}}{96500}$$

$$\therefore \frac{\text{E. C. E. of a substance A}}{\text{E. C. E. of a substance B}} = \frac{\text{Chem. equiv. of A}}{\text{Chem. Equiv. of B}}.$$

This enables us to calculate E. C. E. of a substance when that of any other is known. For example, the E. C. E. of H_2 is known to be 0·00001036 ; then since the chemical equivalents of Cu and of H_2 are respectively equal to 31·5 and 1

$$\text{E. C. E. of Cu} = \frac{31{\cdot}5}{1} \times 0{\cdot}00001036 = 0{\cdot}000329$$

N.B. (1) The E. C. E of a substance can be expressed in practical units as well as in C. G. S. units. Thus E. C. E.

of Cu = 0'000329 gms per coulomb = 0'00329 gms per C. G. S. unit of electricity.

(2) In the equation $W = izt$, if three of the four quantities involved be known, the fourth can be easily determined. If both i and z are supposed to be known, both should be expressed either in practical units or in C. G. S. units. If however one is known and the other unknown the unknown will be obtained in the unit in which the known is expressed.

A current of 0'5 amp produces a deflection of 30° in a tangent galvanometer. If a current passes through this galvanometer and a copper voltameter in series and if the steady deflection of the galvanometer be 60° find the thickness of copper deposited uniformly in 2 hours over an area of one sq. meter. [E.C.E. of Cu = 0'00033 gm per coulomb ; Density of Cu = 8'9]

$$0{\cdot}5 = k \tan 30° = \frac{k}{\sqrt{3}} \quad \therefore \; k = 0{\cdot}5 \times \sqrt{3} = \frac{\sqrt{3}}{2} \text{ practical unit.}$$

When the deflection is 60° the current $i = k \tan 60° = \frac{3}{2}$ amp.

∴ Amount of copper deposited in 2 hours

$$= \frac{3}{2} \times 0{\cdot}00033 \times 2 \times 60 \times 60 = 3{\cdot}564 \text{ gms.}$$

$$\therefore \text{ required thickness} = \frac{3{\cdot}564}{8{\cdot}9 \times 100 \times 100} = 40 \times 10^{-6} \text{ cm.}$$

Art 119. Arrhenius Theory The most successful theory attempting to explain the above phenomenon, is that due to Arrhenius. According to him as soon as an electrolytic substance is dissolved in a suitable liquid, some—not all—of the molecules of the solute are dissociated, *i. e.* broken up into two components or ions. Both these ions are charged with electricity—one ion (basic radical) is charged with +ve electricity and is therefore called positive ion or anion, the other ion (acid radical) with −ve electricity and is therefore known as negative ion or Kation. Thus a NaCl

molecule is broken up into Na^+ and Cl^-, a $CuSO_4$ molecule into Cu^{++} and SO_4^{--} and so on.§

It should be remembered that at ordinary dilution only a fraction of the total number of molecules is dissociated in the above way. The degree of dissociation, *i.e.* the percentage of dissociated molecules depends upon the dilution of the solution. The greater the dilution the greater will be the degree of dissociation. At infinite* dilution the dissociation is complete, *i. e.* all the molecules are dissociated.

As soon as a potential difference is established between the electrodes placed within the solution these charged ions begin to move, positive ions towards the cathode and negative ions towards the anode. As a matter of fact the movement of these charged ions constitutes the electric current through the solution, or in other words, these ions act as carriers of electricity for the passage of the current.

It must not be supposed that at any dilution molecules once dissociated always remain in the dissociated state until an electric current makes the ions move towards the respective electrodes. The process is on the other hand a case of statistical equilibrium. It has been established from various considerations that the molecules of the solute as well as the ions produced by the dissociation of the molecules, behave like the molecules of a gas. They move hither and thither, collide with one another at frequent intervals and travel short distances known as free paths, before meeting with fresh collisions. According to modern ideas an atom consists of a number of electrons, rotating round a central positive nucleus. These electrons are not crowded in one single orbit but are distributed over different orbits.** A Na atom contains one electron in excess of those in satisfied† orbits. A Cl atom on the other hand, has one electron too short to make the outermost orbit satisfied. When a Na atom combines with a Cl atom to form the compound NaCl, the Na atom parts with its excess electron which the Cl atom readily absorbs in order to have all its orbits satisfied. Since

§ Why these ions are charged will be explained later.

* For all practical purposes a large dilution may be regarded as infinite dilution.

** The word orbit has been used here in the sense in which the word "shell" is often used.

† Vide Art 198.

electrons are negatively charged particles, both Na atom and Cl atom become therefore charged with electricity by this process; Na atom having lost one electron is charged positively and Cl atom having gained one electron, is charged negatively. Divalent atoms, such as Cu Cd etc. have two electrons in excess of those in satisfied orbits. These two electrons are transferred to the acid radical when such molecules as $CuSO_4$, $CdCl_2$ etc. are dissociated in a solution. Thus in these cases the ions contain two electronic charges; similarly in the case of trivalent atoms there are three electronic charges in each of the ions. Ordinarily, due to electric forces existing between them these two charged ions—a basic ion and an acidic ion—remain united forming a neutral molecule. In the solution when they are moving hither and thither, these neutral molecules meet with collisions—collisions among themselves and sometimes with already dissociated ions. As a result of these collisions, some of the molecules are disrupted and broken up into two component ions. Simultaneous with this process of dissociation, another process producing recombination of already dissociated ions, also takes place. Two ions (having opposite charges) moving like gas molecules, sometimes mutually come into contact and due to electric force operating between them, re-unite forming an undissociated neutral molecule. What Arrhenius' theory therefore means, is that at any dilution a certain fraction of the total number of molecules remains in the dissociated stage, there being always an exchange between dissociated and undissociated molecules.

Art 120 That Arrhenius' theory is substantially correct is verified from the following :—

1. It is well known that if a solute be dissolved in a solvent (liquid), the boiling point of the solution is higher and the freezing point is lower than those of the pure solvent. This raising of the boiling point or the lowering of the freezing point is proportional to the number of gram molecules of the solute present per litre of the solution and hence to the concentration of the solution. Thus if a known quantity of sugar or a similar organic compound be dissolved in water, the raising of the boiling point or the lowering of the freezing point may be measured and thereby the molecular weight of the solute may be determined. If however the solute dissolved be of the class of electrolytic substances, *e. g.* NaCl, $CuSO_4$ etc. it is found that the lowering of the freezing point or the raising of the boiling point is greater than

what would be expected from the amount of the substance dissolved. At infinite dilution the effect is double of what would be produced by the number of gram molecules of the solute dissolved. These phenomena can be explained on Arrhenius' theory. For, according to this theory at any dilution a fraction of the molecules is dissociated into ions; so far as the lowering of the freezing point or the raising of the boiling point is concerned, these ions are as effective as the neutral molecules themselves. Hence the total number of molecules and ions being greater by the dissociation of some of the molecules, the effect produced by them is also greater. At infinite dilution each of the molecules is dissociated into a pair of ions and hence the effect is doubled.

2. We start with the following definition :—

$$\text{Equivalent conductivity} = \frac{\text{Specific conductivity}}{\text{Concentration}}$$

Specific conductivity is simply the inverse of specific resistance and concentration is defined to be the number of gram equivalents of the solute per c. c. of the solution.

It is found by actual experiment that as an electrolytic solution is gradually diluted the equivalent conductivity at first steadily rises, ultimately at infinite dilution (*i. e.* when the dilution is sufficiently large) the equivalent conductivity becomes constant. The actual relation is shown graphically in Fig. 175. Arrhenius' theory gives a correct explanation of the phenomenon as follows ;—

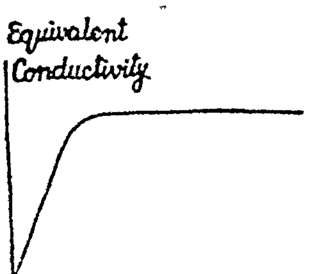


Fig. 175

Suppose for example we take a $CuSO_4$ solution and gradually diminish the concentration by diluting it with water. The total volume being thus increased the number of ions per c. c. decreases and conductivity which depends on the number of these carriers

(ions) per c. c. also decreases. The question is whether the ratio of conductivity to concentration also changes or not by dilution. If fresh ions be not produced by dilution, the number of ions per c. c. decreases exactly in the same ratio as the concentration and in that case equivalent conductivity does not change with dilution. But according to Arrhenius' theory more and more molecules are dissociated as the dilution is increased. Therefore, with gradually increasing dilution the number of ions per c. c. and hence conductivity decreases no doubt but not so rapidly as the concentration; the ratio conductivity to concentration therefore *increases* with dilution. When however all the molecules are dissolved at infinite dilution (*i. e.* at a very large dilution), any further increase in dilution does not produce fresh ions (since there are now no more neutral molecules to be dissociated) and equivalent conductivity does not change any more with further dilution.

Art 121 Long before electrons were discovered the atomic nature of electricity was brought out by the phenomenon of electrolysis as explained below.

We know that one gram atom of a substance contains N (Avogadro's number) atoms. If each of these atoms carries a charge E in the ionised state total amount of charge carried in a gram atom is NE. We also know that chem equiv $= \dfrac{\text{gram atom}}{\text{valency}}$; hence total charge carried by a chem. equiv. of substance is equal to $\dfrac{NE}{\text{valency}}$. And since chemical equivalents are liberated by one Faraday (9650 C. G S. units) of electricity, we must have

$$\frac{NE}{\text{valency}} = 9650 \quad \text{or} \quad NE = 9650 \times \text{valency} \quad \ldots \quad (a)$$

$$\therefore \quad E = \frac{9650}{N} \times \text{valency} = \frac{9650}{60{\cdot}67 \times 10^{22}} \times \text{valency}$$

$$= 1{\cdot}59 \times 10^{-20} \times \text{valency}.$$

Since valency is necessarily a positive integer 1, 2, 3, etc.,

the charge E carried by an ion must always be an integral multiple of $1{\cdot}59 \times 10^{-20}$ C. G. S. unit of charge.

Again if m be the mass of single atom of a substance of atomic weight A,

$$Nm = A \quad \ldots \quad \ldots \quad (b)$$

∴ Dividing (a) by (b), we have

$$\frac{E}{m} = \frac{9650}{A} \times \text{valency} \quad \ldots \quad (44)$$

Thus $\frac{E}{m}$ for any substance can be determined.

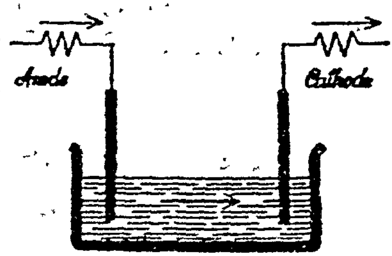


Fig. 176

Art 122. According to what has been stated in the preceding paragraphs we may assume that at any dilution there is an approximately uniform distribution of positive and negative ions throughout an electrolytic solution. With the application of an electric field between two electrodes —the anode and the cathode, these ions begin to move, positive ions towards the cathode and negative ions towards the anode. When the field is first applied ions have accelerated velocities; but since they encounter so many neutral molecules in their paths they soon acquire uniform limiting* velocities. These limiting velocities—different for different kinds of ions—obviously depend upon the strength of the potential gradient between the electrodes. The mobility of an ion is defined to be the uniform velocity generated under the action of unit potential gradient. If u and v be respectively the mobilities of the positive and negative ions and V be the actual potential gradient, *i. e.* potential difference per cm applied between the electrodes, the actual velocities of ions are uV and vV.

Mobility

* This is analogous to the limiting velocities acquired by small droplets of rain falling freely in air under the action of gravity.

The first step towards determination of u and v was made

Determination of $\frac{u}{v}$

by Hittorf who determined the ratio $\frac{u}{v}$ in an ingenius way. Consider the diagram (Fig. 177) originally given by Hittorf. Let us suppose that at the instant of applying the field both ions are

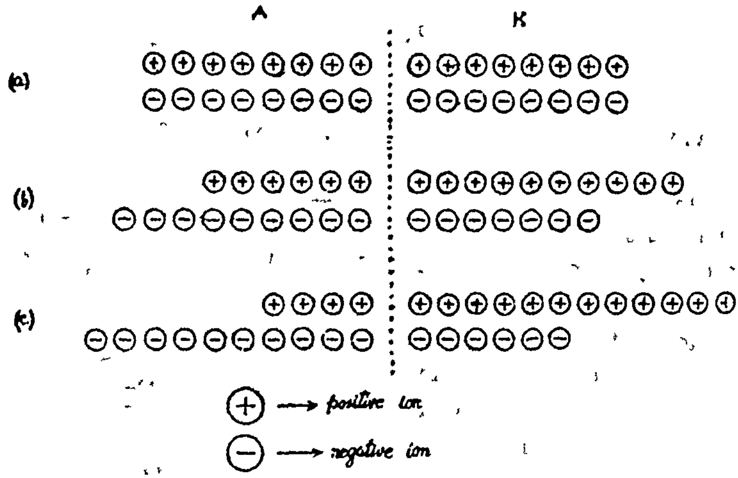


Fig. 177

arranged uniformly as shown in (*a*) : to be more precise let us suppose that there are 16 completely dissociated molecules, 8 molecules on the anode side A and an equal number on the cathode side K. For simplicity let us assume that the velocity of positive ions is double that of negative ions, *i. e.* let $\frac{u}{v}=2.$

An instant after the passage of the current the state of affairs is as indicated in (*b*). The ratio of mobilities being 2 while two positive ions pass to the cathode side only one negative ion is transferred to the anode side. Three ions which are free on either side are deposited on each electrode and there are now 6 molecules on the anode side and 7 molecules on the cathode side. Still another instant later, as shown in (*c*), three more ions (*i. e.* 6 ions in all) are deposited on each electrode ; there are now 4 molecules in the anode

half and 6 in the cathode half. Thus it will be seen that at each step, the loss in concentration of solute on the anode side is twice as large as that on the cathode side. Hence

$$\frac{\text{Rate of diminution in concentration at anode}}{\text{Rate of diminution in concentration at cathode}} = 2 = \frac{u}{v}$$

We have proved this when the ratio is assumed to be 2; this is however true generally. To determine

Experiment. this ratio experimentally two beakers containing the solution are joined by a syphon tube also containing the solution. Two electrodes are immersed in

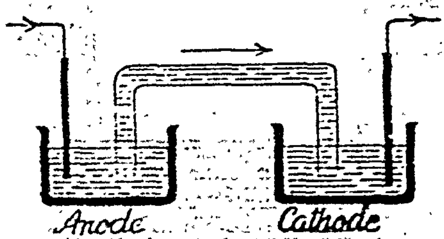


Fig. 178

the two beakers and an electric current is passed as shown in Fig. 178. At the start concentration is the same in the two beakers; but with the passage of the current the concentration diminishes at each of the two electrodes. Portions of the solution are removed from each vessel from time to time and are analysed chemically. Thus the rate of diminution in concentration at each electrode is determined and hence the ratio $\frac{u}{v}$ is obtained.

Art 123
Determination of $u+v$

A further step was due to Kohlrausch who determined $u+v$. Motion of positive ions constitutes a positive current i_1 from the anode to the cathode. Similarly by the movement of negative ions we have a negative current i_2 from the cathode to the anode; but this is equivalen to a positive current i_2 from the anode to the cathode. Thus i_1+i_2 represents the total current flowing through the solution

from the anode to the cathode. As before let V be the potential gradient so that the actual velocities of ions are uV and vV. Let c be the concentration of the solution *i. e.* let there be c gram equivalents of the solute in each c. c. of the solution. If we also suppose that dilution is so large that all the molecules are completely dissociated then there are c gram equivalents of each ion in one c. c. And since one gram equivalent of an ion carries 96500 coulombs, total charge of each kind per c. c. of the solution is c. 96500 coulombs.

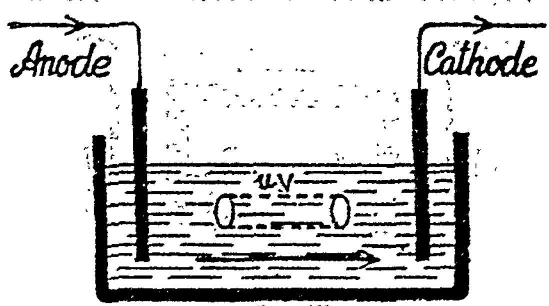


Fig. 129

Consider now a unit area at right angles to the current. Let us imagine a cylinder whose base is this unit area and whose length is equal to uV taken along the direction of the current. The volume of this cylinder being uV the number of gram equivalents of positive ions in this cylinder is cuV and therefore the total amount of positive charge within this cylinder is cuV. 96500. And since all this charge crosses the unit area in one second the current density i_1 per unit area, due to movement of positive ions, is given by

$$i_1 = cuV.\ 96500 \text{ amperes per sq. cm.}$$

Similarly the current density i_2 due to movement of negative ions is

$$i_2 = cvV.\ 96500 \text{ amperes per sq. cm.}$$

Hence total current density

$$i = i_1 + i_2 = cV(u+v).\ 96500 \text{ amperes per sq. cm.}$$

If k be specific conductivity of the solution this current is also equal to kV. For, k being the specific conductivity,

$\frac{1}{k}$ is the resistance between the faces of a centimetre cube of the solution and V (Potential gradient) is the potential difference between the same two faces; hence by Ohm's Law, current per sq. cm $= kV$.

Thus $$cV(u+v).\ 96500 = kV.$$
$$\therefore\ u+v = \frac{k}{c}\frac{1}{96500}$$

$\frac{k}{c}$ is the equivalent conductivity of the solution at infinite dilution and can therefore be measured. Hence $u+v$ can be determined.

Thus knowing $\frac{u}{v}$ and $u+v$, u and v can be separately determined.

Art 124
Conduction of electricity

We may here profitably discuss *how* a current passes through various substances. The passage of a current means flow of electricity; and for electricity to flow through a substance there must be carriers conveying electricity from one point to another.

According to modern ideas all substances contain electrons. In the case of a metallic conductor it is believed that at least some of the electrons are loosely bound with the parent atoms and can be detached quite easily; they are more or less free to move. When a potential difference is established between any two points within the conductor these electrons begin to move from the negative terminal to the positive one, *i. e.* these electro s act as carriers of electric charge. It should be noted that the movement of electrons constitutes a negative current. A negative current from the negative terminal to the positive one, is equivalent to a positive current in the reverse direction.

In the case of an electrolytic substance, as we have seen, some of the molecules of the solute are dissociated into two oppositely charged ions. These act as carriers of electricity and move towards the respective electrodes by the influence of any potential difference established between two points within the solution.

The conduction of electricity through gases is a slightly different process. Ordinarily, gases are non-conductors because of want of any carriers of electricity; a current cannot therefore be made to pass through a gas. But if some of the molecules of the gas be ionised, *i. e.* if one or more electrons be removed from the molecules the remaining portions will be positively charged. These positively charged molecules (*i. e.* positive ions) and the electrons (which have been removed from the molcules) float about in the gas just like ordinary molecules; when there is any potential difference between any two points these ions and electrons move towards the respective electrodes, *i. e.* they act as carriers and their motion constitutes the electric current. This ionisation of gas molecules can be brought about in a variety of ways, *e. g.* by ultra violet rays, by X rays, by γ Rays etc. etc.

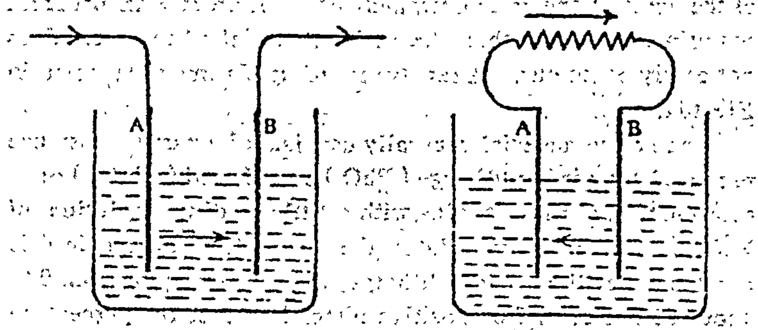
Art 125 Storage battery

We are now in a position to discuss what is called a secondary cell. Secondary cells are also sometimes referred to as storage cells or accumulators. Cells may broadly be divided into two classes—Primary and Secondary. In both these cells certain chemical reactions take place, cells undergo some changes and current is obtained at the expense of chemical energy. In primary cells these chemical reactions are not reversible, *i. e.* if after a current is drawn from the cell a current from *an external source* be passed through the cell in the reverse direction, the cell *is not brought back* to the initial condition. Daniell cell, Leclanche cell, Bunsen cell etc, are common examples of primary cells. Chemical reactions in secondary cells are on the other hand reversible; after a supply of current has been obtained from a secondary cell if a current be passed by an *external agency* through the cell in the opposite direction the cell is restored to the original condition and a fresh supply of current may again be obtained from it. When the cell supplies the current the cell is said to be discharging. When however a current is passed by an external agency through the cell in the

opposite direction we say that the cell is being charged. This process of charging and discharging may be repeated a large number of times. It should be remembered however that the word '*Storage*' is rather a misnomer. Electricity is never *stored* in the cell. When the cell supplies the current chemical energy is converted into electric energy; when the current passes through the cell in the opposite direction electric energy is re-converted into chemical energy and a current may again be obtained from the cell.

Art 126 Storage cells again are of two different types, acid cells and alkaly cells, according as the electrolyte is an acid or an alkaly. We shall consider acid cells first.

In an acid cell the two plates are lead (Pb) and lead peroxide (PbO_2). It is however found that instead of using ordinary lead and ordinary peroxide, if they are specially prepared or *formed* the capacity of the cell is increased materially. According to the earlier Plante process two plates of lead are dipped in a solution of H_2SO_4 *dil* and an electric current is passed for some time through the cell from an external source. A small quantity of lead peroxide is thereby formed on the plate A (positive plate) and on the other plate B (negative plate), a thin layer on the surface is



(a) Fig. 180 (b)

converted into what is called spongy lead. A short current is now drawn from the cell which is thus discharged. By repeating this process of charging and discharging the amount of lead peroxide on the positive plate gradually increases and

more of the negative plate is converted into spongy lead; thus the plates are formed. But as this process requires a long time and as this involves a fairly large cost this method

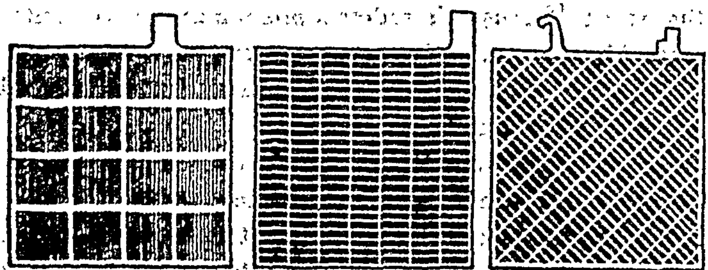


Fig. 181

is seldom used in modern times. In the other method known as Faure method the two electrodes are technically known as Grids on which active material is pasted. Grids are generally made of an alloy of lead and antimony, the percentage of antimony varying between 5 and 12. In batteries required to supply a fairly large current for short duration light grids are generally preferred. When however the discharge is intermittent and the battery is to be designed for long life heavier grids are almost always used. The shape of the grids is specially designed—it is different with different manufacturers—so that the active material when pasted does not easily come out. A few types of grids may be seen in Fig 181.

The active material generally consists of a pasty substance prepared by mixing litharge (PbO) or red lead (Pb_3O_4) or a combination of these oxides with a little of dilute solution of H_2SO_4. For negative plates the proportion of red lead is much smaller than that of litharge, the range varying from 0 to about 25 per cent. For positive plates however the percentage of red lead in the mixture varies between 60 and 80.

When litharge and red lead are each mixed up thoroughly with *dil* H_2SO_4 the following reactions take place

$$Pb_3O_4 + 2H_2SO_4 = 2PbSO_4 + PbO_2 + 2H_2O$$

and $$PbO + H_2SO_4 = PbSO_4 + H_2O.$$

Thus in both the plates $PbSO_4$ is produced.

After the paste is firmly fixed on the grids and after it is thoroughly dried up the grids are immersed in a solution of *dil* H_2SO_4 and the cell is charged.

During charging [Vide Fig 180 (*a*)] as the current passes through the solution water molecules are electrolysed so that H_2^{++} ions move with the current towards the cathode B and O^{--} ions move towards the anode A. The following reactions take place at the electrodes during charging :—

$$\left.\begin{array}{ll} \text{At Anode A} & PbSO_4+O^{--}+H_2O=PbO_2+H_2SO_4+2e \\ \text{At cathode B} & PbSO_4+H_2^{++}=Pb+H_2SO_4-2e \end{array}\right\} \ldots (a)$$

where *e* represents the electric charge.

The cell is now in a position to supply current. If the plates A and B be connected by a resistance (external circuit) [Vide Fig 180 (*b*)], a current passes outside the cell from A to B. It is obvious that *inside* the cell the current now passes from B to A. Accordingly H_2^{++} ions and O^{--} ions formed by the decomposition of water molecules now move towards A and B respectively. The following are the reactions that take place at the two electrodes during discharge :—

$$\left.\begin{array}{ll} \text{At Anode A} & PbO_2+H_2^{++}+H_2SO_4=PbSO_4+2H_2O-2e \\ \text{At Cathode B} & Pb+O^{--}+H_2SO_4=PbSO_4+H_2O+2e \end{array}\right\} \ldots (\beta)$$

It will be seen from equations (α) and (β) that during charging H_2O molecules are absorbed and H_2SO_4 molecules are produced. Reverse is the case during discharge. Thus the sp. gr. of H_2SO_4 solution rises during charging and falls during discharge. When the cell is fully charged the sp. gr. rises to about 1'25. Later on as the cell is discharged the sp. gr. gradually decreases. It is not however safe to allow the sp. gr. to go down to near about 1'18, for in that case it is found by experience that the cell does not take the charge fully later on during charging. From time to time the sp. gr. of the solution is therefore measured. As it tends to approach 1'18 the cell is charged. Thus the sp. gr. of the solution may be regarded as the true index of the condition of the cell.

The sp. gr. of the solution sometimes increases due to evaporation of water from the solution. To counteract this distilled water is often added to the solution so that the level is maintained at a definite marking usually noted on the body of the cell by the manufacturers.

The E. M. F. of an acid cell is approximately 2 volts. It does not change very much during discharge unless of course the cell is discharged much beyond safety limits. The value of the E. M. F. does not therefore tell us much about the condition of the cell.

Art 127 We shall now consider Alkaly cells. The most important type is Edison cell or NIFE cell. As in the case of acid cells, here also active material is attached to two grids immersed in an electrolytic solution. The electrolyte in this case is a solution of potassium hydroxide (KOH) having a sp. gr. of 1·12 to 1·25. A small amount of lithium hydroxide is usually added to the solution. This is found to increase the capacity of the cell. The grids are prepared of nickel-coated steel plates provided with suitable openings in which active material is fastened. The active material in the case of positive plates, consists of nickelous hydroxide which is converted into a higher oxide of nickel during the process of formation. This being a non-conductor, flake nickel or graphite is added to it to provide necessary conductivity. In the case of the negative plate the active material is ferrous oxide. It is prepared by a special method, dried, ground and then mixed with a small percentage of yellow oxide of mercury. When the cell is first charged the oxide of mercury is converted into metallic mercury which increases the conductivity of the mixture. The active material is contained in narrow perforated nickel-plated steel tubes which are ultimately clamped on the grid by hydraulic press.

During charging the potassium hydroxide molecules are decomposed. K^+ ions go towards the cathode B and OH^- ions towards the anode A. The following reactions take place at the two electrodes.

$$\left.\begin{array}{ll}\text{At Anode} & NiO+2OH^- = NiO_2+H_2O+2e \\ \text{At Cathode} & FeO+2K^+ + H_2O = Fe+2KOH-2e\end{array}\right\} \dots (\gamma)$$

During discharge the current passes *through* the solution from B to A. K^+ ions therefore move towards the anode A and OH^- ions towards the cathode B. The reactions are as follows :—

$$\left.\begin{array}{ll}\text{At Anode} & NiO_2+2K^+ + H_2O = NiO+2KOH-2e \\ \text{At Cathode} & Fe+2OH^- = FeO+H_2O+2e\end{array}\right\} \dots (\delta)$$

It will be seen from equations (γ) that during charging a water molecule is absorbed at cathode but released at Anode. The percentage of water in the solution as a whole therefore remains unchanged. The same is true during discharge as will be evident from equations (δ). It is also clear from these equations that during both charge and discharge, as a number of KOH molecules is absorbed by combination with the electrodes, the same number is produced by chemical reactions. Thus the strength of the solution remains unchanged during charging as well as during discharge. The sp. gr. does not therefore vary at all—it is maintained constant somewhere between 1·19 and 1·25. Unlike in acid cells the sp. gr. is accordingly no index of the condition of the cell.

The E. M. F. of the cell on the other hand remains practically constant so long as the cell is in a fairly charged condition. As the cell tends to get discharged to such an extent that charging becomes necessary, the E. M. F. begins to fall. The E. M. F. therefore is the true index of the condition of alkaly cells. Just after the cell is fully charged for the first time the open circuit voltage rises to about 1·48 volts ; but soon after, it drops down to about 1·35 volts at which value the E. M. F. remains more or less steady.

Exercise XIII

1. State Faraday's laws of electrolysis.

If a current of 0·04 amp passing through copper sulphate solution for 2 hours liberates 0·09475 gm of copper, find how much silver will be liberated if a current of 0·05 amp

passes through silver nitrate solution for 3 hours. [At. Wt of Cu = 63; At. Wt of Ag = 108]. Ans. 0·6091 gm.

2. Explain the terms gram molecule, gram equivalent and chemical equivalent.

In a water voltameter 100·6 c. c. of hydrogen at 22°C and 56 cms of pressure are collected over platinum electrodes, when a current of 0·2 amp passes through acidulated water for 50 minutes. Find the E. C. E. of Cu. [Density of hydrogen at N. T. P. = 0·0899 gms per litre].

Ans. 0·000324 gm per coulomb.

3. Write short notes on electro chemical equivalent.

A copper voltameter and a tangent galvanometer are in series in an electric circuit. If 0·4752 gm of copper are liberated in 2 hours and if the steady deflection of the tangent galvanometer during the period be 60°, find the reduction factor of the galvanometer. [At. Wt of Cu = 63; E. C. E. of H_2 = 0·00001036 gm per coulomb]. Ans. 0·0117 C. G. S. unit.

4. What is meant by the mobility of an ion? Explain how it can be measured.

Find how many grams of water are decomposed by a current of 2 amps in 2 hrs. [E. C. E. of silver = 0·001118 gm per coulomb. At. Wt of silver = 108]. Ans. 0·1491 gm.

5. Give an account of Arrhenius theory of electrolytic dissociation and explain how this theory is supported by experimental evidence.

6. A current of 2 amperes is passed through a copper sulphate solution. The area of the cathode surface is 1·5 square meters. Calculate the average increase in the thickness of the copper deposit per minute. [E. C. E. of Cu = 0·0003295; Density of Cu = 8·9]. Ans. $2·96 \times 10^{-7}$ cm.

7. Explain how from the phenomenon of electrolysis we get an idea of the atomic nature of electricity.

Why cannot a single Daniell cell decompose water continuously?

Calculate the minimum E. M. F. necessary in order to decompose water, given the E. C. E. of Hydrogen = 0·0000105 gm/amp. sec. the heat yield of 1 gm of Hydrogen in combining

to form water $= 34500$ calories, $J = 4{\cdot}2 \times 10^7$ ergs and 1 Watt $= 10^7$ ergs/sec. C. U. 1949. Ans. 1·52 volt.

Hints: When 1 gm of hydrogen combines, energy $= 34500$ calories $= 34500 \times 4{\cdot}2 \times 10^7$ ergs. But for 1 gm of hydrogen to combine, charge (Q) required $= \dfrac{1}{0{\cdot}0000105}$ coulomb $= \dfrac{1}{0{\cdot}000105}$ C. G. S. unit. Hence if E be the required E. M. F.

$$EQ = 34500 \times 4{\cdot}2 \times 10^7.$$

$\therefore$ $E = 34500 \times 4{\cdot}2 \times 10^7 \times 0{\cdot}000105 = 1{\cdot}52 \times 10^8$ C. G. S. unit $= 1{\cdot}52$ volt.

8. Write short notes on electrolysis and atomicity of electricity.

Explain how the value of $\dfrac{E}{M}$ of an ion in electrolysis can be determined experimentally.

9. Calculate the number of molecules in one c. c. of a gas at N. T. P. from the following data :—

Chemical equivalent of any element is liberated by 9650 C. G. S. units of electricity. The charge of an electron is $1{\cdot}59 \times 10^{-20}$ C. G. S. units. One gram molecule of Hydrogen occupies 22·4 litres at N. T. P. Ans. $2{\cdot}71 \times 10^{19}$.

Hints: N (Avogadro's number) is the number of molecules in a gm molecule. This occupies 22·4 litres at N. T. P.

$$\text{And } NE = 9650 \quad \therefore \ N = \frac{9650}{E} = \frac{9650}{1{\cdot}59} \times 10^{20}.$$

$\therefore$ Number of molecules in one c. c. at N. T. P.

$$= \frac{9650}{1{\cdot}59 \times 22{\cdot}4 \times 1000} \times 10^{20} = 2{\cdot}71 \times 10^{19}.$$

C. U. Questions.

1961. A current of 2 amperes is passed through a $CuSO_4$ solution. The area of the cathode surface is 1·5 sq. meters. Calculate the average increase in the thickness of the copper deposit per minute. [E. C. E. of Cu $= 0{\cdot}0003294$; density of Cu $= 8{\cdot}9$ gms/c. c.] Ans. $2{\cdot}96 \times 10^{-7}$ cm.

1963. What do you understand by "equivalent conduc-

tivity" of a solution ? Explain the relation between equivalent conductivity and dilution of an electrolytic solution.

Describe a method of measuring electrolytic conductivity of a solution.

1965. State Faraday's laws of electrolysis and explain how they have been experimentally verified.

What is the reason for believing that a monovalent atom carries unit electric charge during electrolysis ?

1966. State Faraday's Laws of electrolysis. What is meant by the electrochemical equivalent of an element and how is it related to the chemical equivalent ?

In calibrating a 0-2 amp ammeter by using a copper voltameter the following data were obtained :—

Initial weight of the cathode......50·120 gm.

Final „ „ „ ......50·701 gm.

Ammeter reading...1 amp. Time...30 minutes.

Determine the true value of the current and hence the percentage error. (E. C. E. of copper = 0·000329)

Ans. 0·981 amp ; 1·94%

1968. State Faraday's Laws of Electrolysis and explain how these laws are verified. What do you mean by electrolytic dissociation ?

1969. Write notes on "Lead accumulator"

CHAPTER XIV

THERMO-ELECTRICITY

Art 128
Seebeck effect

Seebeck a Berlin physicist, discovered the following phenomenon in 1821 :—

If two wires of two dissimilar metals A and B be joined at two ends and if the two junctions be maintained at two different temperatures T_1 and T_2 then a current flows in the circuit. This phenomenon is now known as Seebeck effect. Two metals arranged in this way form a Thermo-couple ; and the current produced by the difference in temperatures is known as Thermo current. The direction and magnitude of the current depend upon the nature of the substances A and B. Seebeck arranged various metals in a series (known as Seebeck series) such that if a couple be made of any two of them, the current flows through the *cold* junction from one occupying higher in series to one coming lower. Seebeck's original series has now been extended and is now known as thermo-electric series This is as follows :—

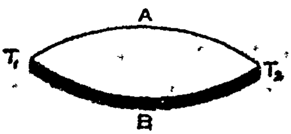


Fig. 182

Selenium (850), Antimony (100), Iron (83), Brass (76), Tin (72), Copper (72), Silver (72), Platinum with 80% Rhodium (71), Gold (71), Zinc (71), Lead (69), Mercury (65), Platinum Nickel (51), Constantan (30), Bismuth (10).

If a couple consists of two of these metals the E. M. F. (expressed in micro-volts) generated in this couple by 1°C difference of temperatures at the two junctions, is approximately equal to the difference in the numbers given in brackets against the corresponding metals.

Art 129
Peltier effect

In 1834 Peltier discovered the converse phenomenon :—

If two dissimilar metals A and B be joined end to end so that initially the two junctions are at the same temperature, and if a current be passed in any direction through this circuit,—say, by inserting a battery in the circuit, one junction gradually becomes heated and the other cooled and thus a difference of temperature is gradually established between the two junctions. Thus in Seebeck effect the current is produced by the difference in temperatures at the two junctions, while in Peltier effect the difference in temperature is produced by the current.

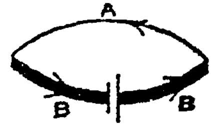


Fig. 183.

Peltier effect in relation to Seebeck effect can be seen best from the following diagrams :—

In Fig. 184 (*a*) the junction P is heated and the junction Q cooled, so that a current flows in the circuit. Let us suppose that the current flows from B to A through the junction Q. In Fig. 184 (*b*) a battery is in the circuit so that the current (due to the battery) flows in the same direction, *i. e.* from B to A through the junction Q. But in this case the junction Q gets heated and the junction P cooled.

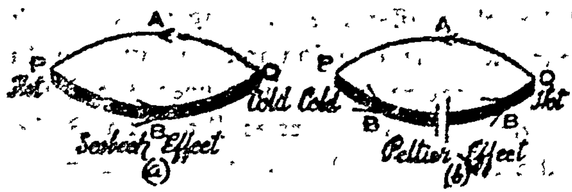


Fig. 184

It may be noted that in Peltier effect as soon as the difference in temperature is created by the current from the battery, a thermo-current tends to be produced by this difference in temperature. The difference in temperature produced in Peltier effect is always such that the thermo-current produced thereby opposes the current from the battery.

Generation of heat in Peltier effect is different from Joule heating (Chapter IX) in two respects; first Joule heating is proportional to the square of the current, whereas Peltier

heating varies as the first power of the current; secondly, Joule heating is independent of the direction of the current, but Peltier effect depends upon the direction.

Art 130
Explanation on electron theory

These two phenomena can be explained in terms of electron theory. We have already learnt that metals contain free electrons. The concentration of these electrons is different in different metals producing thereby difference in conductivity of the metals. Just as when two gases at two different pressures are brought together the pressure tends to be equalised by the flow of gas molecules from higher pressure to lower pressure, similarly when two dissimilar metals are joined together, the concentration of electrons tends to be equalised by the flow of electrons from the higher concentration to the lower one. Thus if A has the greater concentration electrons flow from A to B through the junctions. Now a metal containing free electrons also contains some positive charges, so that as a whole the metal is neutral. If therefore electrons (which are negatively charged particles) pass out from A to B, B gradually acquires negative electricity and A is gradually charged positively. Hence at each junction the two wires on the two sides acquire positive and negative charges and a potential difference (contact E. M. F. or Peltier E. M. F.) is gradually established. The flow of electrons is stopped when the contact E. M. F. (which opposes the flow of electrons) is just sufficiently large.

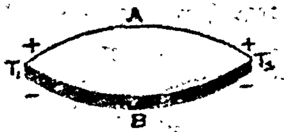


Fig. 185

When the two junctions are at the same temperature the two contact E. M. F.'s are equal and since they are oppositely directed there is no resultant E. M. F. in the circuit. If however the temperature of one of the junctions be changed the flow of electrons at the junction is modified; as a result the contact E. M. F. is altered and a current is generated by the resultant E. M. F. in the circuit.

On the other hand if a current be passed from a battery

by inserting it in the circuit, the current passes down the potential gradient at one junction and up the the potential gradient at the other. At the former heat is evolved or temperature is raised; at the latter heat is absorbed or temperature is lowered. Thus a difference of temperature is created.

Art 131
Thomson effect

In 1851 Sir William Thomson (afterwards known as Lord Kelvin) applied the ideas of Thermodynamics to the thermo-couple. So far we have seen that there are two E. M. F.'s in the couple—the two Peltier E. M. F.'s π_1 and π_2 at the two junctions at temperatures T_1 and T_2. Let us now suppose that a charge Q passes round the circuit, the charge passing from A to B through the hot junction T_2 and let the Peltier E.M.F.'s be directed as shown in the diagram.

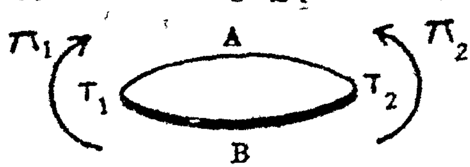


Fig. 186

It is known that energy (in the form of heat) is evolved at places where the charge flows down the potential gradient and is absorbed where the charge flows up the potential gradient. Hence at the junction T_2 energy $\pi_2 Q$ is evolved and at the junction T_1, energy $\pi_1 Q$ is absorbed. Since according to the second Law of Thermodynamics the total change of entropy* is zero, we have

$$\frac{\pi_2 Q}{T_2} - \frac{\pi_1 Q}{T_1} = 0 \quad \text{or} \quad \frac{\pi_2}{T_2} - \frac{\pi_1}{T_1} = 0 \quad \text{or} \quad \frac{\pi_2}{\pi_1} = \frac{T_2}{T_1}$$

$$\therefore \quad \frac{\pi_2 - \pi_1}{\pi_1} = \frac{T_2 - T_1}{T_1} \qquad (a)$$

If π_1 and π_2 be the only E. M. F.'s in the couple the resultant E. M. F. $E = \pi_2 - \pi_1$. If one of the junctions be kept at a fixed temperature T_1 the Peltier E. M. F. π_1 at that

* Entropy is heat evolved or heat absorbed divided by the corresponding temperature; it is positive in one case and negative in the other.

junction is also maintained constant. Hence from (*a*) it is seen that E is proportional to $T_2 - T_1$. But if the temperature T_2 of the hot junction be gradually increased it is easily seen by actual experiment that the E. M. F. E is *not* proportional to the difference of temperature $T_2 - T_1$. Sir William Thomson therefore concluded that π_1 and π_2 are not the only E. M. F.'s in the couple. He then postulated that in any wire if two points are at a difference of temperature dT, there exists an E. M. F. between those two points, equal to $\sigma\, dT$, σ being a constant for the metal. This phenomenon is known as Thomson effect. The constant σ is called Thomson coefficient. It is positive if the higher temperature point is at a higher potential and negative if reverse is the case. For Cu, Sb, Ag, Cd, Zn etc. σ is positive and for Fe, Pt, Bi, Co, Ni, Hg etc. σ is negative. For Pb σ is very approximately equal to zero.

Thus if different points of a wire for which σ is positive, be maintained at different temperatures and if a current flows through the wire, then in some portion of the wire the current flows from the colder part to the hotter part, *i. e.* flows up

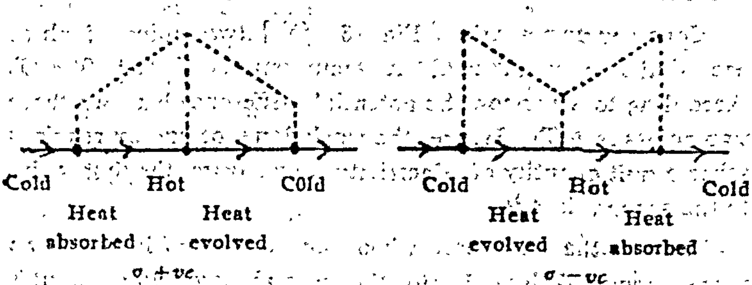


Fig. 187

the potential gradient; heat is therefore absorbed or temperature lowered in this portion. In the other portion where the current flows from the hotter part to the colder part, it flows down the potential gradient; heat is therefore evolved or temperature raised in this portion. The reverse is the effect for metals for which σ is negative.

Specific heat of electricity

Thomson's coefficient σ is sometimes called the specific heat of electricity. This is only by way of analogy.

Consider a narrow tube [Fig. 188 (a)] through which a liquid of sp. heat s flows very slowly—so slowly that liquid acquires the temperature of the tube at every point. If at

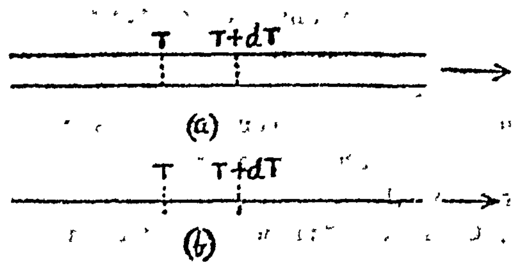


Fig. 188

two points the temperatures are maintained at T and T+dT, the amount of heat energy required when a unit mass of the liquid flows from the first point to the second, is $s\,d$T.

Consider now a wire [Fig. 188 (b)], two points of which are similarly maintained at temperatures T and T+dT. According to Thomson the potential difference between these two points is σdT. Hence the work done or energy required when a unit quantity of electricity flows from the first point to the second, is σdt.

Thus in the first case, when a unit mass of liquid flows energy required is sdT; in the second case, when a unit quantity of electricity flows, energy is σdT. In the first case s is the specific heat of the liquid; therefore in the second case by analogy, σ is called the specific heat of electricity.

Art 132

Demonstration of Peltier effect

A and B are two glass bulbs connected by a narrow tube containing a small pellet of mercury. An iron copper couple is so

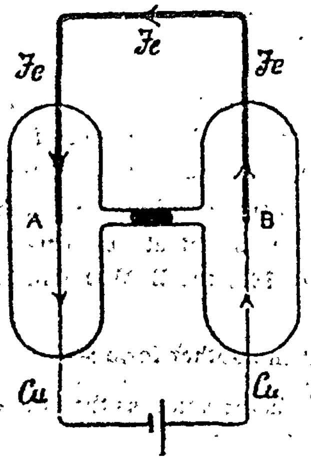


Fig. 189

arranged that the two junctions are one in each bulb. At first pressures of air within the bulbs are equal and the pellet of mercury rests in the centre of the connecting tube. If now a current be passed by inserting a cell in the circuit one of the two junctions is heated and the other cooled; as a result, the pressures of air within the bulbs become unequal and the pellet begins to move along the connecting tube. Thus Peltier effect is demonstrated.

Demonstration of Thomson effect

A thick iron rod is bent at three points A, B and C. A and C are placed in melting ice and B in steam. Two small holes are drilled into the rod at points midway between A and B and between B and C. These holes are filled with mercury and two thermometer bulbs T_1 and T_2 are immersed therein. At first the two thermometers indicate the same temperature. If now a current be passed through the rod in the direction of the arrow, (σ for Fe being negative) heat is absorbed in AB and evolved in BC. T_2 now records a higher temperature than T_1, proving the existence of the Thomson effect.

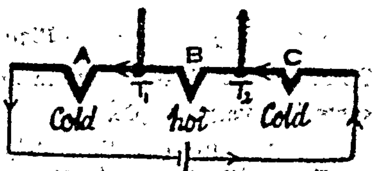


Fig. 190

Art 133
Law of intermediate temperature

The E. M. F. of a thermo-couple between any two temperatures T_1 and T_3 is the sum of two E. M. F's of the same thermo-couple between T_1 and an intermediate temperature T_2 and between T_2 and T_3. Thus

$$\Big[E\Big]_{T_1}^{T_3} = \Big[E\Big]_{T_1}^{T_2} + \Big[E\Big]_{T_2}^{T_3} \qquad \cdots \qquad (45)$$

Law of intermediate metal

The E. M. F. in a thermo-couple consisting of two metals A and B remains unaltered when one of the junctions is opened out and a third metal C is inserted, provided the two junctions at the ends of C are at the same temperature as that at the original junction of A and B. Thus in Fig. 191, the E. M. F's in the two circuits are equal.

This law may also be expressed in another form :—

The E. M. F.'s for a couple A/B is the same as the sum of the E. M. F's for two separate couples—A with a third metal

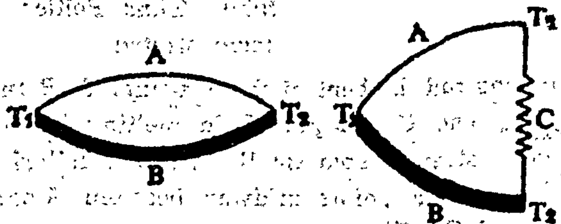


Fig. 191

C and C with B, the temperatures of the junctions being the same for every couple.

$$\text{Thus} \quad E_{T_1}^{T_2}\left(A/B\right) = E_{T_1}^{T_2}\left(A/C\right) + E_{T_1}^{T_2}\left(C/B\right)$$

From this it follows

$$\left.\begin{aligned} E_{T_1}^{T_2}\left(A/B\right) &= E_{T_1}^{T_2}\left(A/C\right) - E_{T_1}^{T_2}\left(B/C\right) \\ \text{or} \quad &= E_{T_1}^{T_2}\left(C/B\right) - E_{T_1}^{T_2}\left(C/A\right) \end{aligned}\right\} \quad \cdots \quad (46)$$

Art 134. If one of the junctions in a thermo-couple be kept at a fixed temperature and the other junction be gradually heated it is found that the E. M. F. of the couple at first

increases, reaches a maximum and then begins to decrease until at a certain temperature it is zero. Beyond this temperature the E. M. F. is reversed and goes on increasing in the reversed direction without showing any sign of again coming to a maximum.

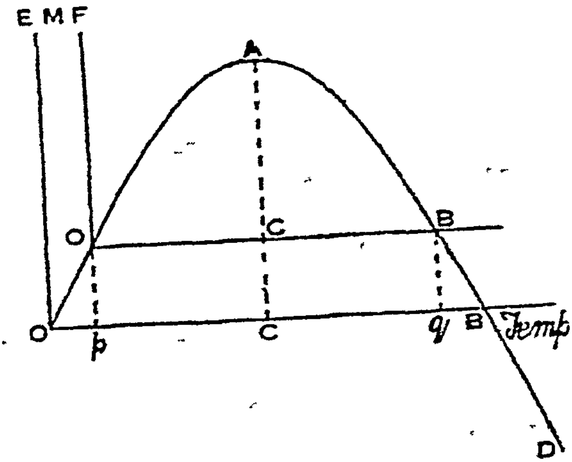


Fig. 192

If the E. M. F. be plotted against the temperature of the hot junction a curve OABD is obtained, approximately of the shape of an inverted parabola. If, as is usually the case, the cold junction be at 0°C, the equation to the curve is approximately $E=at+bt^2$, where a and b are two constants for the given couple.

The temperature C at which the E. M. F. is maximum is called the neutral temperature and the temperature B beyond which the E. M. F. is reversed, is known as the temperature of reversal. If the cold junction be at any temperature higher than 0°C the origin O is shifted to O' corresponding to the temperature Op of the cold junction. The curve is now O'AB'D, so that the neutral temperature remains unchanged but the temperature of reversal B' advances towards the neutral temperature by an amount Bq equal to Op the temperature of the cold junction.

N. B. (1) The neutral temperature is independent of the cold junction temperature and lies midway between the temperature of reversal and the cold junction temperature.

(2) At neutral temperature E is maximum and therefore $\frac{dE}{dt}=0$, Hence differentiating $E^{*}=at+bt^2$, we have $a+2bt=0$

or $t=-\frac{a}{2b}$. This is the neutral temperature and it can therefore be determined when the constants a and b are known.

Art 135 In a thermo couple we have to consider Peltier E. M. F.'s π_1 & π_2 at the two junctions at temperatures T_1, T_2 and Thomson E.M.F.'s $\int_1^2 \sigma_A dT$ and $\int_1^2 \sigma_B dT$ in the two metals A and B. Thus the resultant E.M.F. in the circuit is given by [see Fig. 193]

$$E=\pi_2-\pi_1-\int_1^2 \sigma_A dT+\int_1^2 \sigma_B dT$$

$$=\int_1^2 \frac{d\pi}{dT}dT-\int_1^2(\sigma_A-\sigma_B)dT \quad \ldots \quad (47)$$

$$\therefore \quad \frac{dE}{dT}=\frac{d\pi}{dT}-(\sigma_A-\sigma_B) \quad \ldots \quad (47a)$$

$\frac{dE}{dT}$ is called the thermo-electric power in the circuit.

Let a charge Q pass round the circuit in the clockwise direction [Vide Fig. 193]. Then energy in the form of heat is evolved at places where the charge passes down the potential gradient and is absorbed at places where the charge flows

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* If the cold junction be maintained at a temperature other than 0°C the curve may still be represented by $E=at+bt^2$, but in this case t represents the excess of the temperature of the hot junction over that of the cold one. It should be remembered that this is only an approximate equation. Another equation $\log E=a+b\log t$ has also been suggested ; but this also represents the results only approximately.

up the potential gradient. Thus at two junctions energy $\pi_2 Q$ is evolved at T_2 and energy $\pi_1 Q$ is absorbed at T_1. In the metals for every potential difference $\sigma_A dT$ in the metal A. $Q\sigma_A dT$ is absorbed and for every potential difference $\sigma_B dT$ in the metal B, $Q\sigma_B dT$ is evolved. Since by the second law of

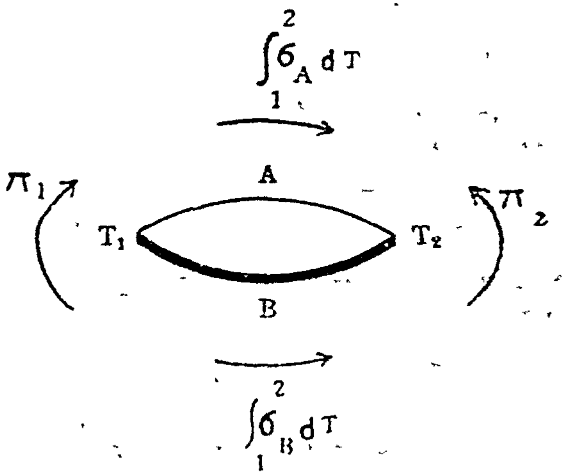

Fig. 193

Thermodynamics, the total change of entropy is zero, we have

$$\frac{\pi_2 Q}{T_2} - \frac{\pi_1 Q}{T_1} - \int_1^2 \frac{Q\sigma_A dT}{T} + \int_1^2 \frac{Q\sigma_B dT}{T} = 0$$

or $$\frac{\pi_2}{T_2} - \frac{\pi_1}{T_1} - \int_1^2 \frac{\sigma_A dT}{T} + \int_1^2 \frac{\sigma_B dT}{T} = 0$$

or $$\int_1^2 \frac{d}{dT}\left(\frac{\pi}{T}\right) dT - \int_1^2 \frac{\sigma_A - \sigma_B}{T} dT = 0$$

$$\therefore \quad \frac{d}{dT}\left(\frac{\pi}{T}\right) - \frac{\sigma_A - \sigma_B}{T} = 0$$

or $$\frac{T\frac{d\pi}{dT} - \pi}{T^2} - \frac{\sigma_A - \sigma_B}{T} = 0$$

or $$T\frac{d\pi}{dT}-\pi=T\left(\sigma_A-\sigma_B\right)$$

$$=T\left(\frac{d\pi}{dT}-\frac{dE}{dT}\right) \quad \text{from (47a)}$$

$$\therefore \qquad \pi=T\frac{dE}{dT} \qquad \ldots \qquad \ldots \qquad (48)$$

Substituting this value of π in (47*a*)

$$\frac{dE}{dT}=T\frac{d^2E}{dT^2}+\frac{dE}{dT}-\left(\sigma_A-\sigma_B\right)$$

or $$\sigma_A-\sigma_B=T\frac{d^2E}{dT^2} \qquad \ldots \qquad \ldots \qquad (49)$$

If the metal B be such that its Thomson coefficient is zero† we have

$$\sigma_A=T\frac{d^2E}{dT^2} \qquad \ldots \qquad \ldots \qquad (49a)$$

Art 136
Thermo-electric diagram

If the thermo-electric power $\frac{dE}{dT}$ $(=P)$ for any couple be plotted against the temperature T a curve—very approximately a straight line—is obtained. Let us consider two points A_1 and A_2 on

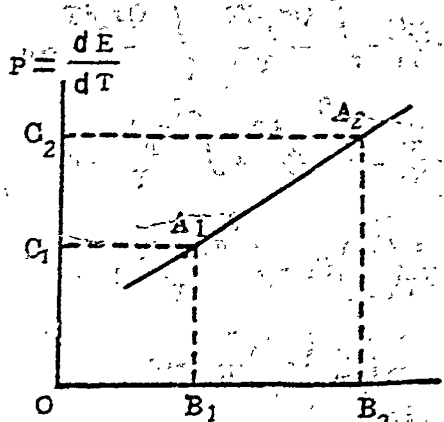

Fig. 194

† Thomson coefficient for Lead is very approximately equal to zero.

this curve, corresponding to temperatures T_1 and T_2. From these points drop perpendiculars A_1B_1 and A_2B_2 to the temperature axis and A_1C_1 and A_2C_2 to the thermo-electric axis.

Then $\quad OB_1 = T_1 \quad OB_2 = T_2 \quad OC_1 = P_1 \quad OC_2 = P_2.$

$$\therefore \quad \text{area } OB_1A_1C_1 = P_1T_1 = \left(T\frac{dE}{dT}\right)_1$$

$$\text{and} \quad \text{area } OB_2A_2C_2 = P_2T_2 = \left(T\frac{dE}{dT}\right)_2$$

Thus if the two junctions of the couple be maintained at temperatures T_1 and T_2

Peltier E.M.F. at T_1 $\quad \pi_1 = \left(T\frac{dE}{dT}\right)_1 = \text{area } OB_1A_1C_1$

Peltier E.M.F. at T_2 $\quad \pi_2 = \left(T\frac{dE}{dT}\right)_2 = \text{area } OB_2A_2C_2$

Also, Thomson E.M.F.

$$= \int_1^2 \left(\sigma_A - \sigma_B\right) dT = \int_1^2 T\frac{d^2E}{dT^2}\,dT$$

$$= \int_1^2 T\frac{dP}{dT}\,dT = \int_1^2 T\,dP = \text{area } A_1C_1C_2A_2$$

Hence resultant E.M.F.

$$= \pi_2 - \pi_1 - \int_1^2 \left(\sigma_A - \sigma_B\right) dT$$

$$= \text{area } OB_2A_2C_2 - \text{area } OB_1A_1C_1 - \text{area } A_1C_1C_2A_2.$$
$$= \text{area } A_1B_1B_2A_2.$$

Thus all the E. M. F.'s are represented by areas on this diagram. Such a diagram is called the thermo-electric diagram for the couple.

Since σ for lead is very approximately equal to zero the thermo-electric diagrams for different metals are usually plotted with lead as the other component of the couple. We

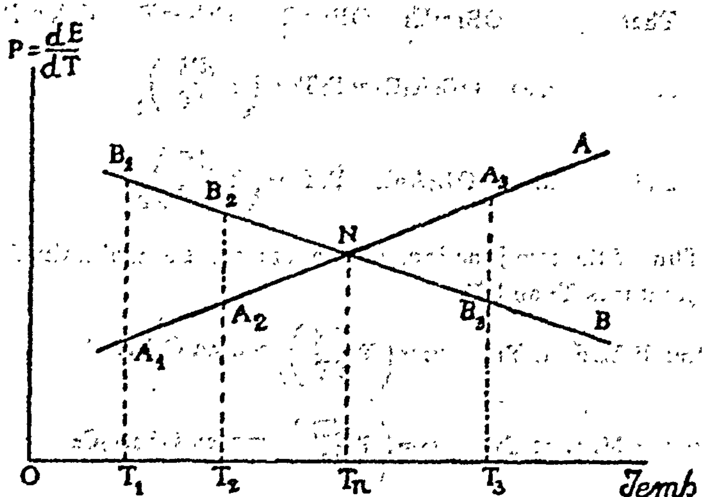

Fig. 195

know $\sigma = T\frac{d^2E}{dT^2} = T\frac{dP}{dT}$; σ is therefore positive or negative according as $\frac{dP}{dT}$ is positive or negative, *i. e.* according as the curve slopes upwards or downwards. Thus in Fig. 195 for the metal A (sloping upwards) $\frac{dP}{dT}$ is positive and hence σ is positive; and for the metal B (sloping downwards) σ is negative because $\frac{dP}{dT}$ is negative.

If a couple be formed with the metals A and B, the junctions being maintained at temperatures T_1 and T_2 [Vide Fig. 195], then

for $\text{Lead}/_A$ couple E. M. F. = area $A_1A_2T_2T_1$

and for $\text{Lead}/_B$ couple E. M. F. = area $B_1B_2T_2T_1$

Hence by the Law of Intermediate metal [Vide Art 133, equation (46)], for the couple A/B,

$$E\left(A/B\right)=E\left(\text{Lead}/B\right)-E\left(\text{Lead}/A\right)$$
$$=\text{area } B_1B_2T_2T_1-\text{area } A_1A_2T_2T_1$$
$$=\text{area } B_1B_2A_2A_1$$

If now the cold junction be maintained at T_1 while the temperature of the hot junction is gradually increased, the area $B_1B_2A_2A_1$ gradually increases and hence the E. M. F. also increases. This goes on until the point N is reached. Beyond this the E. M. F. diminishes; for if we take a temperature T_3 beyond N, the E. M. F. is the difference between the areas $B_1B_3T_3T_1$ and $A_1A_3T_3T_1$ *i. e.* the E. M. F. is equal to the difference between the two triangles NB_1A_1 and NB_3A_3. Thus the temperature T_n corresponding to the point N, gives us the neutral temperature for the couple A/B.

The thermo-electric diagram therefore gives us all possible information with regard to thermo-electric effect.

The thermo-electric power of iron is 1734 micro-volts per degree at 0°C and 1247 at 100°C, that of copper is 136 at 0°C and 231 at 100°C. Calculate the E. M. F. of an iron-copper couple between the temperatures 0°C and 100°C.

If P represents the thermo-electric power at any temperature Pelter E. M. F. $\pi=TP$ and Thomson coefficient $\sigma=T\dfrac{dP}{dT}$.

For the iron-copper couple, at 0°C $P_0=1734-136=1598$ and at 100°C $P_{100}=1247-231=1016$. Hence at 0°C $\pi_1=273\times1598=436254$ and at 100°C $\pi_2=373\times1016=378968$.

Again, for iron $\dfrac{dP}{dT}=\dfrac{1247-1734}{100}=-4{\cdot}87$ and for copper $\dfrac{dP}{dT}=\dfrac{231-136}{100}=0{\cdot}95$. Hence σ_A (for iron) $=-4{\cdot}87T$

and σ_B (for copper) $= +0{\cdot}95T$. $\therefore$ $\sigma_A - \sigma_B = -5{\cdot}82T$.

$$\text{Hence} \int_1^2 (\sigma_A - \sigma_B)\, dT = \int_{273}^{378} -5{\cdot}82\, T\, dT$$

$$= -5{\cdot}82 \left[\frac{T^2}{2}\right]_{273}^{378} = -187986$$

$$\therefore \text{ Required E. M. F.} = \pi_2 - \pi_1 - \int_1^2 (\sigma_A - \sigma_B)\, dT$$

$$= 378968 - 436254 + 187986$$

$= 130700$ micro-volts $= 0{\cdot}131$ volt

This problem can also be solved with the help of thermo-electric diagram.

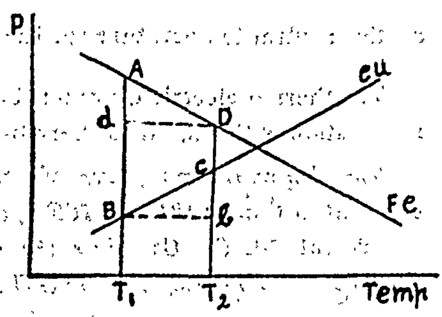

Fig. 196

In Fig. 196 T_1 represents 0°C and T_2 100°C. Hence $AT_1 = 1734$, $DT_2 = 1247$.

$BT_1 = 136$, $CT_2 = 231$ and $T_1T_2 = 100$.

Hence E.M.F. = area ABCD = area AT_1T_2D − area BT_1T_2C

But area AT_1T_2D.
= area AdD + area DT_2T_1d

$$= \tfrac{1}{2} \times 100 \times (1734 - 1247) + 100 \times 1247 = 149050.$$

And area BT_1T_2C = area BbC + area BT_1T_2b

$$= \tfrac{1}{2} \times 100 \times (231 - 136) + 100 \times 136 = 18350.$$

Hence E. M. F. $= 149050 - 18350$

$= 130700$ micro-volts $= 0{\cdot}131$ volt.

Art 137 Applications of thermo-electric phenomena

1. Thermo-couple.

The thermo-couple enables us to measure an unknown

temperature. The E. M. F. generated in a thermo-couple can easily be measured by a potentiometer. The couple is connected to the potentiometer wire AB as shown in the diagram. The temperature T_1 of one of the junctions is maintained at 0°C by placing the junction in melting ice. The temperature

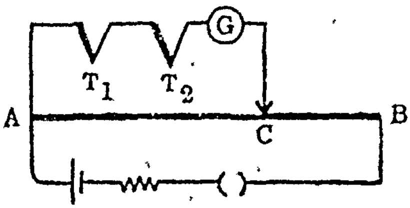

Fig. 197

T_2 of the hot function is gradually increased by placing the junction in a suitable bath. For any temperature of the hot junction the null point C is obtained on the potentiometer wire. If the potential drop V per unit length of the potentiometer wire be previously measured and if the length AC be equal to l the E. M. F. of the couple is equal to lV. Thus the E. M. F. generated in the thermo-couple corresponding to any temperature of the hot-junction is measured. An approximate formula connecting the E. M. F. and the temperature (of the hot junction) is then used. Both the equations $E = at + bt^2$ and $\log E = a \log t + b$ are sufficiently accurate for the purpose. The hot junction is successively placed at two known fixed temperatures and the corresponding E. M. F.'s are measured. Substituting these values in any of the above equations two equations are obtained from which the constants 'a' and 'b' may be solved for. Afterwards the E. M. F. corresponding to any unknown temperature being measured, the unknown temperature itself may be determined.

The range of temperature which can be measured by a thermo-couple is extremely wide—approximately from −200°C to about 1600°C. For temperatures up to 300°C an iron constantan couple or a copper-constantan couple is extremely satisfactory; for they develop quite a large E. M. F. —about 40 to 60 microvolts per degree difference. For high temperatures these base metals quickly get oxidised and cannot therefore be used. A nickel-iron couple may be used up to 600°C; but beyond this temperature a couple of plati-

num and an alloy of platinum with iridium or rhodium must be used.

2. Thermo-Pile.

The E. M. F. generated in a single thermo-couple is usually very small—of the order of a few micro-volts. It may however be multiplied by having a number of couples in series. Two sets of bars of dissimilar metals (usually Antimony and Bismuth) are joined alternately end to end. One set of junctions is exposed to some radation, the other set being kept protected by a metallic cover. Due to the radiation the temperature of the exposed junction is increased. A current is therefore generated in each couple in the same direction; all these currents are added up and a fairly strong current may be made to pass through any sensitive galvanometer. Thus even a faint radiation may be detected and measured by this instrument. Such an instrument is known as thermo-pile.

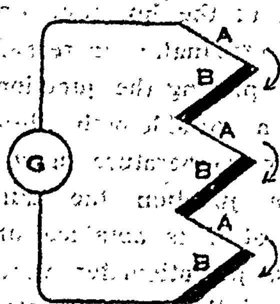

Fig. 198

3. Radio-micrometer.

C. V. Boy's radio-micrometer is a more sensitive instrument; herein the thermo-couple and the galvanometer are combined into one instrument. Between the two poles of a powerful horse-shoe magnet a vertical loop of copper wire is suspended by a quartz fibre. The two terminals of the loop are joined to two small bars of Sb and Bi soldered together at the tips. Radiation incident on this junction raises the temperature. The current thus generated flows through the loop which is therefore deflected by the magnetic field. A small mirror M is attached to the

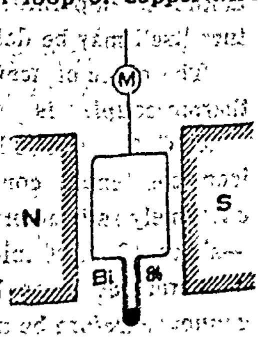

Fig. 199

quartz fibre and the usual lamp and scale arrangement is made to measure the deflection. The instrument is so sensitive that an appreciable deflection is produced by the quantity of heat equal to that received by a 4 anna piece placed at a distance of 1500 ft. from a candle.

4. Thermo-Galvanometer,

Boys' radio-micrometer has been modified by Duddell to form a thermo-galvanometer. Unlike thermo-pile and radio-micrometer this instrument measures a current. A loop of silver wire hangs between the poles of a horse shoe magnet. As in radio-micrometer two bars of Sb and Bi are attached to the two terminals of the loop and the tips of the bars are soldered. Just below the junction a wire is placed through which the current to be measured is passed. This wire is technically known as a 'heater'. The 'heater' gets heated by the current and radiation from it is incident on the Bi-Sb junction. A current is therefore generated in the loop which is consequently deflected by the magnetic field. The 'heaters' are made of various resistances ranging from 4 ohms to 1000 ohms. In the case of lower resistances metallic wires are used, whereas deposit of platinum on quartz is used for heaters of resistances.

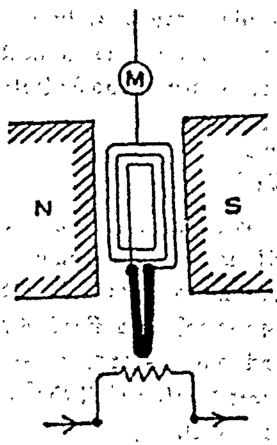

Fig. 200

Exercise XIV

1. Write short notes on Thomson effect. C. U. 1945.

Describe an experiment to illustrate this effect. Why is Thomson coefficient sometimes called specific heat of electricity ?

2. Describe a Thermo-galvanometer. In what way is it different from an ordinary galvanometer ?

3. Explain the thermo-electric power and thermo-electric diagram and show how Peltier E. M. F., Thomson E. M. F., Neutral temperature, temperature of reversal can all be represented in the thermo-electric diagram.

4. What is a thermo pile ? A current from a battery is passed for sometime through a thermo-pile. Immediately afterwards the battery is removed and the thermo pile is connected to a galvanometer. The galvanometer shows some deflection which gradually dies off. Explain this.

5. Establish $\pi = T\dfrac{dE}{dT}$ and $\sigma = T\dfrac{d^2E}{dT^2}$

One junction of a thermo-couple is maintained at 0°C. The E. M. F. developed in the couple is 851 micro-volts when the other junction is at 100°C and 1336 micro-volts when the other temperature is 200°C. Assuming the relation $E = bt + ct^2$, calculate the neutral temperature and the temperature of reversal for the couple. How would these be altered if the cold junction be maintained at 20°C and not at 0°C.

Ans. 282·5°C ; 565°C ; 282·5°C ; 545°C.

6. The thermo electromotive force in a thermo electric circuit is given by $E = 17{\cdot}35\,t - 0{\cdot}025t^2$ where E is expressed in micro volts and t is the temperature (Centigrade) of one junction, the other junction being maintained at 0°C. Determine the neutral temperature and the temperature of reversal for the circuit. If $t = 100$°C, find the Thomson E. M. F. in the circuit and also the Peltier E. M. F's at the two junctions and hence the total E. M. F. in the circuit.

Ans. 347°C ; 694°C ; -1615μV ; $4736{\cdot}55\mu$V ;
$4606{\cdot}55\mu$V ; 1485μV

Hints : Peltier E. M. F $= T\frac{dE}{dT} = T\frac{dE}{dt} = T(17.35 - 0.050t)$

$\therefore \pi_1 = 273 \times 17.35$; $\pi_2 = 373(17.35 - 0.050 \times 100)$

Thomson Coeff $\sigma = T\frac{d^2E}{dT^2} = T\frac{d^2E}{dt^2} = -0.050T$

$$\therefore \text{Thomson E. M. F.} = -\int_{273}^{373} 0.050\, T dT = -0.025(373^2 - 273^2)$$

C. U. Questions

1964. Explain what you understand by (*a*) Peltier effect (*b*) Thomson effect.

Describe an experiment to determine the temperature of a liquid bath utilising any of the above effects.

1964. Write notes on "Thermo-electric Seebeckeffect".

1965. Show that the Peltier Coefficient at a given junction is the product of the absolute temperature and the rate of change of the total E. M. F. in the circuit with temperature.

The E. M. F. of a thermo couple one junction of which is at 0°C is given by $E = at + bt^2$. Determine the Peltier and Thomson coefficients.

1966 Write notes on "Peltier effect."

1968. Write notes on "Thermo-electric effects"

1969. What do you understand by neutral temperature and thermoelectric power of a thermocouple ? Describe briefly Peltier Effect and how it can be demonstrated.

The E.M.F. of a simple thermo-electric circuit, one junction of which is heated while the other is kept at 0°C, is given by $E = bt + ct^2$ where t is the temperature of the hot junction. Determine the neutral temperature of the couple and Peltier and Thomson coefficients.

1972. Explain the following terms : Seebeck effect, Peltier and Thompson effects, thermoelectric power and inversion point.

Describe a thermo-couple and explain how a difference of temperature is measured with the instrument.

CHAPTER XV

MAGNETIC INDUCTION, THEORY OF MAGNETISM

Art 138. In a magnet the intensity of magnetisation* is defined to be the magnetic moment per unit volume. Thus if I be the intensity of magnetisation we have $I=\frac{M}{V}$. But magnetic moment $M=m\times l$, where m is the pole strength and l is the length§ of the magnet. Hence $I=\frac{ml}{V}=\frac{m}{A}$ where A is the area of cross section. Thus the intensity of magnetisation may also be defined as the pole strength per unit area.

If we place a soft iron rod in a magnetic field of strength H the rod is magnetised by induction. The intensity at any point A outside the rod therefore depends on the external field H and also on the field due to the two poles induced in

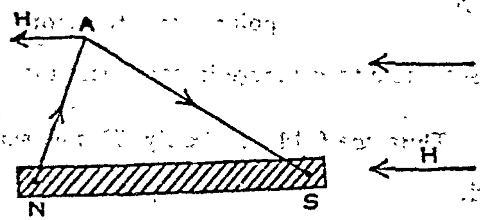

Fig. 201

the rod. This is true for all points *outside* the rod. If we however want to determine the intensity at any point *inside* the rod, the problem becomes much more complicated. For, intensity at a point means force experienced by a unit North pole placed at that point; in order to determine the intensity

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* Intensity of magnetisation must not be confused with Intensity or Field at any point.

§ Strictly speaking, l is the distance between the poles.

at a point within the rod we must place a unit North pole at that point and in order to place the unit pole we must make a hole in the rod. The shape and size of the hole make the problem complicated. We know that if the iron rod (magnetised by induction) be broken into two pieces, each of the two portions is itself a magnet, *i. e.* poles are induced at the face where the rod is broken. In a similar way if a hole be made inside the rod poles are also induced on the surface of the hole. These poles also create a field at the point within the hole.

Let us consider two extreme cases. First consider a hole whose length $2l$ is very large when compared to its cross section α. Then poles of strength $I\alpha$ are induced on the end faces of the hole. These poles produce an intensity $\frac{2I\alpha}{l^2}$ at the centre; and the total intensity at the point is therefore equal to

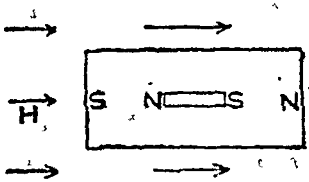

Fig. 202

$H+\frac{2I\alpha}{l^2}$. But l being very large in comparison to α, we may neglect $\frac{2I\alpha}{l^2}$ Thus the field is simply H the same as the external field.

We next consider a hole whose cross-section is very large in comparison to the length North and South poles of strength I per unit area are induced on the end faces. The case is analogous to a parallel plate condenser whose plates have a surface density σ. We know that the electric intensity at any point inside the condenser is $4\pi\sigma$. In a similar way the magnetic intensity at any point inside our hole is $4\pi I$. Hence the resultant intensity at the point is $H+4\pi I$. This is called magnetic induction and is

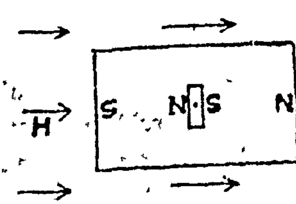

Fig. 203

denoted by the letter B Thus

$$B = H + 4\pi I \qquad \cdots \qquad \cdots \qquad \cdots \qquad (50)$$

We have considered two extreme cases; if the hole be of any other shape the resultant intensity will have other values lying intermediate between H and $H + 4\pi I$.

Dividing (50) by H we have

$$\frac{B}{H} = 1 + 4\pi \frac{I}{H}.$$

$\frac{B}{H}$ is called permeability μ and $\frac{I}{H}$ is known as susceptibility k.

$$\therefore \quad \mu = 1 + 4\pi k \qquad \cdots \qquad \cdots \qquad (50a)$$

Art 139
Hysteresis

It is obvious that induced magnetism in the rod depends on the strength of the external field, *i. e.* I depends on H. If we measure I for different values of H and polt I against H a peculiar curve is obtained. Starting from the origin O the curve traces the line O*ab* for increasing values of H and beyond *b* it follows the path *bc* parallel to H axis. This shows that for a particular value of H corresponding to the point *b*, I becomes maximum; or, in other words magnetism in the rod becomes *saturated*. If we now slowly decrease the strength of H the curve separates from the original path at *b* and ultimately cuts the I axis at *d*. O*d* is therefore the value of I retained when the external field is gone. This is what is known as *residual magnetism*. If now the external field be reversed and gradually increased in the reversed direction, magnetism is at first completely lost at *e* and again ultimately becomes maximum in the reversed direction (*i. e.* with induced polarity reversed) at *f* O*e* — the value of H necessary to destroy the residual magnetism, is called *coercive force*. From *f* the curve passes along the path *f g h b* when the field is again gradually increased in the forward direction. Thus the substance is made to follow the cyclic path. This phenomenon is known as Hysteresis and the cycle is known as the cycle of Hysteresis.

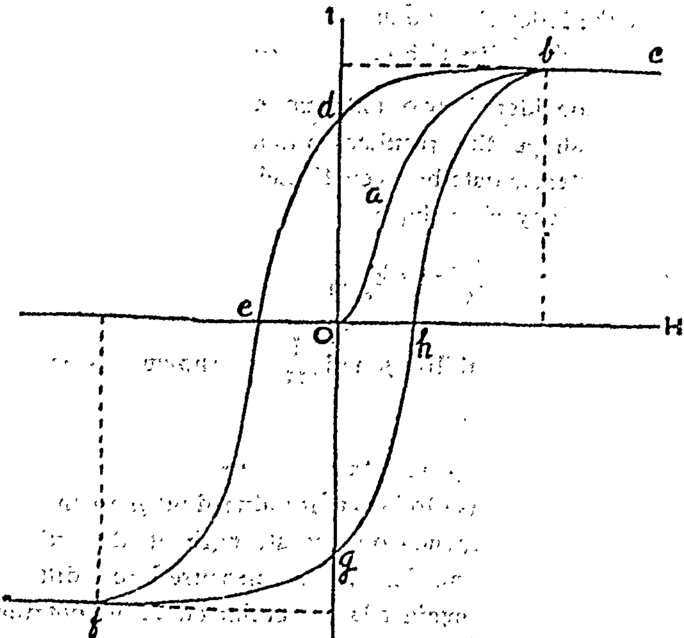

Fig. 204

Art 140
Work done

We will now prove that if a magnetic substance be taken once round a hysteresis cycle work done thereby per unit volume of the substance is equal to the area of the Hysteresis loop. At any instant during the process of magnetisation suppose an elementary magnet (within the substance) of moment m is oriented at an angle θ with the direction of the external field H. Component of this 'm' along H is $m \cos\theta$ and that perpendicular to H is $m \sin\theta$. Since by the application of the field H intensity of magnetisation is manifested only in the direction of H, we have

$$\Sigma\, m \cos\theta = \mathrm{I} \quad \ldots \quad \ldots \quad (a)$$

and $\quad \Sigma\, m \sin\theta = 0$

where the summation is extended over all molecules in a unit volume.

Differentiating (a) $\qquad -\Sigma\, m \sin\theta\, d\theta = d\mathrm{I}.$

Now the couple on the elementary magnet is $mH \sin \theta$ [vide (9) Art 7]. To rotate it further through an angle $d\theta$ in the direction of H, *i. e.* to change the orientation θ by an amount $-d\theta$,

$$\text{work done} = -m\,H \sin\theta\, d\theta.$$

Hence for all elementary magnets in a unit volume

$$\begin{aligned} \text{work done} &= -\Sigma m\, H \sin\theta\, d\theta. \\ &= H \times -\Sigma m \sin\theta\, d\theta. \\ &= H\, dI. \end{aligned}$$

$\therefore$ for a complete cycle,

$$\text{total work done} = \int H dI$$

$$= \text{area of Hysteresis Loop.}$$

Thus if a specimen be taken round a complete cycle work done per unit volume is equal to the area of the hysteresis loop. Energy thus spent is a loss and is known as hysteresis loss. This energy is usually converted into heat which raises the temperature of the specimen. If a rapidly varying alternating current be passed through the coil of an electromagnet the core within is taken rapidly through complete cycles ; the temperature of the core therefore rises.

If we plot B against H we obtain a similar hysteresis loop ; but the area of this loop is bigger than I − H loop. For,

$$B = H + 4\pi I.$$

Multiplying by dH, $\quad BdH = HdH + 4\pi I dH.$

$$\therefore \int BdH = \int HdH + 4\pi \int I\, dH$$

But $\int HdH$ represents the area of the loop when H is plotted against H. Obviously the curve in this case is a straight line. There being no loop at all the area is zero.

Hence $$\int BdH = 4\pi \int I\, dH$$

i. e. the area of the B-H loop is 4π times that of the I-H loop.

Art 141. Since $B = H + 4\pi I$ it is obvious that if B be plotted against H a curve is obtained similar in shape to I-H curve but as B *always* increases with H, the curve never becomes horizontal. If we plot the permeability μ against H we obtain a curve similar to that given in Fig. 205. With increasing values of H, B at first increases much more rapidly than H; μ (which represents the ratio of B to H) therefore increases rapidly at first but afterwards as I attains the saturation value the curve μ-H bends down and is ultimately assymptotic to a line parallel to H-axis.

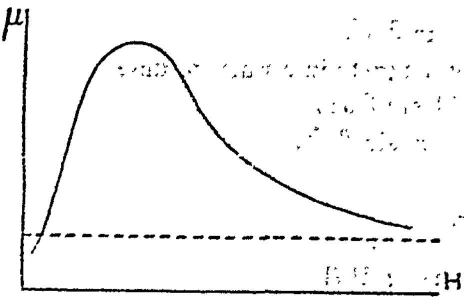

Fig. 205

Hysteresis curve for different specimens of iron, although agreeing generally in the main appearance, differ from one another in many important points. Curves for soft iron and steel are shown in Fig. 206. These curves clearly bring out the following points of difference in the properties of soft iron and steel :—

(1) Maximum induction (*i. e.* saturation value of B) is greater in soft iron than in steel.

(2) Residual magnetism is higher in the case of soft iron than in the case of steel.

(3) Steel has coercive force much greater than that of soft iron.

N. B. The harder the substance the less is the residual magnetism and the greater the coercive force.

(4) The area of the loop and hence hysteresis loss are smaller for soft iron than for steel.

Such curves as shown in Fig 206 give us most of the informations by which materials for different purposes may be selected.

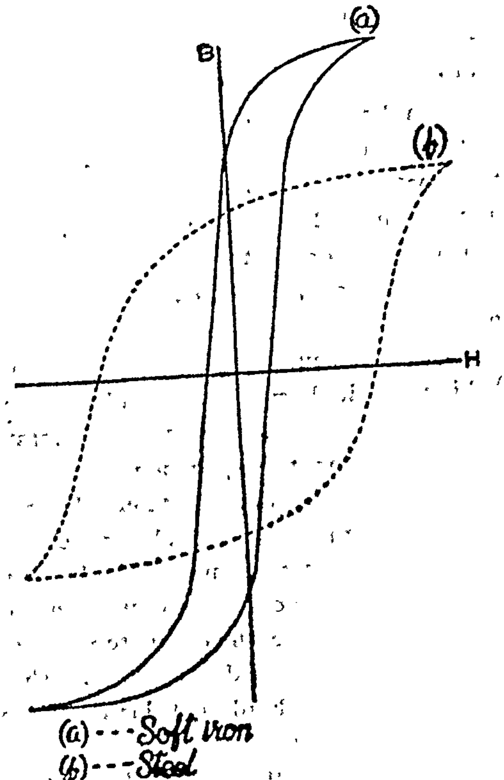

Fig. 206

For the core of an electromagnet we require a material for which (1) hysteresis is minimum and (2) a large maximum induction is obtained with a comparatively small magnetising field. Soft iron satisfies both these conditions and is therefore the most suitable material for this purpose.

Core of an electromagnet

In the case of transformer cores, telephone diaphragms and cores of dynamo coils, the specimen is taken rapidly through hysteresis cycles; essential conditions are therefore (1) small hysteresis loss and (2) large initial permeability for small fields. Soft iron is thus more suitable for this purpose also

Telephone diaphrgam etc

than steel. An alloy known as "transformer steel" is obtained by mixing iron with 4% silicon ; this has a large initial permeability and is therefore more suitable for transformer cores and telephone diaphragms : but as its permeability for greater fields is less than that of iron, it is less satisfactory than pure iron for the cores of dynamo coils. An alloy (known as Permalloy) containing 22% Fe and 78% Ni has a very high initial permeability for extremely weak fields ; these alloys are being extensively used in modern engineering instruments.

Permanent magnet

To construct permanent magnets we require a material having (1) high residual magnetism and (2) high coercivity. Hysteresis loss is of no consideration in this case, for the specimen is never likely to undergo a complete cycle. Coercive force for steel is extremely high and residual magnetism is also not insignificant. Steel is therefore more suitable than soft iron. An alloy known as "Alnico" containing about 18% Ni, 10% Al, 12% Co, 6% Cu, and 54% Fe has a coercive force about 12 times as large as that of ordinary steel. This excellent material is however very hard and brittle so that it cannot be bent and drilled satisfactorily.

Art. 142 Theory of Magnetism

It is a well known fact that when a magnet is broken into two parts each of the two components is itself a magnet with two poles at the two ends. This process of breaking up can be continued (at least theoretically) till the component parts are individual atoms which cannot be broken up any further. We must therefore conclude that individual atoms of a magnetic substance are themselves elementary magnets. In any unmagnetised substance these atoms or elementary magnets are promiscuously oriented so that along any direction the effect due to N poles being exactly counterbalanced by that due to S poles the total effect is nil. With the application of an external field however the atoms gradually tend to orient themselves along the direction of the field so that a net effect is produced, or in other words the substance

is magnetised along this direction. As the magnetising field is gradually increased the atoms rotate more and more along the direction of the field, *i. e.* Intensity of magnetisation I gradually increases. When the field H is sufficiently large all the atoms orient themselves along this external field and I becomes maximum. Further increase in H cannot obviously produce any more increase in I. Thus the phenomenon of magnetic saturation is satisfactorily explained on this simple theory. Difficulty comes in when we attempt to explain residual magnetism and other facts connected with Hysteresis. When atoms rotate by the application of an external field we must assume that there is some opposing or restoring couple; for otherwise the atoms would at once rotate completely by the application of even a very weak external field. Obviously this is not the case. What is therefore the nature of the opposing couple and what is its origin?

Maxwell suggested that this opposing couple (which may also be called restoring couple) is analogous to stresses generated in a strained solid. For a weak external field the rotation of the atoms is small and when the field is removed the restoring couple brings back the atom to their original positions; when however the field is large the rotation of the atoms exceeds a certain limit (corresponding to elastic limit) and the restoration is not complete even when the external field is altogether removed; the phenomenon of residual magnetism is thus exhibited. This theory however does not explain the complete phenomenon of Hysteresis satisfactorily. Wiedemann supposed that the opposing couple is of the nature of friction. This theory although explaining residual magnetism, breaks down on the ground that on this theory a certain minimum value of the field is necessary to overcome this frictional couple and a very weak field therefore will not be able to magnetise a substance. Lord Rayleigh on the other hand showed that the intensity of magnetisation is not zero even when the magnetising field is extremely weak.

Ewing's Theory The theory generally accepted as correct was first given by Sir J. A. Ewing. According

to him the opposing couple on any atom is entirely due to the action of neighbouring atoms. In the absence of any external field the elementary magnets mutually react on one another and hold themselves in some equilibrium position so that they are oriented in all possible directions. With the application of weak external field the magnets are turned slightly. As the applied field increases in strength an unstable mutual arrangement is reached which suddenly passes on to a more stable arrangement producing proportionately a large increase in the intensity of magnetisation. After this state the process is not reversible, *i. e.* residual magnetism is manifested when the external field is removed. Finally when the external field has become sufficiently large all the magnets are arranged parallel to the field. Ewing has tested his theory by experiments with a model of 24 small magnets. He showed that all the phenomena of Hysteresis are exhibited by this model when an external field acting on these magnets is gradually altered.

Art 143
Paramagnetic and Diamagnetic substances

The question still remains as to why an atom of a magnet should itself behave as a magnet. According to modern ideas each atom consists of a positively charged nucleus round which a number of electrons rotate. Rotation of these electrons constitutes electric currents which produce the necessary magnetic effect. This however implies that all substances should have some magnetic effect because within the atoms of all substances there are electrons rotating round the centre. Recent investigation has shown that this is really so. All substances when placed in a magnetic field exhibit magnetic effect to a greater or lesser extent.

So far as magnetism is concerned substances have been broadly divided into two classes (*a*) Para-magnetic and (*b*) Dia-magnetic. In a non-homogeneous external magnetic field certain substances have a tendency to move from the weaker field to the stronger field, as if they are attracted by the external magnet while others behave exactly in the opposite way. Paramagnetic substances belong to the former class and

Diamagnetic substances to the latter. Within the paramagnetic group there are a few substances, Iron, Cobalt Nickel etc., for which the effect is much more pronounced than in the case of other substances. These have been classified into a separate group and are known as Ferromagnetic substances.

Ferromagnetic substances

Examples

Examples of paramagnetic substances are Aluminium, Tin, Sodium, Magnesium, Air, Oxygen etc. The susceptibility k for paramagnetic substances is independent of the magnetic field but decreases with the rise of temperature. For most of these substances Curie-Weiss Law $k^* = \frac{C}{T-\theta}$ is found to be true where C and θ are two constants for the given substance and T is absolute temperature. Bismuth, Gold, Copper, Sulphur, Mercury, Water etc, are examples of diamagnetic substances. k for these substances is independent of the field as well as of the temperature. The effect of temperature on Ferromagnetism is however complicated; it has been discussed in the next article.

Effect of temperature

The difference in behaviour between the two classes was first systematically studied by Faraday. A small thin rod made of a solid substance (whose magnetic behaviour is to be tested) was suspended by a thin thread between the pointed pole pieces of a powerful electromagnet. If the substance was paramagnetic the rod set itself along the line joining the pole pieces; if the substance was diamagnetic the rod was perpendicular to this line.

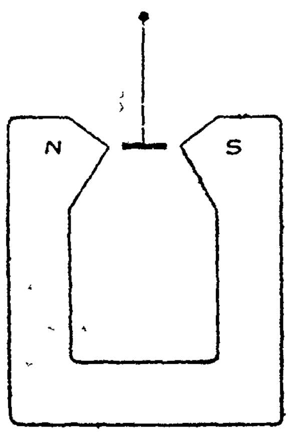

Fig. 207

* Here k represents susceptibility for *unit mass*, *i.e.* $k = \frac{I}{H}$ where I is intensity of magnetisation per *unit mass*.

To test the nature of a liquid substance a small quantity of the liquid was taken in a watch glass resting upon the

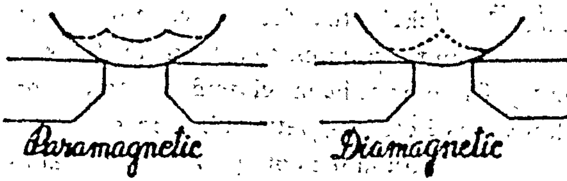

Fig. 208

pole-pieces of a powerful electromagnet. In the case of paramagnetic substances small elevations appeared in consequence of magnetic attraction—above the poles. In the case of a diamagnetic liquid the repulsion gave rise to small depressions above the poles with a corresponding elevation between them.

Art 144
Effect of temperature

The susceptibility of a ferro-magnetic substance depends largely on temperature. The effect however is dependent on the

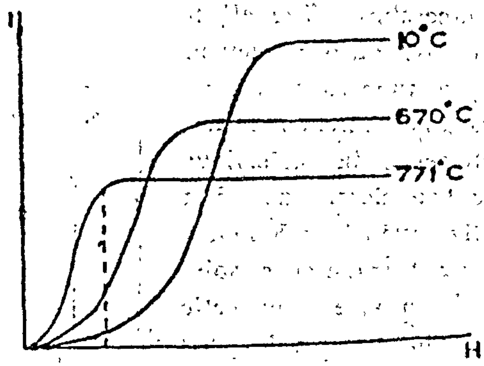

Fig. 209

strength of the field as well. For weak fields susceptibility increases with temperature; reverse is the case when the field is strong. Thus in Fig. 209 when H is small the intensity of magnetisation I increases with increasing temperature. But for large values of H I is smaller for higher temperatures. Thus the maximum value of I (saturation value) is less at higher temperature than at lower one.

As the temperature is raised a peculiar phenomenon is observed with iron at about 785°C. For weak fields the substance is highly magnetic just before this temperature but as soon as this temperature is reached the substance suddenly becomes practically non-magnetic. This is shown in Fig. 210, where μ is plotted against temperature. For large fields the effect is gradual and not so sudden until

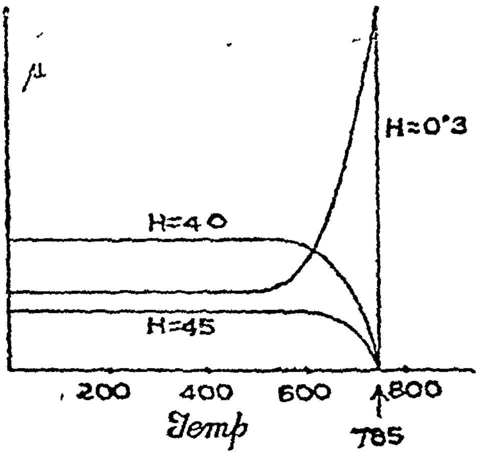

Fig. 210

finally at this temperature 785°C the substance becomes almost non-magnetic. Thus whether the field is high or low all magnetic properties vanish at this temperature. This critical temperature is known as *Curie point.*

That some important molecular changes take place at this temperature is also evident from the fact that this is also the temperature of recalescence, *i. e.* at this temperature a mass of cooling iron suddenly begins to glow again; as soon as this temperature is passed the glowing vanishes.

It is also found that above this temperature ferromagnetics become ordinary paramagnetics obeying Curie Weiss Law.

Art 145 We shall now discuss a method of measuring the susceptibility and hence of drawing the hysteresis curve for a given specimen of iron.

intensity is $I\alpha \left\{ \frac{1}{r^2} - \frac{r}{(r^2+l^2)^{3/2}} \right\}$ along PM. If this be at right angles to the Earth's horizontal component H′ the needle rotates through an angle θ given by

$$\tan\theta = \frac{I\alpha}{H'} \left\{ \frac{1}{r^2} - \frac{r}{(r^2+l^2)^{3/2}} \right\}$$

In this equation all the quantities excepting I are measurable ; Hence I is determined.

To find the magnetising field due to the solenoidal current an ammeter A is placed in series with the solenoid. The current being measured by the ammeter the magnetising field within the solenoid is given by $H = 4\pi ni$, where n is the number of turns of the solenoid per unit length. Thus H is determined.

By altering the resistance R the current strength i and hence the field strength H are altered and corresponding values of I are measured. Thus sets of values of I and H are obtained for direct and reverse currents : the complete hysteresis curve may therefore be plotted. At any stage the ratio I/H gives the value of the corresponding susceptibility. And knowing the susceptibility k the permeability μ may be obtained from the relation $\mu = 1 + 4\pi k$.

Sources of error Several sources of error must be eliminated before correct result can be obtained

First the rod being placed vertically is magnetised by the earth's vertical component. This produces some effect on the magnetometer needle. To eliminate this a vertical circular coil C_1 is placed with its axis passing through the needle. The rod PQ being magnetised by the Earth's vertical component a small deflection of the magnetometer needle is produced even when the solenoidal current is zero. The current in C_1 is adjusted until this deflection of the needle is destroyed. This current is maintained constant throughout the experiment.

Secondly the solenoid itself behaves as a magnet when a current passes through it ; this produces a deflection of the magnetometer needle. To eliminate this a second vertical coil C_2 (also with its axis passing through the needle) is used in series with the solenoid. The

rod is first removed from within the solenoid and a current is passed. The distance of the coil C_2 from the needle is adjusted until the effect on the needle due to the solenoid is exactly counterbalanced by that due to the coil, *i. e.* until the needle is brought back to its undisturbed position. It is obvious that once this balancing is made for any current it holds good for all currents.

Exercise XV

1. Define the terms ; magnetic induction (B), intensity of magnetisation (I), magnetic force (H), permeability (μ) and susceptibility (k). Show that $\mu = 1 + 4\pi k$. C. U. 1954.

2. In what respects do the magnetic properties of iron and steel differ ? Define the terms Intensity of magnetisation (I) Induction (B) and Magnetic force (H). How do you obtain the relation $B = H + 4\pi I$? What is the general character of the magnetic permeability of iron in strong fields ? C. U. 1949.

3. What is meant by 'hysteresis' and a 'cycle of magnetisation' ? Draw the hysteresis curve for (a) soft iron and (b) steel.

Prove that the area of the B-H curve denotes 4π times the energy dissipated per c. c. of the magnetic substance during each cycle of magnetisation.

4. Show that when a specimen is taken round a complete hysteresis cycle work done per c. c. of the specimen is equal to the area of the I-H loop.

An alternating current of 100 cycles per sec is passed through an electromagnet. Calculate the rise in temperature of the iron inside the electromagnet in one minute (assuming that there is no loss of heat) from the following data :—

Area of the hysteresis loop = 50,000.

Sp. heat of iron = 0·12 and Density of iron = 7·7 gms/cm^3

Ans. 7·73°C

Hints :—Heat generated per unit volume per minute

$= 50{,}000 \times 100 \times 60$ ergs $= \dfrac{30 \times 10^7}{4{\cdot}2 \times 10^7} = \dfrac{50}{7}$ calories. If θ be the rise in temperature $\dfrac{50}{7} = 7{\cdot}7 \times 0{\cdot}12\ \theta$. Hence find θ.

5. What is the effect of temperature on (1) permeability (2) maximum intensity of magnetisation of iron? What is Curie point?

6. Give an account of Ewing's theory of magnetism. Show how it explains the various phenomena in connection with hysteresis.

7. Give an account of the different hypotheses by which the various phenomena of hysteresis have been tried to be explained. What is the theory which more or less successfully explains all the facts in this connection?

8. What are diamagnetic and paramagnetic substances? Give examples. How can it be tested whether a given substance (solid or liquid) is paramagnetic or diamagnetic?

9. Describe how the chief characteristics of dia-, para- and ferromagnetic substances are experimentally distinguished, giving a detailed treatment of any method for the determination of magnetic susceptibility. C. U. 1938.

C. U. Question

1962. Show the general nature of a B-H loop and explain the terms retentivity, coercivity and hysteresis with reference to it. Compare the B-H loop for iron and steel.

Prove that the energy dissipated per unit volume of a material during a complete hysteresis cycle is $\frac{1}{4\pi}$ times the area enclosed by the B-H curve.

1964. Explain the terms (*a*) permeability (*b*) susceptibility (*c*) remanence and (*d*) hysteresis. What is a hysteresis loop?

Establish a relation between permeability and susceptibility of a magnetic material.

1963. Define and explain the following terms : (*a*) Intensity of magnetisation (*b*) Magnetic Induction (*c*) Permeability and (*d*) Susceptibility.

Derive the relation connecting permeability and susceptibility of a magnetic substance.

1964. Explain what you understand by (*a*) hysteresis (*b*) retentivity and (*c*) coercivity of a magnetic material. What is hysteresis loop ?

Briefly state how you can distinguish between a ferromagnetic, paramagnetic and diamagnetic material.

1965. (*a*) Define magnetic permeability and susceptibility and show how these are related to each other.

(*b*) Illustrate the nature of the hysteresis loop of a sample of steel and that of a soft iron piece. Indicate from these how the two materials differ in their magnetic behaviours.

1966. Write notes on Hysteresis as applied to magnetic problems.

1967. Write notes on "Ferro-, para- and diamagnetic materials."

1969, 1971. Define the terms: Magnetic induction (B), Intensity of magnetisation (I), permeability (μ), and susceptibility (k). Show that $\mu=1+4\pi k$. What is a Hysteresis loop?

1973. Define magnetic permeability and susceptibility and find the relation between them.

What is a Hysteresis loop? Indicate the nature of hysteresis loop of a sample of steel and that of a soft iron piece and discuss their magnetic behaviour.

1974. Define magnetic induction (B), Intensity of magnetisation (I), magnetic intensity (H), permeability (μ) and susceptibility (k). Deduce the relation $\mu=1+4\pi k$.

CHAPTER XVI

ELECTROMAGNETIC INDUCTION

Art 146
Experimental fact

Consider a closed coil of wire and a magnet at a certain distance. If the magnet be moved towards the coil it is found that an instantaneous current is generated in the coil; if the magnet be moved away from the coil a current is also generated in the coil but this time in the opposite direction. The current exists only so long as the magnet is moving; as the magnet is stopped the current also dies down.

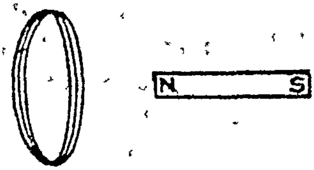

Fig. 212

Instead of moving the magnet towards the coil if we move the coil towards the magnet the same effect is produced; in fact it is the relative motion between the coil and the magnet that produces the current in the coil. If both the coil and the magnet be moved in the same direction with the same speed so that there is no relative motion between them, no current is generated in the coil.

N.B. (1) The induced current exists only so long as there is relative motion.

(2) If the polarity of the magnet presented to the coil be reversed then with the approach or receding away of the magnet, the current in the coil flows in a direction reverse to that in the preceding case.

(3) The generation of the current indicates that an E. M. F. is induced in the coil. When the coil is open, *i. e.* when there is an air gap between the two terminals of the coil, the E. M. F. is still generated by the relative motion of the magnet; but in this case there is no current because the resistance of the coil is infinitely large due to the presence of the air gap. If the induced E. M. F. is sufficiently strong sparking may take place across the air gap between the two terminals (Vide Induction coil, Art 165).

magnet will continue to move towards the coil. Thus by spending a small amount of energy in pushing the magnet slightly towards the coil the magnet may be made to move through a comparatively large distance in going up to the coil. This is evidently against the principle of conservation of energy. It follows that when the magnet be moving towards the coil the direction of the induced current in the coil should be such that the magnet is repelled by the action of this current so that the motion of the magnet towards the coil is checked. Opposite would be the effect if the magnet recedes away from the coil.

Mathematically, the 3rd law is expressed by placing a minus sign before $\frac{dN}{dt}$ in (51). Thus we have

$$\text{Induced E. M. F.} \propto -\frac{dN}{dt}$$

$$\therefore \quad E = -K\frac{dN}{dt} \text{ where K is a constant.}$$

With suitably chosen units K may be made equal to unity. Hence finally

$$E = -\frac{dN}{dt} \qquad \ldots \ldots (51a)$$

N. B. This equation contains all the three laws.

Art 148 Theoretical Proof

Equation (51*a*) may be formally proved as follows :—

Consider an electric circuit carrying a current i placed in a magnetic field of strength H. Let a bar AB—a part of the electric circuit, make an angle θ with H. Then the force on AB is $F = Hil \sin\theta$ where $(AB = l)$ acting in a direction perpendicular to the plane containing H and AB. If AB be capable of sliding in this direction parallel to itself along two parallel conducting rails the force F causes it to do so. In time δt let AB move through a distance δx to the new position A′B′, so that the work done by the force F is $Hil \sin\theta \cdot \delta x$. The energy spent thereby ultimately comes

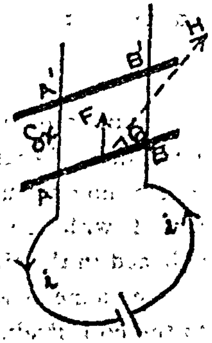

Fig. 215

from the battery which produces the current i in the circuit. If E be the E. M. F. of the battery, total energy spent by the battery in time δt is $Ei\delta t$. Part of this supplies the energy $Hil \sin\theta\ \delta x$ and the remaining portion is utilised in overcoming the resistance of the circuit and is ultimately converted into heat. This latter portion is equal to $i^2R\delta t$, where R is the resistance of the circuit.

Hence $$Ei\delta t = i^2R\delta t + Hil\sin\theta.\ \delta x.$$

Dividing by $i\delta t$, $$E = iR + \frac{Hl\sin\theta\ \delta x}{\delta t}$$

$$\therefore\quad i = \frac{E - \dfrac{Hl\sin\theta.\delta x}{\delta t}}{R}$$

But $l\delta x$ is the area described by AB in moving through δx and $H\sin\theta$ is the component of H perpendicular to this area. The magnetic field at any point being equal to the number of lines of force per unit area, product $H\sin\theta.l\delta x$ represents the number of lines of force crossing the area ABB′A′ in the normal direction. If N be the number of lines originally associated with the circuit, $H\sin\theta\ l\delta x$ is the change in N (by the movement of AB), or is equal to δN.

$$\text{Thus}\quad i = \frac{E - \dfrac{\delta N}{\delta t}}{R}$$

Comparing this with ordinary Ohm's Law equation for a fixed circuit, *viz.* $i = \dfrac{E}{R}$ we conclude that due to the movement of AB, *i. e.* by the variation of the number of lines of force linked with the circuit, an additional E. M. F. equal to $-\dfrac{\delta N}{\delta t}$ is induced in the circuit.

If δt be infinitesimally small we have in the limit.

$$\text{Induced E. M. F.} = -\frac{dN}{dt}.$$

Art 149 **Self-Induction (Inductance)** Consider a circuit containing a battery, a coil of wire and a tapping key. When the key is open there is obviously no current and therefore no magnetic lines of force are linked with the coil. When the key is pressed a current is established in the circuit. This current produces magnetic lines of force, some of which are linked with the coil itself. As the key is pressed the current grows from zero to the maximum value i_0 and the lines of force due to the current also increase in number. Hence during the growth* of the current the number of lines of force linked with the coil increases and an instantaneous E. M. F. is therefore induced in the coil.

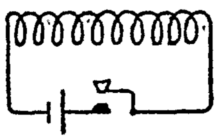

Fig. 216

At any instant during the growth let i be the instantaneous value of the current. Clearly N the number of lines of force linked with the coil at the same instant is proportional to the current i. We have therefore

$$N = Li \quad \ldots \quad \ldots \quad (52)$$

where L is a constant depending on the shape and size of the coil.

$$\therefore \text{ Induced E.M.F. } E = -\frac{dN}{dt} = -L\frac{di}{dt} \quad (52a)$$

Again, as the induced E.M.F. at any instant is $E = -L\frac{di}{dt}$ the rate of expenditure of energy is $Ei = -Li\frac{di}{dt}$. Hence total expenditure of energy *i.e.* total work done in establishing the final current $i_0 = -\int_0^{i_0} Li\frac{di}{dt} = -\frac{1}{2}Li_0^2$. The minus sign

* The time for this growth is extremely small—usually a very small fraction of a second.

only indicates that this work is done by the induced E. M. F. which opposes the E. M. F. of the battery. Due to the final current a magnetic flux is established. Thus the work done in establishing the magnetic flux

$$= \frac{1}{2} L i_0^2 \text{ (neglecting the minus sign)} \qquad \ldots (52b)$$

This constant L is called the self-induction (or simply, induction), or inductance of the coil. We may define the inductance from either of the three equations (52), (52a) or (52b).

From (52), When $i = 1$, $L = N$. Thus

First definition The inductance of a coil is defined as the number of lines of force linked with the coil due to a unit current flowing through the coil.

From (52a), when $\frac{di}{dt} = 1$, $L = E$ (neglecting the minus sign)

Second definition Thus the inductance of a coil is defined as the E. M. F. induced in the coil when the rate of variation of current in the coil is unity.

From (52b) if $i_0 = 1$, $L =$ twice the work done in establishing the magnetic flux. Thus the inductance of a coil

Third definition is defined as twice the work done in establishing the magnetic flux associated with the final steady unit current in the coil.

N. B. (1) The inductance is neither a number (as given by the 1st definition), nor an E. M. F. (as stated in the 2nd definition), nor it is energy (as given by the 3rd definition). Like resistance it is simply a property of the coil. Just as resistance depends on the length, cross section and material of the coil, similarly inductance depends only on the shape and size of the coil. It is *measured* by the number or the induced E. M. F. or the energy as the case may be, as given by the three definitions above.

Fig. 217

(2) A coil of wire has usually both resistance and inductance. Such

a coil having both resistance and inductance is usually represented as shown in Fig. 217 (*a*).

A coil may however be so wound that its inductance is zero. Such winding—known as non-inductive winding—is shown in Fig 217 (*b*). In this case the current in one half of the coil flows side by side with that in the other half; but the currents in the two halves flow in opposite directions. The E. M. F. induced by the variation of the current in one half cancels that generated by the variation of the current in the other half and the resultant induced E. M. F. is zero; or in other words the inductance of the coil is nil. A coil having resistance but no inductance is usually represented as shown in Fig 217 (*c*).

Art 150 The inductance of a solenoidal coil may be determined as follows :—

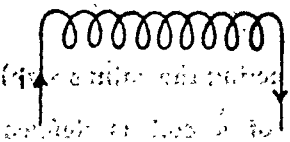

Fig. 218

At any instant let i be the current through the solenoid. Due to this current the field inside the solenoid is $4\pi ni$ where n is the number of turns *per unit length* of the solenoid. But field is equal to the number of lines of force per unit area of cross-section. Hence if A be the area of cross-section of the solenoid the total number of lines of force inside the solenoid is $4\pi nAi$. These lines of force are obviously linked with each turn of the solenoid. If l be the length of the solenoid the total number of turns is nl. Hence if N be the flux (*i. e.* total number of lines of force) linked *with the solenoid*, we have

$$N = 4\pi nAi \times nl = 4\pi n^2 lAi$$

Comparing this with (52) we find that the inductance L is given by

$$L = 4\pi n^2 lA \quad \ldots \ldots \quad (53)$$

$$= \pi^2 n^2 l D^2 \quad \ldots \quad \ldots \quad (53a)$$

where D is the diameter of the solenoid.

A straight wire, however long, has a very small inductance. But if the same wire be coiled up in the form of a solenoid the inductance increases to a great extent because of the term

The mutual inductance between two coils is the E. M. F. induced in one when the rate of variation of the current in the other is unity.

N. B. Instead of the current varying in A if the current i in B varies then the flux linked with A is also Mi, the constant M having the same value as before.

Art 152 The mutual inductance between two coils C_1 and C_2—one being *completely* wound over the other, may be found out as follows :—

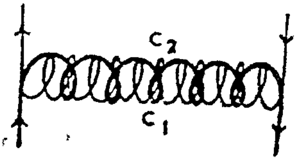

Fig. 220

Let i be the instantaneous current through the coil C_1. Due to this current field inside C_1 is $4\pi n_1 i$ where n_1 is the number of turns per unit length of C_1. If A be the cross-sectional area of C_1 (or of C_2) the total number of lines of force inside C_1 is $4\pi n_1 Ai$. If the coils C_1 and C_2 are so wound over each other that all the lines of force generated by the current in one are linked with each turn of the other then $4\pi n_1 Ai$ lines of force are linked with each turn of C_2 If n_2 be the number of turns per unit length of C_2 and l be the length of either coil the total number of turns of C_2 is $n_2 l$. Hence if N be the flux linked with C_2.

$$N = 4\pi n_1 Ai \times n_2 l = 4\pi n_1 n_2 l Ai$$

Comparing this with (54),

$$M = 4\pi n_1 n_2 l A = \pi^2 n_1 n_2 l D^2 \qquad (54b)$$

where D is the diameter of C_2 (or of C_1)

If L_1 and L_2 be the inductances of the coils C_1 and C_2 we have from (53*a*)

$$L_1 = \pi^2 n_1^2 l D^2 \text{ and } L_2 = \pi^2 n_2^2 l D^2$$

Hence from (54*b*) $\quad L_1 L_2 = M^2 \quad \ldots \quad \ldots \quad (55)$

It is however to be remembered that this equation is strictly true only when the flux produced by the current in one is *entirely* linked with each turn of the other, *i. e.* when there is no leakage of magnetic flux. In practice this is seldom true. Usually there is some leakage and M is less than

$\sqrt{L_1L_2}$. The relation $\frac{M}{\sqrt{L_1L_2}}$ is called the *coefficient of coupling* and two mutually inductive circuits are said to be tightly or loosely coupled according as the ratio $\frac{M}{\sqrt{L_1L_2}}$ approaches unity or is considerably less than unity.

If two mutually inductive resistances are connected in series the joint inductance is given by $L = L_1 + L_2 \pm 2M$, the double sign indicating that the circuits may be connected in two ways, *i. e.* connection may be such that the E. M. F's due to self-inductances may help or may oppose that due to mutual inductance.

Art 153

Circuit Containing L and R

Consider a coil of wire having inductance L and resistance R. Let the coil be in series with a battery and a key. When the circuit is closed by pressing the key the current in the circuit grows from zero to the maximum value and during the growth the E. M. F. $-L\frac{di}{dt}$ is induced in the circuit, i being the current at any instant. Hence if E be the E. M. F. of the battery the total E. M. F. is $E - L\frac{di}{dt}$ and by Ohm's Law this must be equal to Ri. Thus during the growth of the current

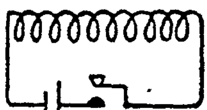

Fig. 221

$$E - L\frac{di}{dt} = Ri \quad \ldots \quad \ldots \quad (56)$$

If now the circuit be broken the current decays from the maximum value i_0 to zero; during the decay the induced E.M.F. $-L\frac{di}{dt}$ is still produced but the battery is out of action.

$$\text{Thus} \qquad -L\frac{di}{dt} = Ri \quad \ldots \quad \ldots \quad (56a)$$

We shall solve (56a) first

Decay of the current

$$-L\frac{di}{dt} = Ri$$

Separating the variables, $\dfrac{di}{i} = -\dfrac{R}{L}\,dt.$

or $\quad \log_e i = -\dfrac{R}{L}t + A_1$ where A_1 is an arbitrary constant

$\therefore \quad i = e^{-\frac{Rt}{L} + A_1} = e^{A_1} \cdot e^{-\frac{Rt}{L}} = Ae^{-\frac{Rt}{L}}$, where A is a new constant equal to e^{A_1}.

To find A we impose the initial condition, *viz*.,

$$\text{at} \quad \left.\begin{matrix} t = 0 \\ i = i_0 \end{matrix}\right\} \qquad \therefore \quad i_0 = A$$

$$\text{Hence} \qquad i = i_0 e^{-\frac{Rt}{L}} = i_0 e^{-\frac{t}{\lambda}} \quad \ldots \quad \ldots \quad (56b)$$

$$\text{where} \quad \lambda = \frac{L}{R} \quad \ldots \quad \ldots \quad (56c)$$

This λ is called the **time constant** of the circuit.

In (56b) if we put $\lambda = t$ we have $i = i_0 e^{-1} = i_0 \times \cdot 368.$

Thus the time constant (λ) is defined to be the time taken by the current to decay to ·368 times the original maximum current.

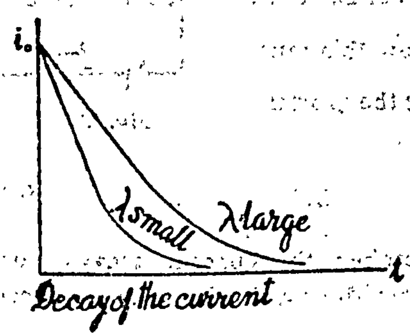

Fig. 222

From (56b), at $t = \infty$, $i = 0$, *i. e.* theoretically at infinite time the current reaches the zero value; but for all practical purposes the current is vanishingly small after a short interval of time. If we plot i against t the curve starts from the point ($t = 0$, $i = i_0$) and meets the t axis assymptotically, as shown in Fig. 222. For small values of λ, $\dfrac{t}{\lambda}$ is fairly large even when t is small, *i. e.* $e^{-\frac{t}{\lambda}}$ decreases rapidly with time

and the current also dies down quickly. On the other hand if λ be large, $e^{-\frac{t}{\lambda}}$ is not small unless t is large, *i. e.* the current i decays slowly [See Fig. 222.]

N. B. 'λ' small means L small and R large.

We shall now solve the equation (56) for the growth of the current.

Growth of the current

$$\left.\begin{aligned} \text{Let } i &= x + \frac{E}{R} \\ \therefore \frac{di}{dt} &= \frac{dx}{dt} \end{aligned}\right\}$$

Substituting these in (56) we have

$$E - L\frac{dx}{dt} = Rx + E$$

$$\text{or} \qquad -L\frac{dx}{dt} = Rx$$

$$\text{Solving as before} \qquad x = Ae^{-\frac{Rt}{L}}$$

$$\text{Adding } \frac{E}{R} \text{ to both sides,} \qquad i = \frac{E}{R} + Ae^{-\frac{Rt}{L}}$$

To find A we impose the inital condition, *viz.*,

$$\text{at} \left.\begin{aligned} t &= 0 \\ i &= 0 \end{aligned}\right\} \quad \therefore A = -\frac{E}{R}.$$

$$\text{Hence} \qquad i = \frac{E}{R}\left(1 - e^{-\frac{Rt}{L}}\right) = i_0\left(1 - e^{-\frac{t}{\lambda}}\right) \quad \ldots \quad (56d)$$

$$\text{where} \qquad i_0 = \frac{E}{R} \text{ and } \lambda = \frac{L}{R} \quad \ldots \quad \ldots \quad (56e)$$

As in the case of decay here also λ is called the time constant of the circuit. In (56d) if we put $t=\lambda$ we have $i=i_0(1-e^{-1}) = i_0(1-0{\cdot}368) = i_0 \times 0{\cdot}632$. Thus the time constant (λ) may also be defined to be the time taken by the current to rise to 0·632 times the maximum steady current.

From (56d), at $t=\infty$, $i=i_0$ *i. e.* theoretically at infinite time the current rises to the maximum value i_0; but for all

practical purposes the maximum value is reached after a short interval of time. The curve showing the relation between i and t starts from the origin ($t=0$, $i=0$) and meets the line $i=i_0$ assymptotically as shown in Fig 223. As in the case of the decay, the growth of the current is rapid if λ is small and slow when λ is large.

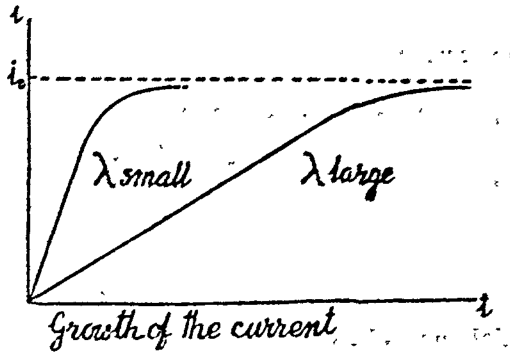

Fig. 223

N. B. 'Large λ' means large L and small R. A large value of L however has no effect on the final maximum value i_0 of the current; it can only delay the attainment of the maximum value.

Art 154 Circuit containing C and R

We now consider a circuit containing a capacity C and a resistance R (but no inductance). Let them be connected in series with a battery of E. M. F. E and a key. When the key is pressed charge begins to flow from the battery to one of the plates A of the condenser. This induces opposite charge on the other plate B and similar free charge flows back to the battery from B. Thus there is a circulation of charge, *i. e.* a flow of current round the circuit. This continues until the condenser is fully charged *i. e.* until the plates attain the Pot. Diff. E; the current then stops.

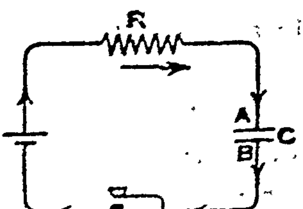

Fig. 224

Let Q be the charge at any instant on the positive plate of the condenser, during the process of charging. This produces an instantaneous P. D. $\frac{Q}{C}$ between the plates. This acts as an E. M. F. and tends to send a current in the opposite direction. Thus during the process of charging the condenser the total E. M. F. in the circuit is $E-\frac{Q}{C}$. If i be the instantaneous value of the current we have by Ohm's Law

$$E-\frac{Q}{C}=Ri=R\frac{dQ}{dt}$$

or $$R\frac{dQ}{dt}+\frac{Q}{C}=E \quad \ldots \quad (57)$$

When the condenser is fully charged let it be disconnected from the battery and be discharged through the resistance R. In this case we have a similar equation excepting that E is zero. Thus for the discharge of the condenser

$$R\frac{dQ}{dt}+\frac{Q}{C}=0 \quad \ldots \quad (57a)$$

We shall solve the second equation first.

Discharge of the Condenser $R\frac{dQ}{dt}+\frac{Q}{C}=0$ or $\frac{dQ}{dt}=-\frac{Q}{CR}$

or $$\frac{dQ}{Q}=-\frac{dt}{CR} \quad \therefore \log_e Q=-\frac{t}{CR}+A_1$$

$$\therefore Q=\epsilon^{-\frac{t}{CR}+A_1}=\epsilon^{A_1}\cdot\epsilon^{-\frac{t}{CR}}=A\epsilon^{-\frac{t}{CR}}$$

where A is a new constant equal to ϵ^{A_1}

To find A we note that

$$\left.\begin{array}{l}\text{at } t=0\\ Q=Q_0, \text{ initial maximum}\\ \text{charge on the condenser}\end{array}\right\} \quad \therefore \quad Q_0=A$$

Hence $$Q=Q_0\epsilon^{-\frac{t}{CR}}=Q_0\epsilon^{-\frac{t}{\lambda}} \quad \ldots \quad (57b)$$

where $\lambda=CR$

The product CR is called the time constant of the circuit. The smaller the value of λ the quicker is the discharge of the condenser and vice versa.

We now consider equation (57)

Charging of the condenser

$$R\frac{dQ}{dt}+\frac{Q}{C}=E$$

$$\left.\begin{aligned}\text{Let}\quad & Q=x+CE\\ \text{or}\quad & \frac{dQ}{dt}=\frac{dx}{dt}\end{aligned}\right\}\quad \therefore\; R\frac{dx}{dt}+\frac{x}{C}+E=E \quad\text{or}\quad R\frac{dx}{dt}+\frac{x}{C}=0$$

Solving as before, $x=Ae^{-\frac{t}{CR}}$

Adding CE to both sides, $Q=CE+Ae^{-\frac{t}{CR}}$

To find A we notice that

$$\left.\begin{aligned}\text{at}\quad t&=0\\ Q&=0\end{aligned}\right\}\quad \therefore\; 0=CE+A \quad\text{or}\quad A=-CE$$

$$\text{Hence}\quad Q=CE\left(1-e^{-\frac{t}{CR}}\right)=Q_0\left(1-e^{-\frac{t}{\lambda}}\right)\quad\ldots\qquad(57c)$$

where $Q_0=CE$ and $\lambda=CR$

As before λ is the **time constant** of the circuit: the rapidity or slowness of the charging of the condenser depends on the smallness or largeness of λ. For the definitions of the time constant see Art. 153.

Art 155

Measurement of capacity

The capacity of a condenser may be compared with that of another condenser by de Sauty's method. Two condensers of capacities C_1 and C_2 and two non-inductive resistances R_1 and R_2 are placed along the four arms of a Wheatstone's net ABCD. A battery E and a ballistic galvanometer G are placed as usual along the two diagonals. As the key is closed the condensers become charged and a momentary current passes along each of the two branches ABC and ADC. Since initially and finally the current is zero the points B and D are at the same potential at the beginning and also at the end. If also the current grows and decays at the same rate in both bran-

ches the points B and D remain at the same potential even during the charging of the condensers and the galvanometer does not show any deflection at any time. The currents however grow and decay at the same rate in both the branches if the time constants of the two branches are equal *i. e* when $C_1R_1 = C_2R_2$ The resistances R_1 and R_2 are adjusted until on pressing the key K no deflection is observed in the galvanometer. In that case $C_1R_1 = C_2R_2$ or $\frac{C_1}{C_2} = \frac{R_2}{R_1}$. Hence

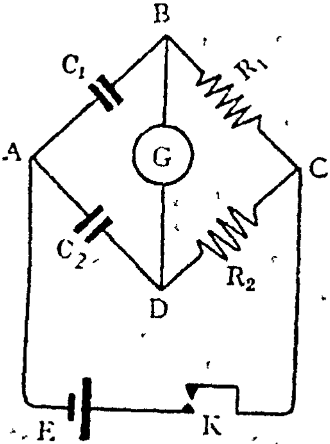

Fig. 225.

the capacities can be compared with each other. If one is known the other can be found out.

[Vide also Arts 58 and 115]

Art 156 **Anderson's Method**

Measurement of Inductance

Let four resistances R_1, R_2, R_3 and R_4 be placed along the four arms of a Wheatstone's net ABCD. The first three resistances are non-inductive but the fourth one, viz. R_4 has an inductance L. As usual a battery with a key is placed along one of the diagonals between A and C. A galvanometer G and a variable non-inductive resistance X are in series along the other diagonal between B and D. A condenser of capacity C is placed between A and O the junction of X and G.

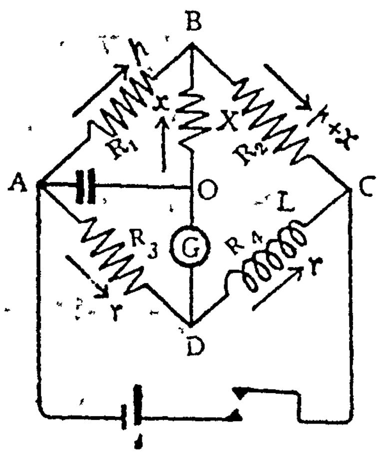

Fig. 226

At first the resistances are so adjusted that when the key is kept closed and a steady current flows round the circuit the galvanometer shows no deflection. In that case, $\frac{R_1}{R_2}=\frac{R_3}{R_4}$ ... (a)

Next, without altering these resistances any more the resistance X is adjusted until even when the key is suddenly closed or opened, *i. e.* when the current is unsteady the galvanometer shows no deflection. In that case the points O and D are at the same potential not only when the current is steady but also during the growth or decay of the current. Let p, r and x be respectively the instantaneous currents along R_1, R_3 and X. Since no current flows through the galvanometer the current through the condenser is also x, the current through R_4 is r and the current through R_2 is $p+x$. Let Q be the instantaneous value of the charge on one of the plates of the condenser C. The current through the condenser is then $\frac{dQ}{dt}$, *i. e.* $\frac{dQ}{dt}=x$. Also $\frac{Q}{C}$ is the potential difference between the plates of the condenser.

Now O and D being at the same potential,

$$V_A-V_O=V_A-V_D$$

$$\left.\begin{array}{l} i.\ e. \quad \frac{Q}{C}=rR_3 \quad \text{or} \quad Q=CrR_3 \\ \therefore \text{ differentiating } x\left(=\frac{dQ}{dt}\right)=CR_3\frac{dr}{dt} \end{array}\right\} \quad \ldots \quad (b)$$

$$\text{Again,} \quad V_A-V_B=\left(V_A-V_O\right)+\left(V_O-V_B\right)$$

$$i.\ e. \quad pR_1=\frac{Q}{C}+Xx$$

$$\therefore \text{ From } (b) \quad pR_1=rR_3+CXR_3\frac{dr}{dt} \quad \ldots \quad \ldots \quad (c)$$

Lastly, O and D being at the same potential,

$$\left(V_O - V_B\right) + \left(V_B - V_C\right) = V_D - V_C$$

i. e. $\quad Xx + (p+x)R_2 = rR_4 + L\dfrac{dr}{dt}$

or $\quad pR_2 + x(X + R_2) = rR_4 + L\dfrac{dr}{dt}$

$\therefore$ Substituting the value of x from (b)

$$pR_2 + CR_3(X + R_2)\frac{dr}{dt} = rR_4 + L\frac{dr}{dt}$$

or $\quad pR_2 = rR_4 + L\dfrac{dr}{dt} - CR_3(X + R_2)\dfrac{dr}{dt} \quad \ldots \quad (d)$

Dividing (c) by (d), $\dfrac{R_1}{R_2} = \dfrac{rR_3 + CXR_3\dfrac{dr}{dt}}{rR_4 + L\dfrac{dr}{dt} - CR_3(X + R_2)\dfrac{dr}{dt}}$

or $R_1\left\{rR_4 + L\dfrac{dr}{dt} - CR_3(X + R_2)\dfrac{dr}{dt}\right\} = R_2\left\{rR_3 + CXR_3\dfrac{dr}{dt}\right\}$

But from (a) $R_1R_4 = R_2R_3$. Hence cancelling rR_1R_4 and rR_2R_3 from both sides we have

$$R_1\left\{L\frac{dr}{dt} - CR_3(X + R_2)\frac{dr}{dt}\right\} = CXR_2R_3\frac{dr}{dt}$$

Now since $\dfrac{dr}{dt}$ occurs in every term we may cancel $\dfrac{dr}{dt}$.

Hence $R_1\{L - CR_3(X + R_2)\} = CXR_2R_3$

$\therefore \quad L - CR_3(X + R_2) = CX\dfrac{R_2R_3}{R_1} = CXR_4 \quad$ [*From* (a)]

$\therefore \quad L = CR_3(X + R_2) + CXR_4$

$\therefore \quad \dfrac{L}{C} = R_2R_3 + X(R_3 + R_4)$

Thus the ratio $\dfrac{L}{C}$ may be determined. If either L or C be known the other may be found out.

We now consider the general case of a circuit containing an inductance L, a capacity C and a resistance R. As before when the key is pressed charge begins to flow round the circuit. If i be the instantaneous value of the current, the total E.M.F.

Art 157
Circuit containing C, L and R

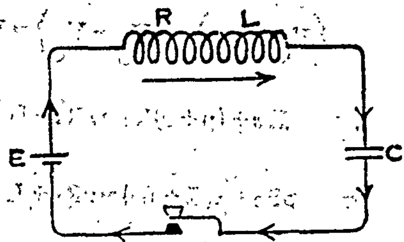

Fig. 227

in the circuit is $E - L\dfrac{di}{dt} - \dfrac{Q}{C}$. And by Ohm's Law this is equal to Ri. Thus $E - L\dfrac{di}{dt} - \dfrac{Q}{C} = Ri$.

$$\text{or} \qquad L\frac{di}{dt} + Ri + \frac{Q}{C} = E \qquad \ldots (58)$$

Similarly for the discharge of the condenser

$$L\frac{di}{dt} + Ri + \frac{Q}{C} = 0 \qquad \ldots (58a)$$

We shall solve the second equation first.

Discharge of the condenser

$$L\frac{di}{dt} + Ri + \frac{Q}{C} = 0$$

We know $i = \dfrac{dQ}{dt}$

$$\therefore \frac{di}{dt} = \frac{d^2Q}{dt^2}$$

$$\therefore \quad L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = 0$$

$$\therefore \quad \frac{d^2Q}{dt^2} + 2k\frac{dQ}{dt} + p^2Q = 0$$

where the new constants k and p are given by

$$2k = \frac{R}{L} \text{ and } p^2 = \frac{1}{CL} \qquad \ldots \qquad (58b)$$

Writing $D \equiv \dfrac{d}{dt}$ $\qquad D^2 + 2kD + p^2 = 0$

$$\text{or} \quad D = -k \pm \sqrt{k^2 - p^2}$$

$$\therefore \quad Q = A_1e^{(-k+\sqrt{k^2-p^2})t} + A_2e^{(-k-\sqrt{k^2-p^2})t}$$

where A_1 and A_2 are two arbitrary constants.

$$=e^{-kt}\left\{ A_1 e^{\sqrt{k^2-p^2}\,t} + A_2 e^{-\sqrt{k^2-p^2}\,t} \right\} \qquad \cdots \qquad (58c)$$

The values of A_1 and A_2 can be determined by imposing initial condititions :—

$$\left.\begin{array}{l} \text{at } t=0 \\ Q=Q_0 \\ \text{(initial maximum} \\ \text{charge)} \end{array}\right\} \text{ and } \left\{ \begin{array}{l} \text{at } t=0 \\ i=\dfrac{dQ}{dt}=0 \end{array}\right.$$

Substituting the 1st of these conditions in (58c)

$$Q_0 = A_1 + A_2 \qquad \cdots \qquad \cdots \qquad (\alpha)$$

Also $$i=\frac{dQ}{dt}=A_1(-k+\sqrt{k^2-p^2})e^{(-k+\sqrt{k^2-p^2})t}$$

$$+A_2(-k-\sqrt{k^2-p^2})e^{(-k-\sqrt{k^2-p^2})t}$$

∴ From the second initial condition

$$0=A_1(-k+\sqrt{k^2-p^2})+A_2(-k-\sqrt{k^2-p^2})$$

or $$A_2=A_1\frac{-k+\sqrt{k^2-p^2}}{k+\sqrt{k^2-p^2}} \qquad \cdots \qquad (\beta)$$

Substituting in (α)

$$A_1\left\{1+\frac{-k+\sqrt{k^2-p^2}}{k+\sqrt{k^2-p^2}}\right\}=Q_0$$

or $$A_1\frac{2\sqrt{k^2-p^2}}{k+\sqrt{k^2-p^2}}=Q_0$$

∴ $$A_1=\frac{Q_0}{2}\cdot\frac{k+\sqrt{k^2-p^2}}{\sqrt{k^2-p^2}}=\frac{Q_0}{2}\left(1+\frac{k}{\sqrt{k^2-p^2}}\right)$$

Hence from (β)

$$A_2=\frac{Q_0}{2}\cdot\frac{k+\sqrt{k^2-p^2}}{\sqrt{k^2-p^2}}\cdot\frac{-k+\sqrt{k^2-p^2}}{k+\sqrt{k^2-p^2}}$$

$$=\frac{Q_0}{2}\left(1-\frac{k}{\sqrt{k^2-p^2}}\right)$$

Thus $$Q=\frac{Q_0}{2}\left\{\left(1+\frac{k}{\sqrt{k^2-p^2}}\right)e^{(-k+\sqrt{k^2-p^2})t}+\left(1-\frac{k}{\sqrt{k^2-p^2}}\right)e^{(-k-\sqrt{k^2-p^2})t}\right\}$$

If $k > p$ equation (58c) cannot be simplified further. If $k < p$ equation (58c) can be reduced to a much simpler form. Thus

$$Q=e^{-kt}\left\{A_1 e^{j\sqrt{p^2-k^2}t}+A_2 e^{-j\sqrt{p^2-k^2}t}\right\}^*$$

$$=e^{-kt}[A_1\{\cos(\sqrt{p^2-k^2}\,t+j\sin(\sqrt{p^2-k^2}.t)\}$$
$$+A_2\{\cos(\sqrt{p^2-k^2}.t)-j\sin(\sqrt{p^2-k^2}.t)\}]$$

$$=e^{-kt}[(A_1+A_2)\cos(\sqrt{p^2-k^2}.t)+j(A_1-A_2)\sin(\sqrt{p^2-k^2}.t)]$$

$$=e^{-kt}\left\{A_3\cos(\sqrt{p^2-k^2}.t)+A_4\sin(\sqrt{p^2-k^2}.t)\right\}$$

where A_3 and A_4 are two new constants given by $A_3=A_1+A_2$ and $A_4=j(A_1-A_2)$

$$=Ae^{-kt}\cos(\sqrt{p^2-k^2}.t-\phi)^{**} \quad \ldots \quad \ldots \quad (58d)$$

where the new constants A and ϕ are given by

$$A=\sqrt{A_3^2+A_4^2} \text{ and } \tan\phi=\frac{A_4}{A_3}$$

To find the values of these two constants A and ϕ, the same two initial conditions may be applied.

Thus at $t=0$, $Q=Q_0$ $\quad\therefore\quad Q_0=A\cos\phi \quad \ldots \quad (\gamma)$

and at $t=0$, $i=\frac{dQ}{dt}=0$

---

* j represents the imaginary quantity $\sqrt{-1}$

** Putting $A_3=A\cos\phi$ and $A_4=A\sin\phi$ in the previous step

we have $\quad Q=Ae^{-kt}\cos(\sqrt{p^2-k^2}.t-\phi)$

Now $\quad Q = Ae^{-kt} \cos(\sqrt{p^2 - k^2}.t - \phi)$

$$\therefore \quad \frac{dQ}{dt} = -Ake^{-kt} \cos(\sqrt{p^2-k^2}.t-\phi)$$
$$-Ae^{-kt}\sqrt{p^2-k^2} \sin(\sqrt{p^2-k^2}.t-\phi)$$

$$\therefore \quad 0 = -Ak \cos\phi + A\sqrt{p^2-k^2} \sin\phi$$

$$\left.\begin{aligned} &\therefore \quad \tan\phi = \frac{k}{\sqrt{p^2-k^2}} \\ &\therefore \quad \sin\phi = \frac{k}{p} \text{ and } \cos\phi = \frac{\sqrt{p^2-k^2}}{p} \end{aligned}\right\} \quad \ldots \quad (\delta)$$

$$\therefore \quad \text{from } (\gamma) \quad A = \frac{Q_0}{\cos\phi} = Q_0 \frac{p}{\sqrt{p^2-k^2}} \quad \ldots \quad (\theta)$$

$$\therefore \quad \text{from (58}d\text{)}, \quad Q = \frac{Q_0 p}{\sqrt{p^2-k^2}} e^{-kt} \cos(\sqrt{p^2-k^2}.t-\phi)$$

$$\text{where} \quad \tan\phi = \frac{k}{\sqrt{p^2-k^2}}.$$

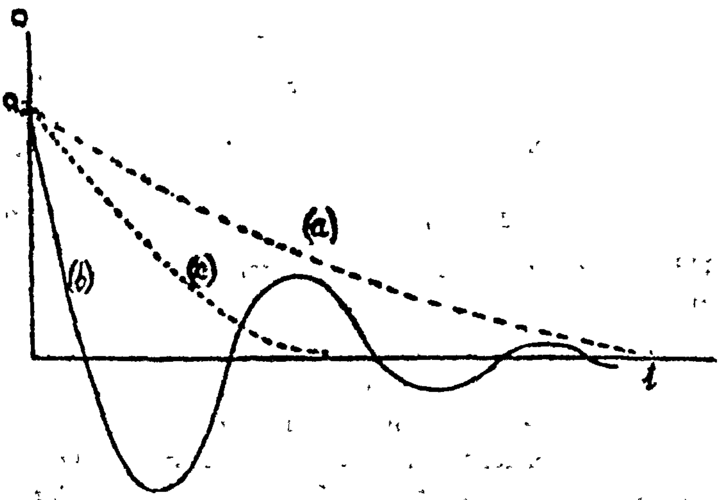

Fig. 228

(a) Dead-beat discharge. (b) Oscillatory discharge.
(c) Dead-beat discharge in the limiting condition.

The expression for Q in (58*d*) contains a cosine term. It is therefore alternately positive and negative with the increase

Oscillatory discharge

of time t. The amplitude again contains the exponential term e^{-kt}. The charge therefore alternately surges backward and forward in the circuit with gradually decreasing amplitude until the condenser is finally discharged. The discharge is graphically represented in Fig. 228(*b*). The current in the circuit is alternating during the process. The oscillatory discharge takes place, as we have seen

$$\text{when} \quad k \text{ is } < p$$

$$i.\ e. \quad \text{when} \quad \frac{R}{2L} \quad \text{is} \quad < \sqrt{\frac{1}{CL}}$$

$$i.\ e. \quad \text{when} \quad R \quad < \quad 2\sqrt{\frac{L}{C}}$$

The frequency of oscillation is given by

$$n = \frac{\sqrt{p^2 - k^2}}{2\pi} \quad \frac{\sqrt{\frac{1}{CL} - \frac{R^2}{4L^2}}}{2\pi} \qquad \ldots \qquad (59)$$

If R be extremely small in comparison to L we have

$$n = \frac{1}{2\pi\sqrt{CL}} \qquad \ldots \qquad \ldots \qquad (59a)$$

This frequency is called the natural frequency of the circuit.

In the case when k is $>p$, there is no such oscillation : the

Deadbeat discharge

charge moves only in one direction *i. e.* from the positive plate to the negative one and the condenser gradually gets discharged. This is known as dead beat discharge. The decay of Q with t is shown in Fig. 228(*a*).

The case when $k = p$ is interesting. In this limiting case equation (58*c*) breaks down; for both A_1 and A_2 become infini-

tely large as would appear from their respective values. To get a solution we first imagine that $\sqrt{k^2 - p^2}$ is not exactly zero but a small quantity h. Equation (58c) is then reduced to

$$Q = e^{-kt} \left\{ A_1 e^{ht} + A_2 e^{-ht} \right\}$$

Expanding e^{ht} and e^{-ht} by the exponential theorem and —since h is small—neglecting square and higher powers of h, we have

$$Q = e^{-kt} \left\{ A_1(1 + ht) + A_2(1 - ht) \right\}$$

$$= e^{-kt}(B_1 + B_2 t).$$

where $B_1 = A_1 + A_2$ and $B_2 = (A_1 - A_2)h$.

To find the values of these constants B_1 and B_2 we apply the same two initial conditions.

At $\left.\begin{array}{l} t = 0 \\ Q = Q_0 \end{array}\right\}$ $\therefore \; Q_0 = B_1$

At $\left.\begin{array}{l} t = 0 \\ i = \dfrac{dQ}{dt} = 0 \end{array}\right\}$ $\begin{array}{l} Q = e^{-kt}(B_1 + B_2 t) \\ \therefore \; \dfrac{dQ}{dt} = B_2 e^{-kt} - ke^{-kt}(B_1 + B_2 t) \end{array}$

$$\therefore \; 0 = B_2 - kB_1$$

$$\therefore \; B_2 = kB_1 = kQ_0$$

Hence $Q = Q_0 e^{-kt}(1 + kt)$ ... ... (58e)

Obviously the discharge is dead-beat in this case also. The variation of Q with t is shown in Fig. 228(c). It is to be noted that the discharge in this case is quicker than when $k > p$.

Art. 158. An expression for the *current* may be obtained by differentiating Q.

Dead-beat discharge.

From (58c) $Q = \epsilon^{-kt}\left\{A_1 \epsilon^{\sqrt{k^2-p^2}.t} + A_2 \epsilon^{-\sqrt{k^2-p^2}.t}\right\}$

$$\therefore \quad i = \frac{dQ}{dt}$$

$$= \epsilon^{-kt}\left\{A_1\sqrt{k^2-p^2}\,\epsilon^{\sqrt{k^2-p^2}.t} - A_2\sqrt{k^2-p^2}\,\epsilon^{-\sqrt{k^2-p^2}.t}\right\}$$

$$-k\epsilon^{-kt}\left\{A_1\epsilon^{\sqrt{k^2-p^2}.t} + A_2\epsilon^{-\sqrt{k^2-p^2}.t}\right\}$$

$$= \epsilon^{-kt}\left\{A_1\left(\sqrt{k^2-p^2}-k\right)\epsilon^{\sqrt{k^2-p^2}.t}\right.$$

$$\left. - A_2\left(\sqrt{k^2-p^2}+k\right)\epsilon^{-\sqrt{k^2-p^2}.t}\right\}$$

$$= \frac{Q_0\epsilon^{-kt}}{2}\left\{\frac{\sqrt{k^2-p^2}+k}{\sqrt{k^2-p^2}}\left(\sqrt{k^2-p^2}-k\right)\epsilon^{\sqrt{k^2-p^2}.t}\right.$$

$$\left. - \frac{\sqrt{k^2-p^2}-k}{\sqrt{k^2-p^2}}\left(\sqrt{k^2-p^2}+k\right)\epsilon^{-\sqrt{k^2-p^2}.t}\right\}$$

substituting the values of A_1 and A_2

$$= -\tfrac{1}{2}.\frac{Q_0 p^2 \epsilon^{-kt}}{\sqrt{k^2-p^2}}\left\{\epsilon^{\sqrt{k^2-p^2}.t} - \epsilon^{-\sqrt{k^2-p^2}.t}\right\}$$

Oscillatory discharge.

From (58d) $\qquad Q = A\epsilon^{-kt}\cos\left[\sqrt{p^2-k^2}.t-\phi\right]$

$$\therefore \quad i = \frac{dQ}{dt} = -Ak\epsilon^{-kt}\cos\left[\sqrt{p^2-k^2}.t-\phi\right]$$

$$-A\sqrt{p^2-k^2}\,\epsilon^{-kt}\sin\left[\sqrt{p^2-k^2}.t-\phi\right]$$

$$= -A\epsilon^{-kt}\Big[k\,\{\cos(\sqrt{p^2-k^2}.t)\cos\phi + \sin(\sqrt{p^2-k^2}.t)\sin\phi\}$$

$$+\sqrt{p^2-k^2}\,\{\sin(\sqrt{p^2-k^2}.t)\cos\phi - \cos(\sqrt{p^2-k^2}.t)\sin\phi\}\Big]$$

$$= -Ae^{-kt}\{(k\cos\phi - \sqrt{p^2-k^2}\sin\phi)\cos(\sqrt{p^2-k^2}.t)$$

$$+(k\sin\phi + \sqrt{p^2-k^2}\cos\phi)\sin(\sqrt{p^2-k^2}.t)\}$$

$$= -Ae^{-kt}\left\{\left[k\frac{\sqrt{p^2-k^2}}{p} - \sqrt{p^2-k^2}.\frac{k}{p}\right]\cos(\sqrt{p^2-k^2}.t)\right.$$

$$\left.+\left[k.\frac{k}{p} + \sqrt{p^2-k^2}.\frac{\sqrt{p^2-k^2}}{p}\right]\sin(\sqrt{p^2-k^2}.t)\right\}$$

substituting the values of $\sin\phi$ and $\cos\phi$ from (δ)

$$= -A^{-kt}\left[\frac{k^2}{p} + \frac{p^2-k^2}{p}\right]\sin(\sqrt{p^2-k^2}.t)$$

$$= -Ape^{-kt}\sin(\sqrt{p^2-k^2}.t)$$

$$= -\frac{Q_0 p^2 e^{-kt}}{\sqrt{p^2-k^2}}\sin(\sqrt{p^2-k^2}.t) \qquad [\text{ from } (\theta)\ ]$$

Dead-beat discharge in the limiting case.

From (58e) $\quad Q = Q_0 e^{-kt}(1+kt)$

$$\therefore\quad i = \frac{dQ}{dt} = Q_0 ke^{-kt} - Q_0 ke^{-kt}(1+kt)$$

$$= kQ_0 e^{-kt}\left\{1-(1+kt)\right\}$$

$$= -k^2 t Q_0 e^{-kt}.$$

Art 159
Charging of the condenser

We shall now consider the equation for th charging of the condenser.

$$L\frac{di}{dt} + Ri + \frac{Q}{C} = E$$

$$\text{or}\quad L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = E$$

Let $Q = x + CR$

$$\therefore \frac{dQ}{dt} = \frac{dx}{dt}$$

and $$\frac{d^2Q}{dt^2} = \frac{d^2x}{dt^2}$$

$$\therefore \quad L\frac{d^2x}{dt^2} + R\frac{dx}{dt} + \frac{x}{C} + E = E$$

or $$L\frac{d^2x}{dt^2} + R\frac{dx}{dt} + \frac{x}{C} = 0$$

This equation can be solved in the same way as before. Having got the value of x, the value of Q may be obtained by adding CE to x. The current i can then be obtained by differentiating Q. In this case also the charging may be dead-beat or oscillatory, the conditions being exactly the same as before. All remarks made in the case of discharge, are applicable to this case also. Growth of the charge under different conditions is shown in Fig 229.

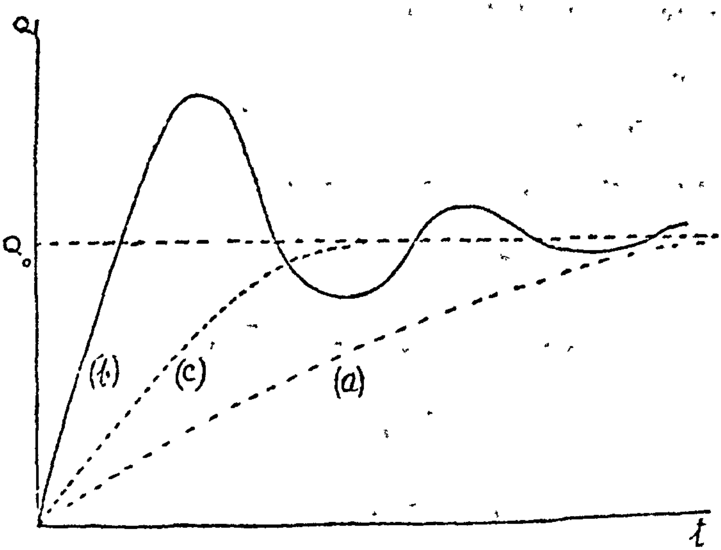

Fig. 229

(a) Dead-bead charging (b) Oscillatory charging (c) Dead-beat charging in the limiting case

In the case of the oscillatory charging—as may be seen from the diagram—the charge on the condenser and hence the potential difference between the plates oscillates a number of times above and below the final maximum value before

reaching it. Thus in the initial stage the potential difference rises much higher than the final steady value. The insulation although it may be sufficiently strong to withstand the final steady potential difference, may break down in the initial stage. This may however be prevented by inserting in the initial stage a resistance (so that the charging is not oscillatory) which may be cut out later on.

Find the inductance of a coil of 100 turns wound on a paper tube 25 cms long and 5 cms radius. If the paper tube be replaced by an iron rod of the same dimensions find how the inductance changes. [Permeability of iron = 1000]

Here $n = \frac{100}{25} = 4$ and $D = 2 \times 5 = 10$

Hence from (53*a*) $L = \pi^2 \times 4^2 \times 25 \times 10^2$ C. G. S. units

$$= \frac{\pi^2 \times 4^2 \times 25 \times 10^2}{10^9} = 3.95 \times 10^{-4} \text{ henry}$$

With iron rod $L = 1000 \times 3.95 \times 10^{-4} = 0.395$ henry

A circuit contains a resistance of 20 ohms, an inductance of 50 milli-henry, and a battery. What is the time constant of this circuit? When the circuit is closed find after what time the current rises to half the maximum value. If the battery be of 8 volts E. M. F. find the maximum current.

Time constant $\lambda = \frac{L}{R} = \frac{50 \times 10^{-3}}{20} = 0.0025$.

Substituting $\frac{i_0}{2}$ for i in (56*d*)

$$\frac{i_0}{2} = i_0\left(1 - e^{-\frac{t}{\lambda}}\right) \text{ or } e^{-\frac{t}{\lambda}} = \tfrac{1}{2} \text{ or } \frac{t}{\lambda} = \log_e 2.$$

$\therefore t = 0.0025 \times \log_{10} 2 \times \log_e 10$

$= 0.0025 \times 0.3010 \times 2.303 = 0.0017$ sec

Maximum current $= \frac{E}{R} = \frac{8}{20} = 0.4$ amp

A condenser of capacity 100 μF is discharged through the leakage resistance of 10^6 ohms. How long will it take to lose half its charge?

Here time constant $\lambda = CR = 100 \times 10^{-6} \times 10^6 = 100$.

Hence substituting $\frac{Q_0}{2}$ for Q_0 in (57*b*)

$$\frac{Q_0}{2} = Q_0 e^{-\frac{t}{\lambda}} \quad \text{or} \quad e^{\frac{t}{\lambda}} = 2 \quad \text{or} \quad \frac{t}{\lambda} = \log_e 2$$

$\therefore \quad t = \lambda \log_e 2 = 100 \times \log_{10} 2 \times \log_e 10 = 69{'}33$ secs.

Exercise XVI

1. Ennnciate and explain on a quantitative basis the laws of induction of currents in a closed circuit by means of external influence. C. U. 1937

2. State Lenz's Law and show how it can be explained on the principle of conservation of energy.

A copper disc of radius 10 cms rotates 25 times a second about an axis passing through its centre and normal to its plane. If the plane of the disc be perpendicular to a uniform field of intensity 100 C. G. S. unit, find the potential difference developed between the centre and a point on the circumference of the disc. Ans. 7'85 milli-volts.

3. How does the indnctance of a circuit affect (*a*) the growth of the current (*b*) the final value of the current?

A circuit contains a resistance of 50 ohms, an inductance of 5 henry and a battery of 4 volts E. M. F. What is the current in the circuit? If the circuit be broken find after what time the current drops down to half its value. Ans. 0'08 amp; 0'069 sec.

4. What is self induction and what is meant by non-inductive winding of a coil?

A coil of self-induction 50 henry is joined to the terminals of a battery of 2 volts E. M. F. through a resistance 10 ohms. What is meant by the time constant of this circuit? What is the maximum current that is finally established in the circuit?

Find also the time in which the current rises to half the maximum value. Ans. 5 ; 0·2 amp ; 3·47 sec.

5. Discuss the theory of discharge of a charged condenser through a non-inductive resistance.

A condenser of capacity 2 micro-farads is charged to a potential difference of 200 volts. The terminals of the condenser are joined by a wire of 100 ohms resistance. Find in what time the potential difference falls to (1) 100 volts (2) 10 volts. Ans. (1) $1{\cdot}39 \times 10^{-4}$ sec. (2) $5{\cdot}99 \times 10^{-4}$ sec.

6. What do you understand by self-inductance and mutual inductance ?

A solenoid wound with 800 turns of insulated wire is of length 25 cms and diameter 5 cms. Find the inductance of the coil. If the coil has a resistance of 10 ohms and if it is connected to a 10 volts battery of negligible internal resistance find how the current grows with time. Ans. 6·32 milli-henry.

7. Calculate the self-inductance of a solenoid of 1200 turns wound on a paper cylinder, the length of the solenoid being 50 cms and the mean radius of the turns being 1 cm. Find also the value of the inductance if the paper cylinder be replaced by an iron cylinder, the permeability of iron being 1000.

Ans. 1·137 milli-henry ; 1·137 Henry.

8. A charged condenser is discharged through a resistance of 10 megohms and the voltage drops from 100 units to 80 units in 5 min. Assuming that there is no leakage through insulation resistance of the condenser find the capacity of the condenser. Ans. 134·5 μF.

9. Due to defective insulation of a condenser the voltage falls from 100 units to 90 units in 5 min. If the terminals of the condenser are connected by a wire of resistance of 10 megohm the voltage drops from 100 units to 20 units in 2 min. Calculate the insulation resistance of the condenser.

Ans. 371·5 megohm.

10. A condenser of capacitance 4μF is discharged through an inductanse of 0·25 Henry. Find the minimum resistance necessary which when inserted in the circuit will prevent the discharge from being oscillatory. Ans. 500 ohms.

11. When is the discharge of a condenser oscillatory ?

A condenser of a capacity 0·75 microfarad is discharged through a coil of resistance 500 ohms and of inductance 0·5 henry. Is the discharge oscillatory ? If so, find the frequency of oscillation of discharge. Ans. 247·4 cycles per sec.

12. A disc of radius 8 cms is rotated inside a long solenoid of 50 turns per cm about an axis perpendicular to the plane of the disc, the axis of rotation coinciding with the axis of the solenoid. If the disc makes 600 rev/min and the current in the solenoid is 1 amp find the potential difference between the centre and circumference of the disc. C U. 1954.

C. U. Questions

1964. If a battery is suddenly connected to a circuit consisting of a resistance and an inductance in series, show that the current grows exponentially. On what factor does the rate of growth of current depend.

1965. Define coefficient of self-inductance of a coil. Show how the current in a circuit containing a coil of inductance L, a resistance R in series with a battery of E. M. F. E builds up when switched on.

A 2 volt battery of negligible internal resistance is applied to a coil of inductance 1 henry and of resistance 1 ohm. Calculate the time required by the current to attain a value half that in the steady state. Ans. 0·69 sec.

1966. (1) Define self-inductance. Derive an expression for self-inductance of a long solenoid.

An air-cored solenoid has 500 turns over a length of 40 cms. The diameter of the solenoid is 3 cms. Calculate its self-inductance. Ans. 0·56 milli-henry.

(2) Find the expression for the current at any instant in a circuit of given resistance and capacitance.

1968. Calculate the growth of charge in a capacitor of capacitance C connected in series with a battery of E.M.F. E and a (non-inductive) resistance R.

If $C = 2.4\ \mu F$, $R = 0.02$ megohm in what time will the charge in the capacitor attain half its final value ($\log_e 2 = 0.6931$).

1969 Explain what is meant by coefficient of self-induction and coefficient of mutual induction. Calculate the value of self-induction in a very long solenoid.

Find the self-inductance of a solenoid 40 cm long and radius 4 cm having 200 turns ($\mu = 1$).

1970. An electric circuit containing a capacitance C and a non-inductive resistance R in series are connected to a battery of E.M.F. equal to E through a plug key. Show that when the battery circuit is closed the charge on the condenser plates grows exponentially.

Explain what is meant by time constant of the circuit.

1971. An electric circuit consists of a coil of inductance L of negligible resistance in series with a non-inductive resistance R with a battery of E.M.F.E. (with negligible internal resistance) and a plug key. The circuit is closed so that a steady current is flowing. Derive an expression for the rate of decay of current when the plug key is suddenly taken off. What is the time constant of the circuit?

1972, 1975. Explain the terms: "Coefficients of self and mutual inductions."

A condenser of capacity C is connected in series with a non-inductive resistance R and a cell of e.m.f. E through a tapping key. Derive an expression to show the nature of development of charge on the condenser with time as the tapping key is pressed. Draw a curve showing the variation of current with time in the above case.

1974. Which has a greater inductance—a straight long wire or a wire of the same length but coiled up?

1976. Define the coefficient of self-inductance of a coil. What is its practical unit and how is it related with the e.m.u. unit?

A circuit contains a coil of self-inductance L, a resistance R, a plug key and a battery of e.m.f. E, all in series. When the circuit is closed find how current grows in it with time. Show in the form of a graph the variation of current with time. What is meant by time constant of the circuit?

CHAPTER XVII

ELECTRIC INSTRUMENTS

Art 160
Dynamo

Suppose a coil rotates between two poles of a powerful horse shoe magnet, the axis of rotation being at right angles to the direction of the magnetic field. At any instant let the plane of the coil make an angle θ with the magnetic field. Since the field strength H is the number of lines of force per unit area, the number linked with each turn of the coil is HA sin θ, A being the area of the coil. Hence if there be n turns of the coil the number of lines of force linked with the coil is given by $N = n\text{HA} \sin\theta$. As the coil rotates this number changes and an E. M. F. is induced in the coil the strength of which is as follows :—

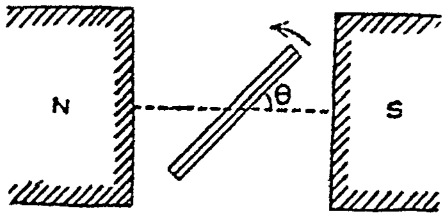

Fig. 230

$$E = \frac{dN}{dt} = n\text{HA}\cos\theta\,\frac{d\theta}{dt}$$ neglecting the minus sign which indicates the direction.

$$= n\text{HA}w\cos\theta, \text{ if } \frac{d\theta}{dt} = w$$ be the constant angular speed of rotation of the coil.

$$= n\text{HA}w\cos wt \quad \ldots \quad \ldots \quad (60)$$

if the angle be measured from the instant when the plane of the coil coincides* with the lines of force.

Obviously with the increase of time t cos wt changes its

---

* If the angle be measured from the instant when the plane of the coil is perpendicular to the field and if the coil be rotated in the opposite direction cos θ is evidently equal to sin wt; if the angle be measured from any other instant, a phase angle α is introduced.

magnitude and sign periodically. The E. M. F. therefore undergoes periodic changes both in magnitude as well as in direction. Such an E. M. F. depending on a cosine or a sine* function of time t is called an alternating E. M. F.; the current produced by such an alternating E. M. F. is also a cosine or a sine function of time and is called an alternating current.

The direction of current induced in a conductor moving in a magnetic field may be found out by Fleming's *Right* hand Rule :—

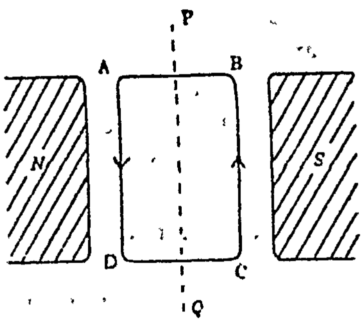

Fig. 231

Stretch the thumb, the forefinger and the middle finger of your *right* hand in such a way that each is perpendicular to the other two. Then if the forefinger indicate the direction of the field and the thumb the direction of motion then the middle finger indicates the direction of the induced current. Thus in Fig. 231 let the coil ABCD rotate about its axis PQ. At any instant let the arm AD be moving downwards towards the plane of the paper and the arm BC upwards away from the plane of the paper. Then by applying Fleming's Right Hand Rule it can easily be seen that the current generated in the coil at the instant is in the direction of the arrow as shown in the diagram.

The two terminals of the coil of wire rotating between the pole pieces N and S [Vide Fig. 230], are connected to two metallic rings (called slip rings) both of which rotate with the same speed and about the same axis as the coil [Vide Fig. 232]. Two brushes pressing against the two rings lead the current from the instrument to the external circuit. Such

* In an actual dynamo the E. M. F. is usually a more complex function of time; but this complex function can always be resolved by Fourier's Theorem into a number of cosine or sine functions.

a machine producing an alternating E. M. F. is known as an alternating current (A. C.) dynamo or an A.C. generator or an alternator. The rotating part is known as the rotor and the stationary portion is called the stator.

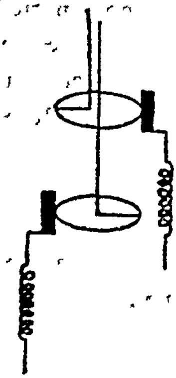

Fig. 232

From (60) $E = E_0 \cos wt$ ... (60a)

where $E_0 = nHAw$ ... (60b)

E_0 is evidently the maximum value of E. To increase E_0 we have got to make one or all of the four quantities n, H, A, w as large as possible. Values of n and A are limited by the consideration of space between the pole pieces. To have large values of n and A we require a large air gap between the poles, but that reduces the other factor H. The angular velocity w can be increased if the coil be rotated by mechanical means, *e. g.* by an oil engine, a steam engine or a steam turbine. It is to be noted that the running cost of a dynamo depends chiefly on that of this mechanical power. If the power obtained from a waterfall be utilised for this purpose the running cost becomes extremely small and electricity may be obtained very cheap. Such hydro-electric power has revolutionised industry in many foreign countries. In India it has just made a beginning.

The magnetic field H can obviously have a large value if the magnet is an electro-magnet. But this necessitates the use of a current obtained from some external supply. We are thus led to the paradoxical problem that in order to generate a current by a dynamo we must have another source producing a current. The solution of this problem depends—as we shall see below—on the residual magnetism discussed in connection with hysteresis.

To produce the electromagnet a coil is wound round the pole pieces; this is known as the field coil. The coil which rotates between the pole pieces is called the armature coil. The field coil and the armature coil are connected,—sometimes in series, often in parallel and in certain cases in mixed

circuit. The dynamo is called series dynamo, shunt dynamo and compound dynamo in the three different cases.

As has been explained in the article on Hysteresis the pole pieces,—once they are magnetised by an external field, retain a small magnetism (residual magnetism) even when the external field is removed. The pole pieces are therefore *once* magnetised by an external source of electric current. The residual magnetism which is left when the current is switched off is then always present and in future no more excitation of the field coil is necessary by an external source. The armature coil rotating in the weak field produced by this residual magnetism generates a weak current; the field coil being connected to the armature coil, part of this current flows through the field coil also. The strength of magnetism is thereby increased somewhat producing a corresponding Increase in the current generated. Thus one helping the other both the field strength and the current steadily increase until the former reaches the saturation value when the current also attains its maximum strength.

As is obvious some energy has to be spent for rotating the armature coil. And when the armature rotates we get energy in the form of an electric current. The ratio of the energy obtained to the energy spent is known as the efficiency of the dynamo.

Art 161 Commutator As has been explained previously the E. M. F. generated by such a dynamo is necessarily alternating, the E. M. F. changing its direction after each half revolution of the armature coil. The E. M. F. *applied to the external circuit* may however be made unidirectional or direct by a device known as the commutator. This consists of a short cylinder split into two halves. The two terminals of the armature coil—instead of being connected to the slip rings, as explained earlier—are attached to these two halves of the cylinder. The cylinder is rigidly attached to the armature and therefore rotates with the same speed and about the same axis.

Two carbon brushes* pressing against the cylinder at

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* These brushes are fixed and do not rotate with the armature.

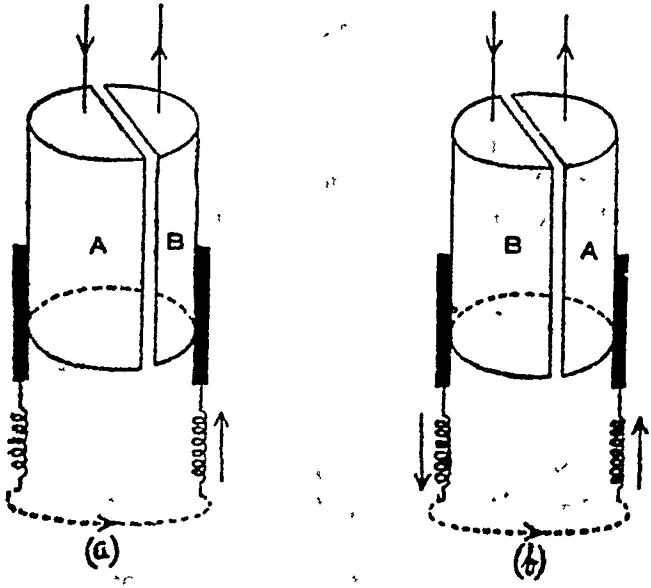

Fig. 233

opposite ends of a diameter, lead the current to the external circuit. In Fig. 233 (a) the E. M. F. is directed from the armature to the half A and from the half B to the armature and is applied to the external circuit in the direction shown in the diagram. After half a revolution the E. M. F. in the armature is reversed in direction, *i. e* it is now directed from the armature to the half B and from the half A to the armature; but as A and B have also interchanged their positions during this half revolution [Vide Fig. 233 (b)], the direction in which the E. M. F. is applied to the external circuit remains unaltered.

It should however be noticed that by this simple device, the E. M. F. in the external circuit is made unidirectional but not of uniform strength. For without the commutator the E. M. F. varies with time as show in Fig. 234 (a). By the commutator the E. M. F. in the opposite direction is reversed ; thereby the E. M. F.-time relation becomes as represented in Fig. 234 (b). Thus the E. M. F. though unidirectional, fluctuates in strength with time ; this is obviously not a satisfactory arrangement.

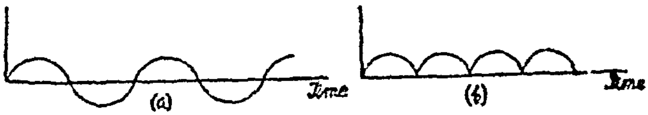

Fig. 234

Let us now consider the effect of a having a second coil in series with the first placed at right angles to it. From (60a) it is clear that when the E. M. F. in one of the coils is maximum that in the other is zero and *vice versa*. In Fig. 235 the curves A and B represent the E. M. F.'s produced separately by the two coils. Since the two coils are in series the resultant E. M. F. is obtained by adding up the two E. M. F.'s algebraically. This is shown by the curve C.

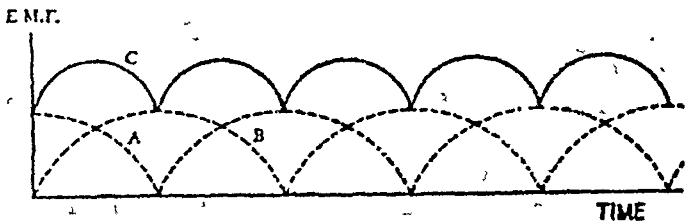

Fig. 235

It is seen that the resultant E. M. F. is never zero. At its minimum it is equal to the maximum of any one of the two component E. M. F.'s. Also the fluctuation, *i. e.* the difference between maximum and minimum values, is now much less—actually it is 41% of the fluctuation in any component E. M. F. produced by a single coil. Thus, with only two coils the fluctuation with time is reduced to 41%. If more coils are added in different angular positions the fluctuation diminishes still further. It can be shown that with 30 such coils the fluctuation practically ceases to exist and the resultant E. M. F. becomes unidirectional as well as steady.

Thus in an actual dynamo instead of a single coil a cylinder rotates between the pole pieces of the electromagnet, with its axis perpendicular to the magnetic field. Along the length of the cylinder a number of slots is cut. In these slots coils of wire are wound so that there is a number of coils in

different angular positions, all rotating with the cylinder. The commutator also,—instead of being divided into two parts—consists of an even number of insulated segments. Two terminals of each of rotating coils are connected to a pair of these segments.

It is so arranged that to each segment of the commutator, two terminals—one of each of two coils, are attached. The coils are thus mutually connected and the resultant E. M. F. generated by these coils is ultimately collected by a pair* of carbon brushes pressing against the rotating segments of the commutator. A dynamo provided with a commutator of this type is known as a direct current (D. C) dynamo or a D. C. generator.

Art 162
Long distance transmission and transformer

In modern times electricity is often generated by utilising the power of a waterfall or of a running stream. In such cases the generating station is miles away from the station where electrical energy is actually consumed. It therefore becomes necessary to transmit electric power through a long distance. Now power is the product of the voltage and the current. If electrical power is transmitted at low voltage a heavy current must flow through the cables** joining the two stations. In order that the cables may withstand this heavy current they must be thick. Thick cables however are very costly. Much cost can therefore be saved if

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* In actual dynamos the number of poles of the electromagnet is generally greater than two. The coils of the armature are wound in either of the two ways,—Wave winding or Lap winding. In the former case whatever be the number of poles, the different coils are so connected that in reality there are only two resistances in parallel and two brushes are used to lead the E. M. F. to the external circuit. In the latter case however, the number of separate resistances in parallel is equal to the number of pairs of poles and the number of brushes is the same as the number of poles. The brushes however are mutually so connected that one set of brushes acts as the positive terminal and the other set as the negative terminal, with respect to the external circuit.

** The wires connecting two stations at a fairly long distance apart are known as cables.

the voltage at which power is transmitted is very high so that the current is comparatively low and thin cables may be used.

Thus at the generating station the voltage has to be raised to a very high value—11,000 volts or much higher. At the other station where electric energy is actually consumed the voltage again must be lowered to 220 volts or 440 volts as desired. The instrument by which the voltage can be increased from a low value to a high value or from a high value to a low value is known as a Transformer. In the former case the instrument is a step-up transformer; in the latter the instrument is known as a step-down transformer.

It is to be noted that it is possible to increase or decrease the voltage by a comparatively simple instrument in the case of A. C. supply. In the case of D. C. supply no such simple instrument is possible.

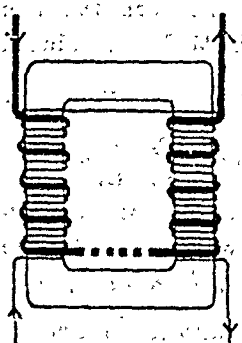


Fig. 286

A simple transformer consists of a closed soft iron laminated core over which are wound two separate coils which are insulated not only from each other but also from the core. The A. C. voltage to be transformed is connected to one of the coils, known as the primary and the transformed voltage is obtained from the other coil known as the secondary. An elementary theory of the transformer is stated as follows :—

The alternating current in the primary produces a varying magnetic flux which circulates round the iron core. This flux being linked with the secondary an E. M. F. is generated there. Since both primary and secondary are wound over the iron core, assuming that there is no leakage the same flux is linked with each turn of the primary or of the secondary. Let n_1 and n_2 be the number of turns in the primary and in the secondary respectively and at any instant let N be the flux linked with each turn. Due to variation of

the flux

E. M. F. E_1 induced in the primary $= n_1 \dfrac{dN}{dt}$

and E. M. F. E_2 „ „ secondary $= n_2 \dfrac{dN}{dt}$

Hence $\dfrac{E_2}{E_1} = \dfrac{n_2}{n_1}$. In a step-up transformer n_2 must therefore be very large in comparison to n_1. Thus the primary consists of a comparatively small number of turns of rather *thick* wire, and the secondary consists of a large number of turns of *thin* wire; reverse is the case in a step-down transformer.

If the resistance of the primary is negligibly small the E. M. F. E_p *applied* to the primary is equal to the E. M. F. E_1 *induced* in the primary. And if the current in the secondary be negligibly small the E. M. F. E_s between the two terminals of the secondary is equal to the E. M. F. E_2 induced in the secondary. The ratio $\dfrac{E_2}{E_1}$ is known as the transformer ratio K. Under ideal conditions stated above $\dfrac{E_s}{E_p} = \dfrac{E_2}{E_1} = K$. If i_p and i_s be respectively the currents in the primary and in the secondary the in-put power, *i. e.* power applied to the primary is $E_p i_p$ and the output power, *i. e.* the power obtained from the secondary is $E_s\,i_s$. The ratio $\dfrac{E_s i_s}{E_p i_p}$ is known as the efficiency of the transformer. Under ideal conditions this is equal to one. Hence $\dfrac{i_s}{i_p} = \dfrac{E_p}{E_s} = \dfrac{n_1}{n_2} = \dfrac{1}{K}$.

The resistance of the primary, however although small, can never be zero. If we take into consideration the resistance R_p of the primary it is obvious that there is a potential drop $i_p R_p$ across the primary. In that case $E_p - i_p R_p = E_1$. Similarly if the current i_s in the secondary be taken into consideration the potential drop is $i_s\,R_s$ where R_s is the resistance of the secondary. In that case

$$E_2 - i_s R_s = E_s \quad \text{or} \quad E_2 = E_s + i_s R_s.$$

Hence $$\frac{E_s + i_s R_s}{E_p - i_p R_p} = \frac{E_2}{E_1} = K.$$

or $$E_s + i_s R_s = K(E_p - i_p R_p)$$

$$\therefore \quad E_s = K(E_p - i_p R_p) - i_s R_s$$

$$= KE_p - K^2 i_s R_p - i_s R_s$$

$$= KE_p - i_s (K^2 R_p + R_s)$$

$$\therefore \quad \frac{E_s}{E_p} = K - i_s \frac{K^2 R_p + R_s}{E_p}$$

Thus the ratio $\frac{E_s}{E_p}$ is not equal to the constant K but has a variable portion depending on the secondary current i_s. The greater is the value of the current i_s drawn from the secondary the less is the output voltage E_s. The efficiency $\frac{E_s i_s}{E_p i_p}$ is in this case necessarily less than one.

Art 163
Losses in a transformer

The efficiency of a transformer is also reduced due to various losses as enumerated below :—

(1) **Eddy current loss** When the flux established by the alternating current varies in the iron core an induced current known as eddy current is generated in the core *at right angles* to the flux This current generates heat ; this heat energy is evidently a loss This is minimised by making the core laminated, *i e.* the core is made up of thin strips of iron insulated* from one another so that due to high resistance of the insulation very little current flows at right angles.

(2) **Hysteresis loss.** Due to alternating current the core is taken very rapidly through Hysteresis cycles. This is made a minimum by using such a material for the core that the area of the Hysteresis Loop for this material is extremely small.

(3) **Copper loss.** As the current flows through the copper wires forming the primary and the secondary heat is

* This insulation is done by varnishing the strips with some paint.

generated. This cannot be entirely eliminated.

(4) **Loss due to leakage.** A few lines of force produced by the primary current escape into air and are not linked with the secondary. This is known as *primary leakage flux.* Similarly some of the lines of force generated by the secondary do not cut the primary. This is called *secondary leakage flux.* The efficiency of the transformer is reduced due to both the losses. These losses are made a minimum if there is no gap between the primary & the secondary and also between these two coils and the iron core.

Art 164
Electric motor

The action of a direct current (D. C.) motor depends on the principle illustrated in a suspended coil galvanometer. Suppose a coil is placed between the pole pieces of a horse-shoe magnet with its plane parallel to the magnetic field and suppose the coil is capable of rotating freely about an axis perpendicular to the magnetic field. If a current be now passed through the coil, the coil rotates because of the couple exerted on the current by the magnetic field (Vide Art 89). This couple vanishes when the plane of the coil becomes perpendicular to the magnetic field. Due to inertia however the coil overshoots this position (known as dead point); at the same instant if the current through the coil be reversed in direction the coil continues to rotate further in the same direction. After rotating through 180° the coil again reaches the dead point, *i. e.* is again perpendicular to the magnetic field. If as it passes through this position due to inertia the current through the coil be again reversed the coil continues in its rotation. Thus if the current be reversed after each half revolution when the coil overshoots the dead point the coil goes on rotating continuously.

It is now clear that a dynamo provided with a commutator may also act as a direct current electric motor. Let us first suppose that there is a single coil in the armature and the commutator consists of only two segments [Vide Fig. 233]. The current from an external source is led to the brushes from which the current ultimately passes through the arma-

ture coil. This produces the rotation of the armature. The positions of the brushes are so adjusted that when the armature coil crosses the dead point, the brushes pass on to the other segment of the commutator and the current through the armature is reversed in direction. This happens after every half revolution when the armature overshoots the dead point. Thus the rotation of the armature is produced continuously.

In an actual motor (which is the same as a generator), there is a number of coils (armature coils) placed in slots cut along the length of a cylinder rotating between the pole pieces. The commutator also, as in a generator, is divided into an even number of segments. Two terminals of each of the coils are connected to an opposite pair of these segments and two brushes press against the commutator. In the dynamo the cylinder is rotated continuously by a steam engine, an oil engine or by some such agency and the current produced is led by the brushes to the external circuit. In the motor, on the other hand, the current from an external source is led by the brushes to armature coils and continuous rotation of the coils and hence of the cylinder is produced.

Back E. M. F.

It must however be remembered that while the armature is rotating due to the current from the external source, it must also be producing an E. M. F. (as in a generator) because it is rotating in a magnetic field. By Lenz's Law this E. M. F. (known as back E. M. F.) is in opposition to the external E. M. F. Thus if E be the external E. M. F. and e the back E. M. F. generated, $E-e$ is the resultant E. M. F. which ultimately sends current through the armature. Thus the current is less after a few revolutions than at the start; for, at the start the current is due to the external E. M. F. E alone, but after a few revolutions it is due to $E-e$. Sometimes the current at the start is so large that there is a chance of the armature coil being burnt out, although it (armature) is quite capable of standing the final current. In that case at the start a resistance is used in series with the armature. This is known as 'starting resistance'. After a few revolutions of the

armature the resistance is removed from the circuit; an automatic arrangement may be made for the purpose.

We have here discussed in an elementary way the principle of a D. C. motor. The principle of an A. C. motor is different and is beyond the scope of this book.

Art 165 An induction coil essentially consists of
Induction coil two coils of wire, one wound over the other; the inner one is known as the primary and the outer one as the secondary. An automatic arrangement is made whereby

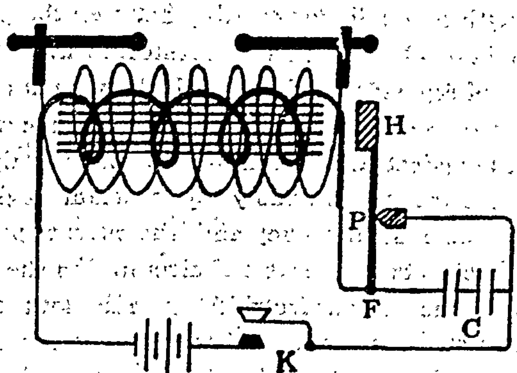


Fig. 237

the primary circuit is made and broken alternately. At each make the primary current grows and at each break the primary current decays and hence the lines of force due to the primary current correspondingly increase and decrease. These lines being linked with the secondary an induced E. M. F. is generated in the secondary at each make and at each break of the primary.

The primary consists of a coil of comparatively thick wire wound over a number of laminated iron rods—known as iron core. The secondary consists of a *large* number of turns of thin wire wound over the primary. A soft
Description iron hammer H placed in front of the iron core is held by a lever whose fulcrum is F. A platinum point just touches the lever at P. Electric connections are made as shown in Fig. 237. Usually a condenser C is placed in parallel with the platinum contact at P.

When the key K is closed the primary circuit is complete. Due to the primary current thus established, the iron core becomes magnetised and attracts the iron hammer H. The lever carrying the hammer is thereby pulled towards the primary thus breaking the primary circuit at P. The primary current therefore stops and the iron core is demagnetised. The force of attraction on the hammer being thereby removed the lever springs back to its original position thus again establishing the primary circuit. The action goes on repeating as before.

Action

E. M. F. at break is greater

As the primary circuit is thus made and broken an induced E. M. F. is generated in the secondary at each make and at each break of the primary. The strength of the E. M. F. varies as the rate at which the primary current grows at make and decays at break. This rate again depends on the time constant $\left(=\frac{L}{R}\right)$ of the primary circuit [Vide Art 153]. When the primary circuit is closed the resistance R is practically equal to that of the primary coil; on the other hand when the circuit is broken a small air gap appears at the platinum contact at P, thereby increasing the total resistance enormously. The time constant being thus reduced to a large extent at break, the primary current decays very rapidly. The rate of decay of the primary current at break, is therefore much greater than the rate of growth at make. The E. M. F. induced in the secondary is necessarily much greater at break than at make of the primary circuit.

The E. M. F. in the secondary is usually utilised in sending an electric discharge through a gas. Ordinarily, the E. M. F. at make is not sufficiently large to send the discharge. Thus although E. M. F.'s are developed in the secondary, at make as well as at break of the primary—and these E. M. F.'s are oppositely directed—discharge through the gas takes place

in one direction only, due to the E. M. F. at break. Usually a commutator is included in the primary circuit, whereby the current in the primary and therefore the discharge through the gas by the secondary, may be reversed in direction.

The presence of the condenser helps to increase the strength of the E. M. F. at break still more. When the primary circuit is closed the condenser is charged; when it is broken the condenser tends to get discharged, sending a current in the opposite direction. Thus without the condenser, at break of the primary the current simply drops from the maximum value i_0 down to zero value; whereas with the condenser, the current is reversed in direction, *i. e.* changes from $+i_0$ to $-i_0$. As a result the current varies practically at double the previous rate and the corresponding E. M. F. developed in the secondary is also almost doubled.

Action of the condenser

The condenser also acts as a shunt for the spark gap. When the primary circuit is broken there is a tendency for an arc to be formed across the air gap between the platinum tips at P. This rapidly wears away the platinum points. The condenser however absorbs almost all the electrical energy and prevents the arc from forming; the platinum tips therefore last longer.

The function of the iron core is two fold. First when it is magnetised by the primary current it attracts the iron hammer and thereby causes the primary circuit to be broken. When it is demagnetised the attraction on the hammer being gone the primary circuit is again made. The make and break of the primary circuit therefore essentially depend on the action of this iron core. There is another important aspect in which the iron core affects the working of the instrument. When it is magnetised lines of force are generated due to its magnetism and these lines are also linked with the secondary: when it is demagnetised only a small residual magnetism

Action of the iron core

remains in the core so that almost all the lines of force previously associated with the secondary are removed. If the core has a large permeability μ these lines are much more numerous than those produced by the primary current alone. The strength of the E. M. F. generated in the secondary therefore mainly depends on the variation of these lines of force due to the iron core. A core having a large permeability and a low hysteresis loss is evidently highly suitable for use in the induction coil; for in that case the variation of the lines of force is very large. The core is therefore made of soft iron (Vide Art 141),

Further, induced currents are also generated in the core itself due to the variation of the primary current. These induced currents contribute nothing towards the working of the instrument and therefore represent a loss. This wastage of energy is reduced to a minimum if the iron core consists of laminated rods, *i. e.* rods which are long but extremely thin and insulated from one another.

It may be noted that an induction coil is in reality a step-up transformer. Although direct current from a battery is applied to the primary the primary current is never steady, it is either growing or decaying. Due to the variation of the primary current an E. M. F. is generated in the secondary. As the number of turns of the secondary is extremely large in comparison to that of the primary the secondary E. M. F. is extremely high. The induction coil is thus a step-up transformer. By applying a battery of 4 volts to the primary an E. M. F. of 10,000 volts or even more may easily be obtained from the secondary.

Art 166
Telegraph

In order to transmit messages from one station to another two instruments are necessary :—(1) a transmitter and (2) a receiver. In the transmitter a lever (made of a conducting substance) has its fulcrum in the middle. Two small

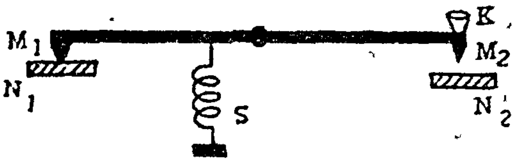


Fig. 238

metal pieces M_1 and M_2 are attached to the bottom of the lever at the two ends and below them there are two other fixed metal pieces N_1 and N_2. A spring S pulls the lever downwards on one side so that M_1 ordinarily remains in contact with N_1. At the other end there is a knob K. When this knob is pressed M_1 goes up and contact is made between M_2 and N_2. When the knob K is released, by the action of the spring S the other end of the lever comes down so that contact is again made between M_1 and N_1.

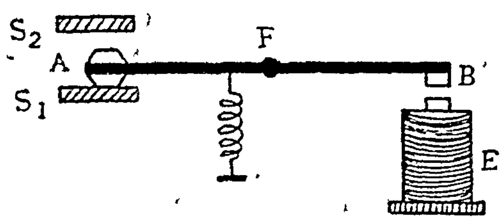
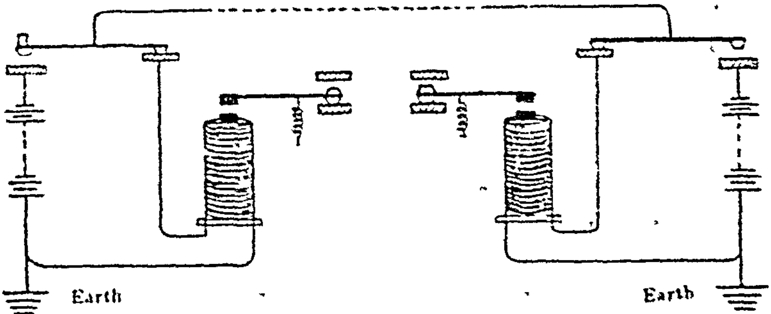


Fig. 239

In the receiver also there is a lever AB having the fulcrum in the middle at F. Near the end A just above and just below the lever there are two fixed metallic studs S_1 and S_2. A spring pulls the lever downwards on one side so that the end A is ordinarily in contact with S_1. At the other end B a small *iron* piece is attached to the lower face of the lever; and below this there is an electromagnet E. When a current passes through the electromagnet the iron piece is attracted and the end B comes down; the other end A therefore goes up and comes in contact with S_2 and a short sound is thereby produced. When the current through the electromagnet is stopped the force of attraction on the iron piece is gone and by the action of the spring the end A of the lever comes down. As the end A comes in contact with S_1 another short sound is produced.

The complete connection is now shown in Fig 240. Let us suppose that a message is to be sent from the station A to the station B. At A as the knob is pressed the circuit is



Station A Fig. 240 Station B

completed* and a current from the battery at A passes through the electromagnet at B and a sound is heard there. When the knob is released at A the circuit is broken, the current through the electromagnet stops and another sound is heard at B. Thus at the station A as the knob is pressed and released short sounds are heard at B. The interval between any two successive sounds can evidently be adjusted. If the interval is short it is known as a *dot* and if the interval is comparatively long it is called a *dash*. Different letters of the English alphabet are represented by different combinations of these dots and dashes. Thus in Morse code the letter A corresponds to — . B to — ... O to — — . and so on. It is clear from Fig 240 that in a similar way a message may be sent from the station B to the station A

Art 167 Microphone

The function of a microphone depends upon the fact that carbon granules when loosely packed in a box offer tremendous resistance to the current tending to pass through the box, so that practically no current passes. If however the sides of the box are pressed the carbon granules are more compressed, the resistance diminishes and a current passes through the box. The strength of the current depends upon the compression of

* It will be seen that there is one wire connecting the two stations. The Earth serves the purpose of the second wire so that the circuit is completed through the Earth.

the carbon granules and hence on the pressure applied to the sides of the box.

A microphone† essentially consists of a box B containing carbon granules. On two sides of the box there are two polished carbon plates C_1 and C_2 the other sides being covered up by non-conducting screens. The carbon plate C_1 in front of the box is rigidly attached to an *extremely thin* metallic plate P. By means of a cone C sound waves are concentrated on P. As shown in the diagram an electric circuit is completed by connecting the terminals of a battery to C_1 and C_2. The circuit

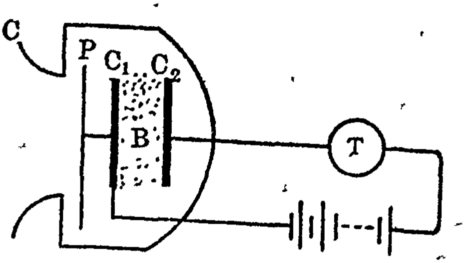
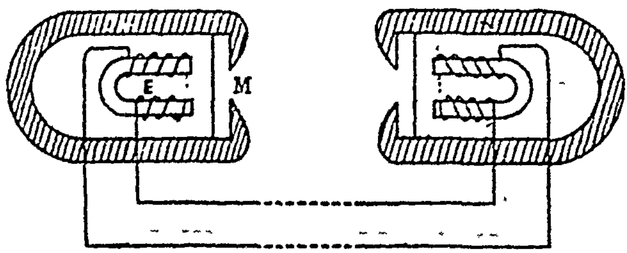


Fig. 241

also contains an instrument T which responds when an electric current passes through it. Ordinarily due to high resistance in the box B no current passes through the circuit. But as sound waves are incident on the plate P it begins to vibrate. C_1 being rigidly attached to P also vibrates, pressure on the carbon granules undergoes variation and a current of varying strength passes through T. The variation of the current obviously corresponds to the variation of pressure on carbon granules produced by sound waves and hence on the nature of the sound. If the instrument T be the receiver of a telephone [Vide Art 168]. the same sound is reproduced; a person attending the telephone may easily hear the sound. If however the instrument be a loudspeaker sound is reproduced with increased intensity so that a large audience may hear the sound.

† A microphone is popularly known as "mike".

Art 168 Telephone The telephone was first invented by Alexander Graham Bell in 1878. In Bell's original form the transmitter and the receiver were identical. A thin steel membrane M is stretched before the poles of a horse-shoe magnet E A coil of wire is wound over this magnet. One such instrument is used as the transmitter and an exactly similar one as the receiver. Two such instruments are therefore used—one at each station—and the coils wound over the horse shoe magnets in the two instruments are connected in series. Suppose a message is to be sent from the station A to the station B. At A some one speaks in front of the membrane M. Due to sound waves M vibrates. Being in the presence of the horse shoe magnet the membrane M is itself magnetised by induction. Due to its vibration

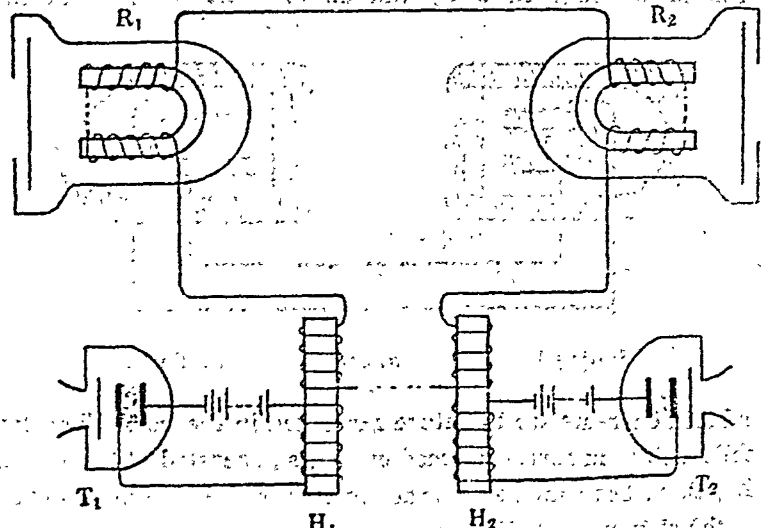


Station A Fig. 242 Station B

induced currents are therefore generated in the coil. The two coils being in series induced currents generated at the station A pass through the coil wound over the horse shoe magnet at the station B. At B as the currents pass through the coil the strength of the magnet undergoes fluctuations and necessarily the force of attraction on the steel membrane varies. The membrane accordingly vibrates and speech is reproduced, Obviously in a similar way a message may be sent from the station B to the station A.

In modern times with minor modifications the receiver is practically the same as in Bell's telephone. But the transmitter is nowadays a microphone transmitter. In Fig 241 if the

instrument T be the receiver of a telephone sound produced in front of the microphone generates currents which when passed through the coil of the receiver produce vibrations of the steel membrane and sound is reproduced. Thus at each station there must be a microphone transmitter and also a receiver. The complete connection is shown in Fig 243. Usually two transformers H_1 and H_2 are also used, one at each station. The secondaries of the two transformers and the coils of receivers are all in series. At the station A when sound is produced before the microphone transmitter T_1 currents are generated and these pass through the primary of the transformer H_1.



Station A Fig. 243 Station B

Currents are therefore produced in the secondary and as these currents pass through the coil of the receiver R_2 at the second station B sound is reproduced there. Similarly sound produced before the transmitter T_2 is reproduced in the receiver R_1.

Exercise XVII

1. A 1000 turn circular coil of wire of radius 10 cms. is rotated 50 times a second about a vertical diameter. Find the E. M. F. generated. [H = 0·36]. Ans. 0·25 volt (virtual)

2. Explain the action of a generator. How can the current produced by a generator be made direct ?

3. Discuss in an elementary way the physical principles on which the working of an ordinary electric motor is based.

4. Describe the construction and explain the action of an induction coil. Explain why the sparking between the terminals of the secondary is usually unidirectional, although E. M. F's generated between these terminals at make and break of the primary are in opposite directions.

5. Explain fully the purpose served by the bundle of iron wires which form the core of an induction coil. Has the kind of iron used any influence on the working of the coil ?

6. Explain in details the working of an induction coil with special reference to the parts played by the iron core and the condenser. Can you call it a transformer ? Give reasons for your answer. C. U. 1934

7. Explain the action of (*a*) a microphone (*b*) a telephone.

C. U. Questions.

1968. Explain the principle of action of a transformer and some of its uses. Obtain a relation between the turns ratio and voltage transformation ratio for an ideal transformer.

1970 (a) What is transformer ? Explain the principle of its action. Discuss its importance in power transmission.

(b) Explain the working of a D.C. motor. Why is it necessary to have a starter in a large D.C. motor ?

1974. Describe the construction of a Direct Current Generator and give a sketch diagram. Explain how the output current is made unidirectional and the advantage of having a number of coils wound on the same armature. What is meant by the efficiency of a generator?

CHAPTER XVIII

ALTERNATING CURRENT

Art 169 We have seen that the E. M. F. produced by an A. C. dynamo is expressed by $E = E_0 \cos wt$ or $E = E_0 \sin wt$. Graphically, the E. M. F. is represented as shown below.

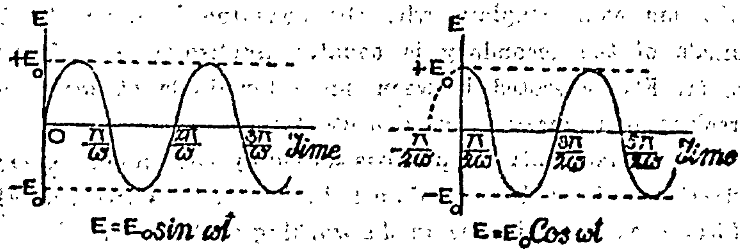


Fig. 244

The current produced by an alternating E. M. F. is also alternating, *i. e.* is a sine or cosine function of time t; it can also be represented graphically as above.

The period for a complete cycle is $\frac{2\pi}{w}$ and that for half a cycle is $\frac{\pi}{w}$.

Average E. M. F. Average value of E. M. F. durring a complete cycle is obviously zero; for, during one half of a complete cycle the E. M. F. is positive and during the other half it is negative. When however we speak of average E. M. F. we mean the average over half a complete cycle.

In the case of a sine function the time for half a cycle extends from 0 to $\frac{\pi}{w}$. In the case of a cosine function it is from $\frac{\pi}{2w}$ to $\frac{3\pi}{2w}$; it is however more usual to take it from $-\frac{\pi}{2w}$ to $+\frac{\pi}{2w}$ [See Fig. 244].

Thus, for a sine function,

$$\text{Average E.M.F.} = \frac{\int_0^{\frac{\pi}{w}} E_0 \sin wt \, dt}{\int_0^{\frac{\pi}{w}} dt} = \frac{-\frac{E_0}{w}\Big[\cos wt\Big]_0^{\frac{\pi}{w}}}{\frac{\pi}{w}}$$

$$= -\frac{E_0}{\pi}\Big\{-1-1\Big\} = \frac{2E_0}{\pi}$$

For a cosine function,

$$\text{Average E.M.F.} = \frac{\int_{-\frac{\pi}{2w}}^{+\frac{\pi}{2w}} E_0 \cos wt \, dt}{\int_{-\frac{\pi}{2w}}^{+\frac{\pi}{2w}} dt} = \frac{\frac{E_0}{w}\Big[\sin wt\Big]_{-\frac{\pi}{2w}}^{+\frac{\pi}{2w}}}{\Big[t\Big]_{-\frac{\pi}{2w}}^{+\frac{\pi}{2w}}}$$

$$= \frac{\frac{E_0}{w}\Big\{1-(-1)\Big\}}{\frac{\pi}{w}} = \frac{2E_0}{\pi}$$

Thus in either case average E. M. F. $= \frac{2E_0}{\pi}$

More usually however we find the average in another way. We square the E. M. F. (so that for both halves of the complete cycle the result is positive), next we find the average value of this squared E. M. F. and lastly we extract the square root of this average. This is what we call Root Mean Square E.M.F. or R.M.S. E.M.F. or virtual E.M.F.

For a sine function,

$$\text{Average squared E. M. F.} = \frac{\displaystyle\int_0^{\frac{\pi}{w}} E_0^2 \sin^2 wt \, dt}{\displaystyle\int_0^{\frac{\pi}{w}} dt}$$

$$= \frac{\dfrac{E_0^2}{2}\displaystyle\int_0^{\frac{\pi}{w}} (1-\cos 2wt)\,dt}{\dfrac{\pi}{w}} = \frac{E_0^2 w}{2\pi}\left[t - \frac{\sin 2wt}{2w}\right]_0^{\frac{\pi}{w}}$$

$$= \frac{E_0^2 w}{2\pi}\left\{\frac{\pi}{w}\right\} = \frac{E_0^2}{2}$$

For a cosine function,

$$\text{Average squared E. M. F.} = \frac{\displaystyle\int_{-\frac{\pi}{2w}}^{+\frac{\pi}{2w}} E_0^2 \cos^3 wt \, dt}{\displaystyle\int_{-\frac{\pi}{2w}}^{+\frac{\pi}{2w}} dt}$$

$$= \frac{\dfrac{E_0^2}{2}\displaystyle\int_{-\frac{\pi}{2w}}^{+\frac{\pi}{2w}} (1+\cos 2\,wt)\,dt}{\Big[\,t\,\Big]_{-\frac{\pi}{2w}}^{+\frac{\pi}{2w}}} = \frac{\dfrac{E_0^2}{2}\left[t + \dfrac{\sin 2wt}{2w}\right]_{-\frac{\pi}{2w}}^{+\frac{\pi}{2w}}}{\dfrac{\pi}{2w} - \left(-\dfrac{\pi}{2w}\right)}$$

$$= \frac{\frac{E_0^2}{2}\left\{\frac{\pi}{2w} - \left(-\frac{\pi}{2w}\right)\right\}}{\frac{\pi}{w}} = \frac{E_0^2}{2}.$$

Thus in either case

$$\text{Average squared E. M. F.} = \frac{E_0^2}{2}$$

$$\therefore \quad \text{Virtual E. M. F.} = \frac{E_0}{\sqrt{2}}$$

N. B. Here we have taken the average over half a cycle. But since in this case the E. M. F. is first squared, the same result would follow if we find the average over a complete cycle, *i. e.* if the limits of integration be taken to be 0 and $\frac{2\pi}{w}$ for both sine and cosine functions.

Thus we have Maximum E. M. F. $= E_0$

$$\text{Average E. M. F.} = \frac{2E_0}{\pi}$$

$$\left.\begin{array}{r}\text{Virtual} \\ \text{or R. M. S.}\end{array}\right\} \text{E. M. F.} = \frac{E_0}{\sqrt{2}}$$

Numerically, average E. M. F. is slightly smaller than virtual E. M. F. For

$$\text{Average E. M. F.} = \frac{2E_c}{3{\cdot}14} = 0{\cdot}637\, E_0$$

$$\text{and Virtual E. M. F.} = \frac{E_0}{\sqrt{2}} = \frac{E_0\sqrt{2}}{2}$$

$$= \frac{E_0 \times 1{\cdot}414}{2} = 0{\cdot}707\, E_0$$

In an A. C. supply when we say that the supply E. M. F. is 220 volts we always mean that it is virtual E. M. F. The maximum E. M. F. is therefore $= 220\sqrt{2} = 311$ volts approx. It is now clear why A. C. is more dangerous than D. C. For, if we get a shock from 220 volts D. C. the shock is from 220 volts and not more; but if the shock is from 220 volts A. C. the maximum shock is due to an E. M F. of more than 300 volts and hence it is more severe.

Since in an A. C. supply the current is also a sine or a cosine function of time the above results hold good for an alternating current also.

Thus, if the maximum current $= i_0$

$$\text{average current} = \frac{2i_0}{\pi}$$

$$\left.\begin{array}{l}\text{and virtual}\\ \text{or R. M. S.}\end{array}\right\} \text{current} = \frac{i_0}{\sqrt{2}}$$

These results may be obtained exactly in the same way as before.

Art 170 We shall now find an expression for the power absorbed in an A. C. circuit. We shall see later on that generally speaking, there is always a phase difference between the E. M. F. and the current. Thus if the E. M. F. be $E_0 \sin wt$, the current $= i_0 \sin (wt+\theta)$*. Hence average power

$$= \frac{\int_0^{\frac{\pi}{w}} E_0 \sin wt.\, i_0 \sin (wt+\theta).\, dt}{\int_0^{\frac{\pi}{w}} dt}$$

* In some cases it may be $i_0 \sin (wt-\theta)$

$$= \frac{E_0 i_0 \int_0^{\frac{\pi}{w}} \sin wt (\sin wt \cos \theta + \cos wt \sin \theta)\, dt}{\frac{\pi}{w}}$$

$$= \frac{wE_0 i_0}{\pi} \int_0^{\frac{\pi}{w}} (\sin^2 wt \cos \theta + \sin wt \cos wt \sin \theta)\, dt$$

$$= \frac{wE_0 i_0}{\pi} \int_0^{\frac{\pi}{w}} \tfrac{1}{2}\{(1 - \cos 2\, wt) \cos \theta + \sin 2wt \sin \theta\}\, dt$$

$$= \frac{wE_0 i_0}{2\pi} \left\{ \left[t - \frac{\sin 2wt}{2w} \right]_0^{\frac{\pi}{w}} \cos\ \theta - \left[\frac{\cos 2wt}{2w} \right]_0^{\frac{\pi}{w}} \sin \theta \right\}$$

$$= \frac{wE_0 i_0}{2\pi} \left\{ \left(\frac{\pi}{w} - 0 \right) \cos \theta - \left(\frac{1}{2w} - \frac{1}{2w} \right) \sin \theta \right\}$$

$$= \frac{wE_0 i_0}{2\pi} \cdot \frac{\pi}{w} \cos \theta = \frac{E_0 i_0}{2} \cos \theta = \frac{E_0}{\sqrt{2}} \times \frac{i_0}{\sqrt{2}} \times \cos \theta$$

$=$ virtual E. M. F. $\times$ virtual current $\times \cos \theta$.

Power factor

In a D. C. supply power $=$ E. M. F. $\times$ current; but we find that in an A. C. supply in order to find the power absorbed the product virtual E. M. F. $\times$ virtual current has got to be multiplied by the factor $\cos \theta$. This factor $\cos \theta$ is known as power factor.

N. B. (1) Here we have taken both E. M. F. and current to be sine functions of time. The same result however follows if we use cosine functions, but in this case the limits of integration are to be taken as $-\frac{\pi}{2w}$ and $+\frac{\pi}{2w}$

(2) The power factor is the cosine function of the phase difference; so in any particular case if there be no phase difference the power factor becomes equal to unity.

Art 171

We may consider the problem from another standpoint which is perhaps more illuminating. As before let the current be $i_0 \sin(wt+\theta)$ produced by an E. M. F. $E_0 \sin wt$.

The current $i = i_0 \sin(wt+\theta)$

$$= i_0 (\sin wt \cos\theta + \cos wt \sin\theta)$$

Thus the current consists of two portions $i_0 \cos\theta \sin wt$ and $i_0 \sin\theta \cos wt$. Of these the first portion is in phase with the E. M. F. (both containing the term $\sin wt$) and the second portion is at right angles to the E. M. F. $\left(\text{the phase difference being } \frac{\pi}{2}\right)$

Due to the second portion the power

$$= \frac{\int_0^{\frac{\pi}{w}} i_0 \sin\theta \cos wt\, E_0 \sin wt\, dt}{\int_0^{\frac{\pi}{w}} dt}$$

$$= \frac{\frac{1}{2} E_0 i_0 \sin\theta \int_0^{\frac{\pi}{w}} \sin 2wt\, dt}{\frac{\pi}{w}} = -\frac{w E_0 i_0}{2\pi} \sin\theta \left[\frac{\cos 2wt}{2w}\right]_0^{\frac{\pi}{w}}$$

$$= -\frac{w E_0 i_0}{2\pi} \sin\theta . \left(\frac{1}{2w} - \frac{1}{2w}\right) = 0$$

Wattless current

Thus the second portion contributes nothing towards the power absorbed. This is what is known as "Wattless Current". It may be noted that the power corresponding to the first portion of the

current viz. $i_0 \cos\theta \sin wt$ is $\frac{1}{2} E_0 i_0 \cos\theta$—the same as obtained for the total current. The first portion is therefore the power component.

Art 172 We shall now determine the current produced by an alternating E. M. F. under different conditions.

We first consider a circuit containing a non-inductive resistance R and a source of alternating E. M. F. $E_0 \sin wt$.

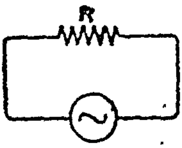


Fig. 245

Here the equation is obviously $E_0 \sin wt = Ri$ by Ohm's Law.

$$\therefore \quad i = \frac{E_0}{R} \sin wt$$

$$= i_0 \sin wt$$

In this case the current is in phase with the E. M. F. The phase difference is zero and the power factor is therefore one.

Art 173
Circuit contlaning L & R

Next we consider a circuit containing an inductance L a resistance R and a source of alternating E. M. F. $E_0 \sin wt$. If i be the current at any instant, the E. M. F. due to self-induction is $-L\frac{di}{dt}$. Thus total E. M. F. being $E_0 \sin wt - L\frac{di}{dt}$ we have

$$E_0 \sin wt - L\frac{di}{dt} = Ri$$

or $$L\frac{di}{dt} + Ri = E_0 \sin wt$$

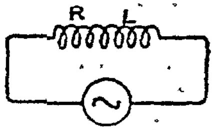


Fig. 246

This equation may be remembered in the following way

E. M. F. due to inductance	+	Ohmic E. M. F.	= Total E. M. F.

and can be written down at once.

Writing $D \equiv \frac{d}{dt}$*, we have

$$(LD+R)\, i = E_0 \sin wt$$

$$\therefore \quad i = \frac{E_0 \sin wt}{LD+R} = \frac{E_0 (R-LD) \sin wt}{R^2 - L^2D^2}$$

multiplying both numerator and denominator by $R-LD$.

$$= \frac{E_0 (R \sin wt - Lw \cos wt)}{R^2 + L^2w^2}$$

$$= \frac{E_0 (A \sin wt \cos \phi - A \cos wt \sin \phi)}{R^2 + L^2w^2}$$

$$= \frac{E_0 A \sin (wt - \phi)}{R^2 + L^2w^2}$$

where $R = A \cos \phi$

and $Lw = A \sin \phi$

$\therefore \quad \tan \phi = \frac{Lw}{R}$

and $A^2 = L^2w^2 + R^2$

$\therefore \quad A = \sqrt{L^2w^2 + R^2}$

$$= \frac{E_0}{\sqrt{R^2 + L^2w^2}} \sin ((wt - \phi)$$

$$= i_0 \sin (wt - \phi).^{**} \quad \ldots \quad \ldots \quad \ldots \quad (61)$$

$$\text{where } i_0 = \frac{E_0}{\sqrt{R^2 + L^2w^2}} \quad \ldots \quad \ldots \quad (61a)$$

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* $D \equiv \frac{d}{dt}$ is known as an operator In solving differential equations this operator can be used as an algebraic quantity. The following property of this operator is also utilised.

$D (\sin wt) = w \cos wt$ Thus D^2 may be replaced

$D^2 \sin wt) = (-w^2) \sin wt$ by $-w^2$.

** This solution known as 'Particular Integral' is only a part of the complete solution. The other part known as 'complementary function is $i = C e^{-\frac{Rt}{L}}$ where C is a constant. Obviously with the increase of time 't' the term $e^{-\frac{Rt}{L}}$ gradually and quickly becomes vanishingly small. Hence this part is not finally effective and has not therefore been considered.

Thus there is a phase difference ϕ between the E. M. F. and the current. Since ϕ is negative the current *lags behind* the E. M. F. *i. e.* maximum and minimum values of the current are reached a short time *after** the corresponding values of the E. M. F. (Vide Fig. 247). The phase difference ϕ is called the angle of lag.

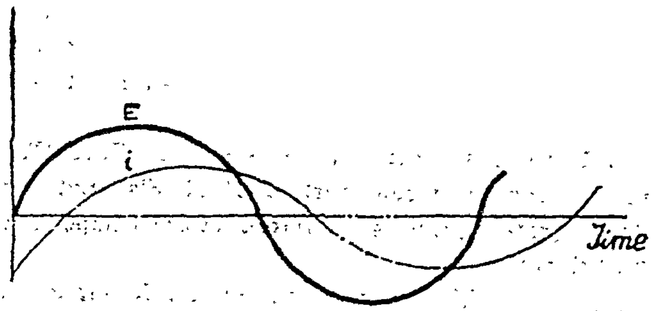

Fig. 247

In an A. C. circuit R is the resistance, $L\omega$ is called the reactance and $\sqrt{R^2+L^2\omega^2}$ is known as the impedance of the circuit. Hence from the relation $\tan\phi = \frac{L\omega}{R}$ we have

$$\text{tangent of the phase difference} = \frac{\text{Reactance}}{\text{Resistance}} \quad \ldots \quad (61b)$$

Reactance is usually represented by X and impedance by Z. Also from (61a).

$$\frac{i_0}{\sqrt{2}} = \frac{E_0/\sqrt{2}}{\sqrt{R^2+L^2\omega^2}}$$

---

* The E. M. F. $E_0 \sin \omega t$ is zero at times $t=0, \frac{\pi}{\omega}, \frac{2\pi}{\omega}, \ldots$ whereas the current $i_0 = \sin(\omega t - \phi)$ is zero when $t = \frac{\phi}{\omega}, \frac{\pi}{\omega}+\frac{\phi}{\omega}, \frac{2\pi}{\omega}+\frac{\phi}{\omega} \ldots$ Thus the zero values of the current occur at times $\frac{\phi}{\omega}$ *after* the corresponding values of the E. M. F. Similar is the case with maximum values.

i. e. Virtual current $=\dfrac{\text{Virtual E. M. F.}}{\text{Impedance}}$

$$i.\,e. \quad I=\frac{E}{Z} \quad \dots \tag{61c}$$

where I and E are virtual current and virtual E. M. F. respectively. Equation (61*c*) corresponds to Ohm's Law in continuous current circuits. Thus in A. C. circuits impedance Z plays the same part as resistance R in D. C. circuits. It may be noted that Z has for its limiting values R for very low frequency and $L\omega$ for very high frequency.

In A. C. circuits whenever we speak of E. M. F's and currents we always mean virtual E. M. F's and virtual currents. Equation (61*c*) is therefore of great importance in working out problems.

Choke coil The current in an A. C. circuit may therefore be diminished by increasing the impedance. Since impedance depends on resistance R as well as on inductance L, the current can be decreased by increasing R or by increasing L. The latter is however more economical; for heat generated in a wire depends upon the resistance R. If R be increased heat generated is also increased. This heat energy cannot usually be utilised and is therefore a loss. On the other hand if we insert in the circuit a coil of thick wire of a fairly large number of turns, inductance is increased but resistance is not appreciably altered and hence there is no corresponding loss of energy. Such a a coil is known as a 'choke coil'.

An alternating E. M. F. of 200 volts, 50 cycles is applied to a coil of 20 ohms resistance and 0'2 henry inductance. Find the current and the angle of lag. What is the power factor of this circuit? Find also the P.D. across the inductance and the P.D. across the resistance.

Here $\omega=2\pi.50=100\pi$

Hence from (61*c*),

$$\text{current} \quad i=\frac{200}{\sqrt{20^2+(0{\cdot}2\times 100\pi)^2}}=\frac{200}{20\sqrt{1+\pi^2}}=3{\cdot}033 \text{ amp}$$

If θ be the angle of lag $\tan\theta = \frac{0{\cdot}2 \times 100\pi}{20} = \pi \quad \therefore \theta = 72°21'$

$\therefore$ Power factor $= \cos\theta = 0{\cdot}3048$

P. D. across the inductance $= i.\omega L = 3{\cdot}033 \times 100\pi \times 0{\cdot}2 = 190{\cdot}5$ volts. P. D. across the resistance $= iR = 3{\cdot}033 \times 20 = 60{\cdot}66$ volts.

Impedance triangle

If we form a right angled triangle ABC with sides AB and BC equal to $L\omega$ and R respectively, the hypotenuse AC is equal to $\sqrt{R^2 + L^2\omega^2}$. Thus the resistance R, the reactance $L\omega$ and the impedance $\sqrt{R^2 + L^2\omega^2}$ form the three sides of the right angled triangle. Such a triangle is known as impedance triangle. Also from Fig. 248, $\tan \angle ACB = \frac{L\omega}{R}$. Comparing this with (61b), we have $\angle ACB = \phi$, *i. e.* in the impedance triangle the angle between the resistance and the impedance is the phase difference ϕ.

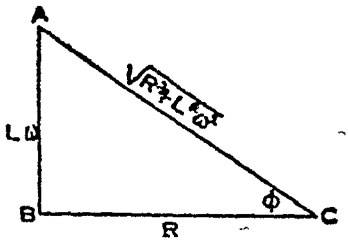

Fig. 248

Further, from (61)

$$i = i_0 \sin(\omega t - \phi) = i_0 \sin\omega t \cos\phi - i_0 \cos\omega t \sin\phi$$

$$= \frac{E_0}{\sqrt{R^2 + L^2\omega^2}} \cdot \frac{R}{\sqrt{R^2 + L^2\omega^2}} \sin\omega t$$

$$- \frac{E_0}{\sqrt{R^2 + L^2\omega^2}} \cdot \frac{L\omega}{\sqrt{R^2 + L^2\omega^2}} \cdot \cos\omega t$$

[Substituting the values i_0, $\cos\phi$ and $\sin\phi$]

$$= \frac{E_0 R}{R^2 + L^2\omega^2} \sin\omega t - \frac{E_0 L\omega}{R^2 + L^2\omega^2} \cos\omega t$$

Thus the current i consists of two parts $\frac{E_0 R}{R^2 + L^2\omega^2} \sin\omega t$

and $\frac{E_0 L\omega}{R^2+L^2\omega^2}\cos \omega t$. The former is in phase with and the latter is at right angles to the E. M. F. $E_0 \sin \omega t$. The former is the power component and the latter is wattless [Vide Art 171].

Art 174
Vector Diagram

Currents and E. M. F's in an A. C. circuit may conveniently be represented in what is known as a Vector Diagram. In Fig 249 let OX and OY be two axes and let the constant vector OA representing the maximum E. M. F. E_0 rotate with a constant angular velocity ω in the direction of the arrow. If t be the time in which the $\angle$AOX is described we have $\angle AOX = \omega t$.

The projection Oa of OA along OY $= E_0 \sin \omega t$. Oa therefore represents the E. M. F. at any instant. Let $\angle$AOB be

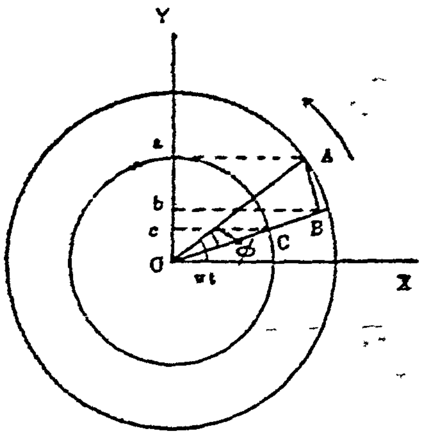

Fig. 249

equal to the angle of lag $\phi \left[= \tan^{-1}\frac{L\omega}{R} \right]$ and let AB be drawn perpendicular to OB.

$$\text{Then OB} = \text{OA}\cos\phi^{*} = \frac{E_0 R}{\sqrt{L^2w^2+R^2}}$$

$$\text{and AB} = \text{OA}\sin\phi = \frac{LwE_0}{\sqrt{L^2w^2+R^2}}$$

Projections of OB and AB on Oy are Ob and ab respectively. They are given by

$$Ob = \text{OB}\sin(wt-\phi) = \frac{E_0 R}{\sqrt{L^2w^2+R^2}}\sin(wt-\phi)$$

$$= Ri \text{ [From (61)]} = \text{Ohmic E. M. F.}$$

$$\text{and } ab = \text{AB}\cos(wt-\phi) = \frac{LwE_0}{\sqrt{L^2w^2+R^2}}\cos(wt-\phi)$$

$$= L\frac{di}{dt}^{\dagger} = \text{E. M. F. due to inductance.}$$

Again from the diagram we have

$$Ob + ab = Oa$$

i. e. Ohmic E. M. F. + E. M. F. due to inductance = Total E. M. F.

Further, if OC be equal to $\frac{\text{OB}}{R}$,

$$\text{OC} = \frac{E_0}{\sqrt{L^2w^2+R^2}} = i_0$$

Projection Oc of OC along Oy

$$= \text{OC}\sin(wt-\phi) = i_0\sin(wt-\phi)$$

$= i$ i.e. current at any instant.

---

* Since $\tan\phi = \frac{Lw}{R}$ we have $\cos\phi = \frac{R}{\sqrt{L^2w^2+R^2}}$ and $\sin\phi = \frac{Lw}{\sqrt{L^2w^2+R^2}}$.

† Since $i = \frac{E_0}{\sqrt{L^2w^2+R^2}}\sin(wt-\phi)$

$$\therefore L\frac{di}{dt} = \frac{LwE_0}{\sqrt{L^2w^2+R^2}}\cos(wt-\phi)$$

As the vector OA r ates with a constant angular velocity w, the vectors OB and OC also rotate with the same angular velocity and projections of OA, OB, AB and OC on Oy represent the total E. M. F., Ohmic E. M. F., E. M. F. due to inductance and the current at any instant. Thus the diagram gives us all information about the E. M. F. and the current at every instant. *Such a diagram is known as a Vector Diagram.*

Art 175. Circuit containing C & R

We next consider a circuit containing a non-inductive resistance R, a condenser of capacity C and a source of alternating E.M.F. $E_0 \sin wt$. As the condenser is charged it develops a P. D. which is in opposition to the E. M. F. of the alternating source. Thus at any instant if Q be the charge on the condenser, $\frac{Q}{C}$ is the corresponding

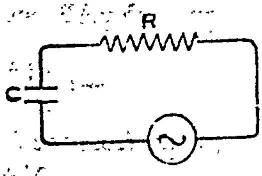

Fig. 250

E. M. F. and the total E. M. F. in the circuit is $E_0 \sin wt - \frac{Q}{C}$

If i be the instantaneous value of the current we have

$$E_0 \sin wt - \frac{Q}{C} = Ri$$

$$\text{or} \quad Ri + \frac{Q}{C} = E_0 \sin wt$$

i.e, $\underset{\text{E. M. F.}}{\text{Ohmic}} + \underset{\text{due to capacity}}{\text{E. M. F.}} = \underset{\text{E. M. F.}}{\text{Total}}$

Differentiating and remembering that $\frac{dQ}{dt} = i$, we have

$$R\frac{di}{dt} + \frac{i}{C} = E_0 w \cos wt$$

Writing $D \equiv \frac{d}{dt}$ $\quad \left(RD + \frac{1}{C}\right) i = E_0 w \cos wt$

or $i = \dfrac{E_0 w \cos wt}{RD + \frac{1}{C}} = \dfrac{E_0 w \left(\frac{1}{C} - RD\right) \cos wt}{\frac{1}{C^2} - R^2 D^2}$ multiplying both

numerator and denominator by $\frac{1}{C}-RD$

$$=\frac{E_0 w\left(\frac{1}{C}\cos wt+Rw\sin wt\right)}{\frac{1}{C^2}+R^2w^2}$$

$$=\frac{E_0 w^2\left[\frac{1}{wC}\cos wt+R\sin wt\right]}{\frac{1}{C^2}+R^2w^2}$$

$$=\frac{E_0(A\sin\theta.\cos wt+A\cos\theta.\sin wt)}{\frac{1}{w^2C^2}+R^2}$$

where $\frac{1}{wC}=A\sin\theta$ and $R=A\cos\theta$

$$=\frac{E_0(\sin wt\cos\theta+\cos wt\sin\theta)}{\sqrt{\frac{1}{w^2C^2}+R^2}}$$

$\therefore\ \tan\theta=\frac{1}{wCR}$

and $A^2=\frac{1}{w^2C^2}+R^2$

$\therefore A=\sqrt{\frac{1}{w^2C^2}+R^2}$

$$=\frac{E_0\sin(wt+\theta)}{\sqrt{\frac{1}{w^2C^2}+R^2}} \quad \ldots \quad \ldots \quad (62)$$

$=i_0\sin(wt+\theta)$*

where $i_0=\dfrac{E_0}{\sqrt{\frac{1}{w^2C^2}+R^2}}$

The phase difference θ is in this case positive. The

---

* As in Art 173 this solution is the Particular Integral. The complementary function is $i=Ae^{-\frac{t}{CR}}$. As this vanishes very quickly with increase of time this has not been considered.

current therefore *leads* the E. M. F. *i. e.* maximum and

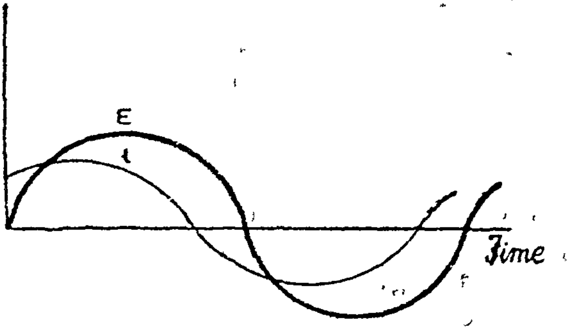

Fig. 251

minimum values of the current occur a little *before* the corresponding values of the E. M. F. [Vide Fig. 251] The phase difference θ is called the angle of lead. In (62*a*) R is the resistance $\frac{1}{wC}$ the reactance and $\sqrt{\frac{1}{w^2C^2}+R^2}$ is the impedance.

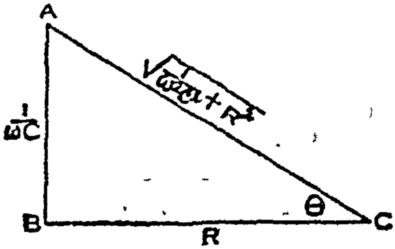

Fig. 252

The current in an A.C. circuit may therefore be reduced by inserting a suitable capacity. The impedance triangle is as before a right angled triangle with sides equal to resistance, reactance and impedance. The angle between resistance and impedance is again equal to θ the phase difference.

Lastly, if $R=0$ the impedance is reduced to $\frac{1}{wC}$ and θ to $\frac{\pi}{2}$. Hence from (62) and (62*a*)

$$i=E_0 wC \sin (wt+\pi/2)$$

$$=E_0 wC \cos wt$$

This current—the charging current of the capacity is at right angles to the E.M.F., $E_0 \sin wt$ and is therefore wattless.

Again from (62a),

$$i_0/\sqrt{2}=\frac{E_0/\sqrt{2}}{\sqrt{\frac{1}{w^2C^2}+R^2}}$$

i. e. Virtual current $=\dfrac{\text{Virtual E. M. F.}}{\text{Impedance}}$ ... (62c)

A 20 volt 5 watt lamp is run from a source of alternating E M. F. of 200 volts and 50 cycles. Find the capacity of the condenser used in series.

The current i (through the lamp) $=\frac{5}{20}=0.25$ amp

$\therefore$ R (of the lamp) $=\frac{20}{0.25}=80$ ohms. And $\omega=2\pi.50=100\pi$.

Hence if C be the capacity of the condenser, we have from (62c).

$$0.25=\frac{200}{\sqrt{80^2+\frac{1}{C^2.100^2\pi^2}}} \qquad \therefore \quad 80^2+\frac{1}{C^2.100^2\pi^2}=800^2$$

Solving, $C=4.0\times10^{-6}$ Farad $=4\ \mu fd.$

Art 176 Circuit containing L, C and R

We now consider a circuit containing a resistance R, an inductance L, a capacitance C and a source of alternating E. M. F. $E_0 \sin wt$.

If at any instant Q be the charge on the condenser and i be the strength of the current we have

$$L\frac{di}{dt}+Ri+\frac{Q}{C}=E_0 \sin wt$$

E. M. F. due to inductance + Ohmic E.M.F. + E.M.F. due to capacity = *Total E. M. F.*

Differentiating $\quad L\frac{d^2i}{dt^2}+R\frac{di}{dt}+\frac{i}{C}=E_0 w \cos wt$

Writing $\quad D\equiv\frac{d}{dt} \qquad \left(LD^2+RD+\frac{1}{C}\right)i=E_0 w \cos wt$

$$\therefore\ i=\frac{E_0 w \cos wt}{LD^2+RD+\frac{1}{C}}=\frac{E_0 w \cos wt}{-Lw^2+RD+\frac{1}{C}}$$

$$= \frac{E_0 w \left\{\left(\frac{1}{C} - Lw^2\right) - RD\right\} \cos wt}{\left(\frac{1}{C} - Lw^2\right)^2 - R^2D^2}$$ multiplying both

numerator and denominator by $\left(\frac{1}{C} - Lw^2\right) - RD$

$$= \frac{E_0 w \left\{\left(\frac{1}{C} - Lw^2\right) \cos wt + Rw \sin wt\right\}}{\left(\frac{1}{C} - Lw^2\right)^2 + R^2w^2}$$

$$= \frac{E_0 w^2 \left\{\left(\frac{1}{wC} - Lw\right) \cos wt + R \sin wt\right\}}{\left(\frac{1}{C} - Lw^2\right)^2 + R^2w^2}$$

$$= \frac{E_0 \left\{ A \cos wt \sin \phi + A \sin wt \cos \phi \right\}}{\left(\frac{1}{wC} - Lw\right)^2 + R^2}$$

$$= \frac{E_0 \left\{ \sin wt \cos \phi + \cos wt \sin \phi \right\}}{\sqrt{\left(\frac{1}{wC} - Lw\right)^2 + R^2}}$$

$$= \frac{E_0 \sin (wt + \phi)}{\sqrt{\left(\frac{1}{wC} - Lw\right)^2 + R^2}} \qquad (63)$$

$= i_0 \sin (wt + \phi)$,* where

$$i_0 = \frac{E_0}{\sqrt{\left(\frac{1}{wC} - Lw\right)^2 + R^2}} \qquad (63a)$$

where $A \sin \phi = \frac{1}{wC} - Lw$

and $A \cos \phi = R$

$$\therefore \tan \phi = \frac{\frac{1}{wC} - Lw}{R} \qquad \ldots \qquad (63b)$$

and $A^2 = R^2 + \left(\frac{1}{wC} - Lw\right)^2$

$$\therefore A = \sqrt{R^2 + \left(\frac{1}{wC} - Lw\right)^2}$$

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* As in Arts 176 and 177 this solution is the Particular integral. The complementary function is $i = C_1 e^{\alpha t} + C_2 e^{\beta t}$ where C_1 and C_2 are two constants and α & β are the roots of $Lx^2 + Rx + \frac{1}{C} = 0$. α and β are essentially negative. Hence the complementary function dies off quickly with increase of time 't' and has not therefore been considered.

We know that $\frac{1}{wC}$ is the reactance due to capacity and Lw that due to inductance; when both are present the effective reactance is $\frac{1}{wC} - Lw$ and the impedance is $\sqrt{\left(\frac{1}{wC} - Lw\right)^2 + R^2}$. Due to inductance the current lags behind the E. M. F. and due to capacity the current leads the E. M. F.; when the circuit contains both inductance and capacity, the current lags or leads according as the reactance *due to the former is greater or less than that due to the latter.* This is evident from equation (63*b*). If $\frac{1}{wC} > Lw$, ϕ is positive and the current leads the E. M. F.; on the other hand if $\frac{1}{wC} < Lw$, ϕ is negative and the current lags behind the E. M. F.

If $\frac{1}{wC} = Lw$, $\phi = 0$ and from (63) $i = \frac{E_0}{R} \sin wt$ *i. e.* in this case the *effect of capacity is exactly counterbalanced by that due to inductance and the circuit behaves as a non-inductive circuit.*

This is also known as the case of resonance—*resonance for series circuit.* For, from equation (63*a*), it is obvious that i_0 is maximum, when

$$\frac{1}{wC} = Lw$$

i. e. when $w^2 = \frac{1}{CL}$ $\therefore$ $w = \frac{1}{\sqrt{LC}}$

If f be the frequency of the alternating E. M. F.

$$f = \frac{w}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

Equation (59*a*) in Art 160 tells us that when R is negligibly small $\frac{1}{2\pi\sqrt{LC}}$ is called the natural frequency of the circuit.

We thus learn that when the frequency of the imposed E. M. F. is equal to the natural frequency of the circuit the current produced is maximum. (*Compare this with resonance in sound.*)

Art 177 **Parallel Resonance** Let us now consider that in one branch of a parallel circuit there is an inductance L and in the other a capacitance C. Let us also suppose that the resistance in either of the two branches is negligibly small. The current i from the alternating source S is divided into two currents, i_1 along the inductance L and i_2 along the capacitance C. Obviously $i = i_1 + i_2$.

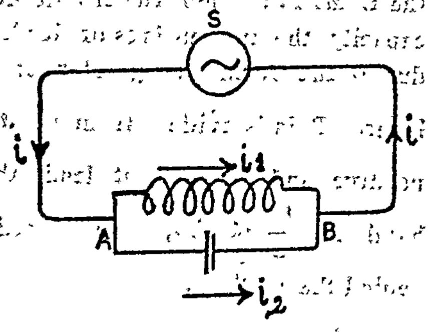


Fig. 253

Let us suppose that an E.M.F. $E = E_0 \sin wt$ is applied to the terminals A and B of the parallel circuit, *i. e.* the E. M. F. $E = E_0 \sin wt$ is applied to the terminals of the inductance as well as to those of the capacitance. Then for the inductance branch

$$L\frac{di_1}{dt} = E_0 \sin wt. \quad \text{or} \quad i_1 = -\frac{E_0}{Lw}\cos wt.$$

And for the capacitance branch $\dfrac{Q}{C} = E_0 \sin wt.$

Differentiating, $\dfrac{i_2}{C} = E_0 w \cos wt$ or $i_2 = E_0 wC \cos wt.$

Hence $\quad i = i_1 + i_2 = E_0\left(wC - \dfrac{1}{Lw}\right)\cos wt.$

If $Cw = \dfrac{1}{Lw}$, *i. e.* if $w = \dfrac{1}{\sqrt{LC}}$ the current i from the alternating source is zero. The corresponding frequency is given by $f = \dfrac{w}{2\pi} = \dfrac{1}{2\pi\sqrt{LC}}$. Thus for this frequency of the alterna-

ting E.M.F the circuit acts as a perfect choke, *i.e.* no current is allowed to pass through. It will be seen that when there is no resistance this frequency is also the natural freqhency of the circuit. [Vide (59*a*) Art 160] In the previous article we have seen that in a series circuit, for this frequency the current is maximum. And now we find that in a parallel circuit, for the same frequency the current is zero. The series circuit is called a resonant circuit and the parallel circuit is known as an anti-resonant circuit.

It should be noted however that the resistance in either of the two branches, although small, is never zero. If we suppose that there is a small resistance R in the inductive branch the current in this branch is $i_1 = \dfrac{E_0}{\sqrt{R^2+L^2w^2}} \sin(wt-\theta)$ where $\tan\theta = \dfrac{Lw}{R}$ The wattless component of this current is

$$-\frac{E_0}{\sqrt{R^2+L^2w^2}} \sin\theta . \cos wt = -\frac{E_0 Lw}{R^2+L^2w^2} \cos wt$$

substituting the value of $\sin\theta$. In the capacitance branch if there is no resistance the current is $E_0 \, wC \cos wt$ and this is obviously wattless. Thus the total wattless current in the main circuit is $E_0 \, wC \cos wt - \dfrac{E_0 Lw}{R^2+L^2w^2} \cos wt$

This is zero when $E_0 wC = \dfrac{E_0 Lw}{R^2+L^2w^2}$ or $\dfrac{L}{C} = R^2+L^2w^2$

$$\text{or} \quad w^2 = \frac{1}{L^2}\left(\frac{L}{C} - R^2\right) = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$\therefore \quad f = \frac{w}{2\pi} = \frac{1}{2\pi\sqrt{\dfrac{1}{LC} - \dfrac{R^2}{L^2}}} \qquad \dots \quad \dots \qquad (64)$$

Thus for this frequency of the **alternating E. M. F.** the wattless component in the main current is **zero**. The **power** component of the current in the impedance is **however**

$$\frac{E_0}{\sqrt{R^2+L^2w^2}}\cos\theta\sin wt=\frac{E_0R}{R^2+L^2w^2}\sin wt,$$

substituting the value of $\cos\theta$. This component passes through the parallel circuit as the power component of the main current. This is however quite small if R is extremely small. Thus when the frequency of the alternating E. M. F. is given by (64) a small current passes through the parallel circuit. *i. e.* there is a small leakage of current through the circuit. It should be noted however that in this case the frequency as given by (64) is not exactly the same as the natural frequency of the circuit.

Similarly if there is a small resistance also in the capacitance branch there is a power component of the current in this branch also and the main current is also increased by this amount, *i. e.* the leakage current increases.

Problem. A coil of negligible resistance and inductance 0·02 henry is in series with a wire of zero inductance and resistance 12 ohms. An E. M. F. of 130 volts 40 cycles is applied. Calculate (1) the current (2) the potential difference across the resistance (3) the potential difference across the inductance and (4) the angle of lag.

$$w=2\pi f=2\pi\times 40=251.2$$

Inductive reactance $=Lw=0{\cdot}02\times 251{\cdot}2=5$ ohms

$\therefore$ Impedance $=\sqrt{R^2+L^2w^2}=\sqrt{12^2+5^2}=13$ ohms

$\therefore$ Current $=\dfrac{130}{13}=10$ amps.

P.D. across resistance = Resistance × current = 120 volts.

P.D. across inductance = Reactance × current = 50 volts.

Angle of lag $=\tan^{-1}\dfrac{Lw}{R}=\tan^{-1}\dfrac{5}{12}=22°\ 37'$

An alternating E.M.F. is represented by the equation E= 200 sin (100πt) volts. What are the frequency and amplitude of the E.M.F.? If this E.M.F. is applied across a series combination of a resistance of 20 ohms and an inductance of 0.15 henry.

Calculate the R.M.S. value of the current in the circuit and the phase lag of the current. C. U. 1970

Here $w=100\pi$ $\therefore$ Frequency$=\dfrac{w}{2\pi}=50$ c/s

Amplitude$=200$ volts $\therefore$ Virtual E.M.F.$=\dfrac{200}{\sqrt{2}}$

$=100\sqrt{2}$ volts.

$\therefore$ Virtual current $i=\dfrac{100\sqrt{2}}{\sqrt{20^2+(\cdot 15)^2 .\ 100^2\pi^2}}=2.76$ amp

If θ be the phase lag

$$\tan\theta=\frac{(\cdot 15)\times 100\pi}{20}=2\cdot 356$$

$$\therefore\ \theta=67^\circ$$

Calculate the inductance that should be used in series with a 100 volt 1000 watt electric lamp so that the combination may be connected to a 200 volt (peak value) A.C. source of frequency 50 c.p.s. C.U. 1972

When the lamp is connected to the source of 100 volts the number of watts consumed is 1000.

$\therefore$ The current through the lamp $=\dfrac{1000}{100}=10$ amp

and the resistance of the lamp$=\dfrac{100}{10}=10$ ohm

When this lamp is connected to a source of 200 volt (peak value) the inductance used must be such that the current through the lamp is still 10 amp.

Now 200 volt (peak value)$=\dfrac{200}{\sqrt{2}}=141.4$ R.M.S. volts.

and $w=2\pi f=2\pi\times 50=100\pi$

From equation $i_0=\dfrac{E_0}{\sqrt{R^2+L^2w^2}}$

$$10=\frac{141.4}{\sqrt{10^2+L^2.100^2\pi^2}}$$

Simplifying, $L=0.032$ henry.

Art 178
Distribution of current

We may now discuss how current produced by a generator is distributed over a large area where electric energy is actually consumed. The wires by which the current is led from the generator to the consumers are known as *Leads*. Lamps, fans and other instruments placed in parallel between the leads are run by the potential difference between the leads. Now although the resistance of the leads is kept quite small by using fairly thick wires, it cannot be entirely eliminated ; consequently there must be some drop of potential along the leads. It is therefore obvious that if the current is to be supplied over a large area, the potential difference between the leads at the remoter parts must necessarily be considerably reduced. Let us now suppose that the voltage at which the current is transmitted is fairly high ; then, for the same consumption of power the current through the leads must necessarily be less and hence potential drop along the leads is also considerably less. Thus when the voltage is high the potential drop along the leads is not very serious. There are also other weighty reasons why it is necessary to have a high voltage when electric energy is to be transmitted to a fairly large distance.

With A. C. supply voltage may be stepped up or stepped down at any stage by means of suitable transformers. Thus

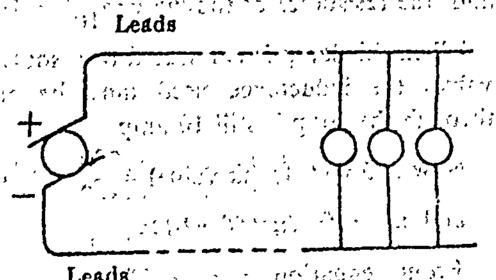
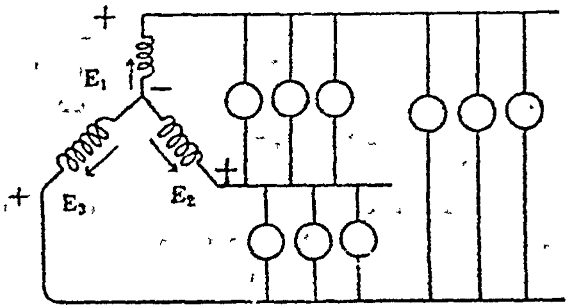


Fig. 254

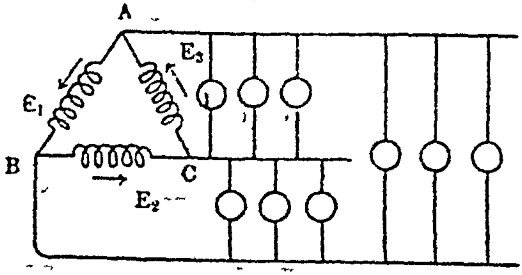
when alternating current is generated at the generating station voltage is increased to a high value by a step-up transformer. At this high voltage electric energy is transmitted to a number of sub-stations suitably scattered over the large area. At each such station voltage is lowered to any desired value by means

of a step-down transformer and current is supplied to the consumers at this low voltage. As each of the substations serves a comparatively smaller area loss of voltage along the leads does not become serious.

Three phase system The coils in the armature of an A. C. generator are connected in such a way that there are usually three different circuits. These circuits are so wound that the E. M. F. generated in any one of them has a phase difference of 120° with that in any other. Such a generator is known as a *three phase** A C. generator. The E. M. F's—known as phase E. M. F's —may be represented by $E_1 = E_0 \sin \omega t$, $E_2 = E_0 \sin (\omega t + 120°)$, $E_3 = E_0 \sin (\omega t + 240°)$. These E. M. F's may be applied separately to the external circuits. But in that case two leads for each E.M.F. *i. e.* six leads in all



(a)



(b)

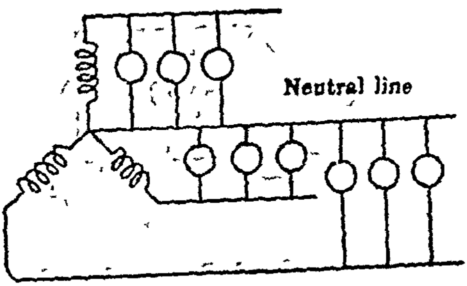
Fig. 255

* Sometimes there are more than three circuits in the armature. The generator in that case is called *Polyphase*.

are necessary. To reduce the number of leads the circuits are connected as a *Star* or as a *Mesh*. In the former case [Fig. 255 (*a*)] one terminal of each circuit is connected to a point—called the neutral point and the other terminals are connected to the leads. In the latter case the circuits are respectively connected to the points A & B, B & C and C & A of a mesh ABC [*Fig.* 255 (*b*)]. The points A, B, C are connected to the leads. Any two of these three leads supply current to the consumers. This arrangement is known as *three phase three wire* system.

Distribution of leads is so made that power consumed between any two leads is approximately the same, *i. e.* the load is balanced. In cases where load is unbalanced, especially when power at a comparatively high voltage is to be supplied to a few consumers the *three phase four wire* system is used. In this case a fourth line—called *Neutral* line—is connected to the neutral point [Vide Fig. 256]. The potential difference between any one lead and the neutral line—this is in reality a phase E.M.F.—supplies current to ordinary consumers. Where a comparatively high voltage is required the potential difference between any two leads is used. If the phase E. M. F. is 220 volts it can easily be proved that the potential difference between any two leads is equal to $220\sqrt{3}$, *i.e.* to about 380 volts.

Fig. 256



Art 179
Earth Inductor

We are now in a position to understand the action of an Earth Inductor. We know that the Earth is a huge magnet producing magnetic field everywhere in space in its

neighbourhood. The Earth's field at any place may however be resolved into two components, the horizontal component H and the vertical component V. Thus at any place we may suppose that there are two sets of lines of force—horizontal lines due to H running north and south and vertical lines due to V. If a coil be rotated in this magnetic field due to the Earth, number of lines of force cutting the coil varies and an induced E. M. F. is generated in the coil.

First, let us suppose that the coil rotates about a vertical axis V V. The E. M. F. generated is then due to the variation of horizontal lines cutting the coil, vertical lines linked with the coil remaining constant in number.

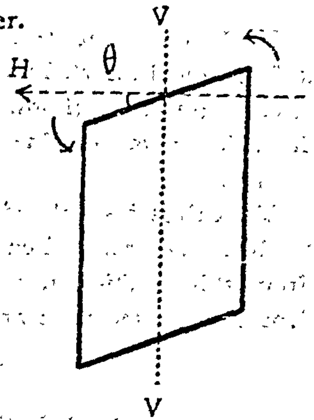


Fig. 257

In any position let the plane of the coil make an angle θ with the horizontal component H. Flux linked with the coil is then given by

$$N = n\,AH \sin\theta,$$

(where n = no. of turns and A = area of the coil.)

If the coil rotates through a small angle $d\theta$ in a short time dt, an instantaneous but short-lived current i is generated. A current flowing for a short time means that a small quantity of charge dQ flows through the coil. Since $\frac{dQ}{dt} = i$, we have $dQ = i\,dt = \frac{E}{R}\,dt$, where E is the E. M. F. generated and R is the resistance of the coil.

But $$E\,dt = \frac{dN}{dt}\,dt = dN = nAH \cos\theta\, d\theta$$

$$\therefore \quad dQ = \frac{nAH}{R} \cos\theta\, d\theta$$

Let the coil be first placed perpendicular to horizontal lines of force. From this position let the coil be rotated very quickly through 180°. The total charge that flows

through the coil by this process is given by

$$Q = \int_{-\pi/2}^{+\pi/2} \frac{nAH}{R} \cos\theta \, d\theta = \frac{nAH}{R}\Big[\sin\theta\Big]_{-\pi/2}^{+\pi/2} = \frac{2nAH}{R}$$

If there be a ballistic galvanometer in series with the coil the galvanometer shows a deflection ϕ_1. Since the charge flowing through a ballistic galvanometer is proportional to the deflection

$$\frac{2nAH}{R} = k\phi_1 \quad \ldots \quad (\alpha)$$

Next let us suppose that the coil rotates with its axis horizontal and directed north and south. The horizontal lines linked with the coil now remain constant in number and the E. M. F. generated is due to the variation of the vertical lines of force.

The coil is first placed with its plane in the horizontal position (*i. e.* perpendicular to the vertical lines of force). From this position it is rotated very quickly through 180°. Proceeding in the same way as before, we have

$$\frac{2nAV}{R} = k\phi_2 \quad \ldots \quad (\beta)$$

where ϕ_2 is the deflection now produced in the ballistic galvanometer.

Dividing (β) by (α). $\dfrac{V}{H} = \dfrac{\phi_2}{\phi_1}$. But we have seen in Art 11 that $\dfrac{V}{H} = \tan\delta$, where δ is the dip at the place.

Hence $\quad \tan\delta = \dfrac{\phi_2}{\phi_1}$

Thus the dip δ at any place may be measured. The coil rotating in Earth's field is known as Earth Inductor.

If the various constants involved in equations (α) and (β) be known actual values of H and V and hence the resultant intensity may also be obtained by Earth Inductor.

Exercise XVIII

1. Explain the apparent increase in resistance of a wire with the frequency for a rapidly alternating current.

2. Write short notes on (*a*) choke coil (*b*) wattless current (*c*) impedance triangle (*d*) power factor.

3. An alternating E.M.F. of 40 volts at 300 cycles per sec. is applied to a solenoid of 500 turns, radius 5 cms, length 25 cms and of 20 ohms resistance. Find the current and also the power consumed. Ans. 1·53 amp ; 4·47 watts.

4. An alternating pressure of 100 volts (virtual) is applied to a circuit of resistance 0·5 ohm and self-inductance 0·01 henry, the frequency being 50 cycles per sec. What will be the reading of an ammeter included in the circuit ? Ans. 31·44 amp.

5. An alternating E.M.F. of 250 volts, 100 cycles is applied to a non-inductive resistance of 25 ohms. Find the inductance of the choke coil which when inserted in the circuit reduces the current to half its value. Ans. 0·069 Henry

6. A circuit contains a non-inductive resistance of 20 ohms and a choke coil. When an alternating E M.F. of 25 cycles is applied to the circuit it is found that the power factor is $\frac{1}{2}$. Find the inductance of the choke. Ans. 0·22 Henry

7. A solenoid of diameter 10 cms and length 20 cms consists of two layers of wire of 1000 turns each. The wire has a resistance of 0·1 ohm per metre. Calculate its inductance. If an E.M.F. of frequency 50 cycles per sec. be applied to it, find (*a*) reactance (*b*) impedance (*c*) power factor of the circuit How will these quantities be affected if an iron core ($\mu = 1000$) is inserted within the solenoid ?

Ans. 0·197H ; 62·01 ohms ; 88·28 ohms : 0·71 ;

197H ; 62030 ohms ; 62030 ohms ; zero.

8. A 40 watt 100 volt electric bulb is to run by A.C. supply of 200 volts and frequency 50 c/s. Calculate the capacity of the condenser which must be used in series with the bulb.

Ans. 7·35 μF

9. A 5 watt 50 volt electric bulb is to be run by the town supply of 220 volts, 50 cycles. Explain how this can be done with the help of a condenser. Find the capacitance of the condenser and the reactance of the circuit. Ans. 1·49 μfd ; 2142 ohms.

10. A 50 watt 200 volt bulb is in series with a condenser of $2\mu F$ capacity. If the bulb be lighted up by an alternating supply of 50 cycles frequency, find the supply voltage.

Ans. 445·2 volts.

11. An alternating supply of 200 volts and 50 cycles is used to light up a 20 volt bulb. If the condenser used in series be of capacity $4\mu F$ find the resistance of the bulb and hence the power absorbed by the bulb, Ans. 79·97 ohms ; 5 watts

12. A 100 cycle alternating E. M. F. is applied to a circuit containing a resistance 15 ohms, an inductance 0·025 henry and a capacity 250 microfarads. Find the impedance of the circuit. Does the current lag or lead and by what angle ?

Ans. 17·67 ohms ; Angle of lag 31°55′

13. What is meant by the resonance of an electric circuit ? A circuit has a resistance of 50 ohms, an inductance of 0·25 henry and a capacity of 100 micro-farads. For what frequency of the applied alternating E. M. F. the circuit behaves as a non-inductive resistance ? Ans. $\frac{100}{\pi}$ cycles.

14. An alternating supply of 200 volts and 50 cycles is used to send a current through a circuit containing a capacity of 5 mfd and a non-inductive resistance of 50 ohms. If a choke coil be now inserted in the circuit it is found that the current is increased. Explain this and find the inductance of the choke coil when the current in the circuit is maximum.

Ans. 6·37 henry

15. In an electric circuit the inductance is 10 mH, resistance is 100 ohms. If the angular velocity of the rotating vector be 1000 radians per sec calculate the capacity of the condenser which when inserted in the circuit makes the circuit non-inductive. Ans. 100 μF

16. A circuit consists of an inductance, a resistance of 100 ohms and a condenser of capacity 10 μF. If an A. C. supply of 220 volts 50 c/s be connected to the circuit the current is found to be in phase with the E. M. F. Find (1) the inductance (2) the current (3) the voltage across the inductance (4) the voltage across the resistance and (5) the voltage across the condenser. What will be the phase angle if the capacity be doubled ?

Ans. 1·013 H ; 2·2 amp ; 700·1 volt : 220 volts ; −700·1 volt ; angle of lag 57° 52′.

C U. Question

1963. A resistance of 10 ohms is in series with an inductance of 0·1 henry. If a P. D. of 100 V (R. M. S.) at 50 cycles is applied, calculate the effective current. Ans. 3·03 amps.

1964. An alternating E. M. F. $E = E_0 \sin wt$ is applied to the ends of a series circuit consisting of resistance R, an inductance L and a capacitance C. Find the current through the circuit at any instant. Explain what is meant by the impedance of the circuit and establish the condition for which resonance occurs.

1965. A series circuit consisting of 0·01 henry inductance and 10 ohms resistance is connected across an alternating E. M. F. of 100 V (R. M. S.) at 50 c/s. Find (i) R. M. S. current through the circuit and (ii) the voltage across the inductance.

Ans. 9·54 amps ; 29·96 volts.

1966. Write notes on "Earth Inductor."

1967. Obtain a relation between the current and the voltage in an alternating current circuit containing an inductance and a resistance in series.

Calculate the inductance of a choke coil to be introduced in an A. C. circuit to light a 10 watt 20 volt bulb on 200 volts 50 c. p. s. mains. Ans. 1·067 H.

1968. What is meant by the term electrical impedance ? Find the electrical impedance due to an induction of 1 henry at 50 c p.s.

Derive an expression for the impedance of a self-inductance L, a capacitance C and a resistance R in series when fed by a sinusoidal E.M.F.

1969 (a) Obtain expressions for the mean value and the R.M.S. value of a sinusoidal current.

(b) Obtain an expression for the average power of an A.C. circuit. Calculate the power factor of a 50 cycles/sec A.C. circuit in which an inductance of 0.1 henry and a 20 ohm. resistance are connected in series.

1969. Write notes on "Parallel resonance in A.C"

1970 (1) Explain the terms impedance and reactance of an A.C. circuit.

(2) Write notes on "series resonance in A C."

1973. Define mean value and root mean square value of an alternating potential. Obtain expressions for the same in the case of simple harmonic potential. Explain how the potential difference across a resistance and an inductance connected in series may be represented by a vector diagram.

1974. Explain the following terms: Resistance, Impedance and Power factor as applied to an A.C. circuit.

A 40 watt lamp works on 120 volts. What should be the value of the inductance of a coil of negligible resistance which when connected in series with the lamp, would make it burn at the rated voltage when connected across 230 volts 50 cycles A.C. mains?

1975. What do you mean by the r.m.s. value of an alternating electromotive force?

An alternating sinusoidal e.m.f. of 100 volts r.m.s. value and frequency 200 cycles per sec is applied to a circuit containing an inductance of 1.0 henry and a resistance of 1000 ohms in series Calculate the peak values of the current in the circuit and potential drop across the inductance.

CHAPTER XIX

UNITS AND DIMENSIONS

In Physics whenever we take a reading we really measure one or more of the three things—length, mass and time. Dimensions of physical quantities can therefore be reduced to these three fundamentals,—Length, Mass and Time. Thus since a velocity represents a distance divided by time its dimension is LT^{-1}; an acceleration is velocity divided by time and is therefore of dimension LT^{-2}; and so on. We give below dimensions of a number of physical quantities obtained in this simple way.

Art 180

N. B. It should be clearly understood that whenever we speak of the dimension we do not say anything about the actual value.

$$\text{Angle} = \frac{\text{Arc}}{\text{Radius}} = \frac{L}{L} = 1$$

$$\text{Angular velocity} = \frac{\text{Angle}}{\text{Time}} = T^{-1}$$

$$\text{Angular acceleration} = \frac{\text{Angular velocity}}{\text{Time}} = T^{-2}$$

$$\text{Area} = \text{Length} \times \text{Length} = L^2$$

$$\text{Volume} = \text{Length} \times \text{Length} \times \text{Length} = L^3$$

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} = ML^{-3}$$

$$\text{Velocity} = \frac{\text{Distance}}{\text{Time}} = LT^{-1}$$

$$\text{Acceleration} = \frac{\text{Velocity}}{\text{Time}} = LT^{-2}$$

$$\text{Force} = \text{Mass} \times \text{acceleration} = MLT^{-2}$$

Energy = Work done = force × distance = ML^2T^{-2}

$$\text{Power} = \frac{\text{Energy}}{\text{Time}} = ML^2T^{-3}$$

Pressure = Force per unit area

$$= \frac{\text{Force}}{\text{Area}} = ML^{-1}T^{-2}$$

Surface tension = Force per unit length

$$= \frac{\text{Force}}{\text{Length}} = MT^{-2}$$

Art I81 In any equation dimensions of quantities involved on both sides of the equation, must be the same. Hence the dimension of any unknown quantity may sometimes be determined.

Thus, Young's Modulus is given by

$$Y = \frac{mgL}{\pi r^2 l}$$

Writing [Y] for the dimension of Y, we have

$$\left[\, Y \,\right] = \frac{MLT^{-2}L}{L^2L} = ML^{-1}T^{-2}$$

Similarly, for Bulk Modulus,

$$k = -\frac{dp}{\frac{dv}{v}} = -\frac{vdp}{dv}$$

i. e. so far as dimension is concerned,

$$\text{Bulk Modulus} = \frac{\text{Volume} \times \text{Pressure}}{\text{Volume}}$$

$$\therefore \left[\, k \,\right] = ML^{-1}T^{-2}$$

dp = change in pressure ; its dimension is the same as that of pressure.
dv = change in volume ; its dimension is the same as that of volume.

For Rigidity, $n = \frac{T}{\theta}$ where T is the tangential force per

unit area and θ is the angle of shear ; the dimension of T is therefore the same as that of pressure and the dimension of θ is unity. Hence

$$\left[n \right] = ML^{-1}T^{-2}$$

It is to be noticed that all the three quantities, Young's modulus, Bulk modulus and Rigidity have got the same dimension, as it must necessarily be ; for, each of them is the ratio of stress to strain and is therefore fundamentally the same.

For Viscosity $\quad F = \eta \frac{dv}{dr}$

Here F is force per unit area ; its dimension is therefore $ML^{-1}T^{-2}$; dv is small velocity and dr is small length.

Hence $\quad ML^{-1}T^{-2} = \left[\eta \right] \frac{LT^{-1}}{L} = \left[\eta \right] T^{-1}$

$$\therefore \quad [\eta] = ML^{-1}T^{-1}$$

Art 182 We shall now determine the dimensions of various electric and magnetic quantities. Here we can start with either of the two fundamental equations, viz.,

(1) Force between two electric charges

$$F = \frac{Q_1Q_2}{kr^2} \qquad [\text{ Vide Art } 23]$$

or (2) Force between two magnetic poles

$$F = \frac{m_1m_2}{\mu r^2} \qquad [\text{ Vide Art } 2]$$

The former is known as the electrostatic system (E. S. system) and the latter as electromagnetic system (E. M. system).

Electrostatic System

Art 183
Charge

The fundamental equation is $F = \frac{Q_1Q_2}{kr^2}$.

Unfortunately, in this equation the dimension

of k is not known to us and cannot be determined. We therefore write the dimensional equation as

$$MLT^{-2}=\frac{[Q]^2}{kL^2} \quad \text{or} \quad [Q]^2=kML^3T^{-2}$$

$$\therefore \quad [Q]=k^{\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}$$

Electric Intensity Intensity $F=\dfrac{Q}{kr^2}$

$$\text{or} \quad [F]=\frac{k^{\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}}{kL^2}=k^{-\frac{1}{2}}M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}$$

Current A current is the rate of flow of charge.

$$\therefore \quad [i]=\frac{\text{Charge}}{\text{Time}}=k^{\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}$$

E. M. F. or Potential difference Work done in carrying a charge Q between two points at a potential difference V is QV.

i. e. Work done $=QV$

$$\text{or,} \quad ML^2T^{-2}=[Q].[V]$$

$$\therefore \quad [V]=\frac{ML^2T^{-2}}{k^{\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}}=k^{-\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}$$

Capacity We know $Q=CV$

$$\text{or} \quad [C]=\frac{[Q]}{[V]}=\frac{k^{\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}}{k^{-\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}}=kL.$$

Thus, ignoring k the dimension of a capacity is length; a capacity is therefore sometimes measured in centimeters.

Resistance From Ohm's Law, resistance $=\dfrac{\text{E. M. F.}}{\text{current}}$

$$\therefore \quad [R]=\frac{k^{-\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}}{k^{\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}}=k^{-1}L^{-1}T=\frac{1}{kLT^{-1}}$$

Ignoring k, the dimension of a resistance is the inverse of that of a velocity.

Magnetic Intensity From Laplace's Law, $H = \frac{ids \cos\theta}{r^2}$

This is the connecting link between electric and magnetic quantities.

ds is a length; its dimension is therefore L, $\cos\theta$ has got no dimension.

Hence $[H] = \frac{[i]\,L}{L^2} = k^{\frac{1}{2}} M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-2}$

Magnetic pole Force on a pole m placed in a magnetic field of strength H is mH

i. e. $\text{Force} = mH$

$$MLT^{-2} = [m].[H]$$

$$\therefore \quad [m] = \frac{MLT^{-2}}{k^{\frac{1}{2}} M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-2}} = k^{-\frac{1}{2}} M^{\frac{1}{2}} L^{\frac{1}{2}}$$

Magnetic moment Magnetic moment M
= pole strength × distance between poles

or $[M] = [m] \times L = k^{-\frac{1}{2}} M^{\frac{1}{2}} L^{\frac{3}{2}}$

Inductance We know E. M. F. $= -L\frac{di}{dt}$

Dimensionally speaking, E.M.F. = Inductance × $\frac{\text{Current}}{\text{Time}}$

$$\therefore \quad [L] = \frac{\text{E. M. F.} \times \text{Time}}{\text{Current}} = \text{Resistance} \times \text{Time}$$

$$= k^{-1} L^{-1} T^2 = \frac{1}{kLT^{-2}}$$

Ignoring k, the dimension of inductance is the inverse of that of an acceleration.

Art 184 Electromagnetic system.

Pole Force between two poles. $F = \frac{m_1 m_2}{\mu r^2}$

As in the case of electrostatic system here also the dimension of μ is unknown and unknowable.

$$\therefore \quad MLT^{-2} = \frac{[m]^2}{\mu L^2} \qquad \text{or} \quad [m]^2 = \mu ML^3T^{-2}$$

$$\therefore \quad [m] = \mu^{\frac{1}{2}} M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1}$$

Magnetic moment

Magnetic moment M = pole strength × distance between poles.

$$\therefore \quad [M] = [m] \times L = \mu^{\frac{1}{2}} M^{\frac{1}{2}} L^{\frac{5}{2}} T^{-1}$$

Magnetic Intensity

Intensity, $H = \dfrac{m}{\mu r^2}$

or
$$[H] = \frac{\mu^{\frac{1}{2}} M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1}}{\mu L^2} = \mu^{-\frac{1}{2}} M^{\frac{1}{2}} L^{-\frac{1}{2}} T^{-1}$$

Current

From Laplace's Law, $H = \dfrac{i ds \cos\theta}{r^2}$.

$$[H] = \frac{[i]L}{L^2} \qquad \therefore \quad [i] = [H] \times L = \mu^{-\frac{1}{2}} M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-1}$$

Charge

We know current $= \dfrac{\text{Charge}}{\text{Time}}$

or
$$[i] = \frac{[Q]}{T} \qquad \therefore \quad [Q] = [i] \times T = \mu^{-\frac{1}{2}} M^{\frac{1}{2}} L^{\frac{1}{2}}$$

Electric Intensity

Force on a charge Q placed in an electric field of strength F is QF.

i. e. Force = QF or $MLT^{-2} = [Q].[F]$.

$$\therefore \quad [F] = \frac{MLT^{-2}}{\mu^{-\frac{1}{2}} M^{\frac{1}{2}} L^{\frac{1}{2}}} = \mu^{\frac{1}{2}} M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-2}$$

E.M.F. or Potential difference

Work done in carrying a charge Q between two points at a difference of potential V is QV.

i. e. work done = QV

$$\therefore \quad ML^2T^{-2} = [Q].[V]$$

$$\therefore \quad [V] = \frac{ML^2T^{-2}}{\mu^{-\frac{1}{2}} M^{\frac{1}{2}} L^{\frac{1}{2}}} = \mu^{\frac{1}{2}} M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-2}$$

Capacity We know $Q = CV$

or $[Q] = [C].[V]$

$$\therefore \quad [C] = \frac{\mu^{-\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{1}{2}}}{\mu^{\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}} = \mu^{-1}L^{-1}T^{2} = \frac{1}{\mu LT^{-2}}$$

Ignoring μ, the dimension of a capacity is the inverse of that of an acceleration.

Resistance From Ohm's Law, resistance $= \frac{\text{E. M. F.}}{\text{Current}}$

$$\therefore \quad [R] = \frac{\mu^{\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}}{\mu^{-\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}} = \mu LT^{-1}$$

Ignoring μ, the dimension of a resistance is the same as that of a velocity. Resistance is therefore sometimes measured in centimeters per second.

Inductance We know $E = L\frac{di}{dt}$

Dimensionally speaking, E. M. F. = inductance $\times \frac{\text{Current}}{\text{Time}}$

$$\therefore \quad [L] = \frac{\text{E. M. F.} \times \text{Time}}{\text{Current}} = \text{resistance} \times \text{Time} = \mu L$$

Ignoring μ, the dimension of an inductance is length; inductance is therefore sometimes measured in centimeters.

Art 185 One important point emerges out from the previous discussion. The dimension of a physical quantity is a fundamental property of the quantity itself; obviously it cannot be different in different systems. That the dimensions are apparently different in Electrostatic and Electromagnetic systems, is due to the fact that the dimensions of k and μ are unknown to us. If we could express the dimensions of k and μ in terms of M, L and T the dimensions of the various quantities would have been the same in both the systems.

Obviously, we can equate the dimension of any of the

physical quantities in E. S. system with that in E. M. system and thereby we can get an equation connecting μ and k. If we do this for each of the physical quantities we can get as many equations involving μ and k. It may be supposed that from any two of these equations, we may solve for the dimensions of μ and k separately. Unfortunately this is not possible. For, whatever quantity we choose, the equation connecting μ and k always comes out to be the same. For example, equating the dimensions of capacity in E. S. and E. M. systems, we have

$$kL = \mu^{-1}L^{-1}T^{2}$$

$$\text{or} \qquad \mu k = L^{-2}T^{2} \quad \therefore \frac{1}{\sqrt{\mu k}} = LT^{-1}$$

The dimension of $\frac{1}{\sqrt{\mu k}}$ is therefore that of a velocity.

Instead of a capacity if we conside any other quantity, the result is always the same ; we can never get a second equation connecting μ and k. The individual dimensions of μ and k therefore remain unknown to us.

Art 186 So long we have not discussed anything about the units of various physical quantities. The length of a rod 12 ft long, is represented by 12 if the unit of length is a foot, by 4 if the unit be an yard and by 144 if an inch be the unit. Clearly in expressing electric and magnetic quantities, proper attention must similarly be paid to the exact definitions of the units of the quantities involved. Here again two systems of definitions are possible. In Art 23, we have defined a unit charge thus :

If two charges of equal strength are placed in air at a distance of one cm apart and if the force between them is one dyne, then each of the two charges is said to be a unit charge.

Clearly, this definition is based on the fundamental electrostatic equation $F = \frac{Q_1Q_2}{kr^2}$. Starting with this definition

a unit charge, units all other electric and magnetic ntities can be gradually defined. These units are known Electrostatic units. In a similar way, based on the fundaital magnetic equation $F = \frac{m_1 m_2}{\mu r^2}$, definition of a unit pole y be derived (Art 2). This may also be the starting point he definitions of units of every other electric and magnetic intity. These units are called Electromagnetic units. us any electric or magnetic quantity can be represented by different numbers according as the unit* chosen is ectrostatic or Electromagnetic

If C_e and C_m be the *measures* of the capacity of a given ndenser in Electrostatic and Electromagnetic system, the mplete expressions for the capacity in the two systems, are $_e\left(kL\right)$ and $C_m\left(\mu^{-1}L^{-1}T^2\right)$. Since they represent the same ondenser, we must have

$$C_e\left(kL\right) = C_m\left(\mu^{-1}L^{-1}T^2\right)$$

$$\text{or} \qquad \frac{C_e}{C_m}\left(L^2T^{-2}\right) = \mu^{-1}k^{-1} = \frac{1}{\mu k}$$

$$\text{or} \qquad \sqrt{\frac{C_e}{C_m}}\left(LT^{-1}\right) = \frac{1}{\sqrt{\mu k}}$$

Thus $\frac{1}{\sqrt{\mu k}}$ is of dimension LT^{-1} and its *value* is $\sqrt{\frac{C_e}{C_m}}$. If we write v for $\sqrt{\frac{C_e}{C_m}}$ we have $\frac{1}{\sqrt{\mu k}} = v$ cms per sec.

Art 187 The value of this velocity v can be found out by determining C_e and C_m. To determine C_e a guard ring air

* It should be clearly understood that both Electrostatic and Electromagnetic units are C. G. S. units, so that one is called C. G. S. electrostatic unit and the other C. G. S. Electromagnetic unit.

Table

Physical Quantity	Dimension		Ratio of E. M. unit to E. S. unit	Practical	
	Electrostatic (E. S.)	Electromagnetic (E. M.)		Name	Ratio of practical unit to E. M. unit
Charge	$k^{\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}$	$\mu^{-\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{1}{2}}$	v	Coulomb	10^{-1}
Electric Intensity	$k^{-\frac{1}{2}}M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}$	$\mu^{\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-2}$	v^{-1}	Volt/cm	10^{8}
Capacity	kL	$\mu^{-1}L^{-1}T^{2}$	v^{2}	Farad	10^{-9}
Current	$k^{\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}$	$\mu^{-\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}$	v	Ampere	10^{-1}
E. M. F.	$k^{-\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}$	$\mu^{\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}$	v^{-1}	Volt	10^{8}
Resistance	$k^{-1}L^{-1}T$	μLT^{-1}	v^{-2}	Ohm	10^{9}

Physical Quantity	Dimension		Ratio of E. M. unit to E. S. unit	Practical	
	Electrostatic (E. S.)	Electromagnetic (E. M.)		Name	Ratio of practical unit to E. M. unit
Inductance	$k^{-1}L^{-1}T^{2}$	μL	v^{-2}	Henry	10^{9}
Pole	$k^{-\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{1}{2}}$	$\mu^{\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}$	v^{-1}		
Magnetic Intensity	$k^{\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-2}$	$\mu^{-\frac{1}{2}}M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}$	v	Oersted†	1
Magnetic Moment	$k^{-\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{3}{2}}$	$\mu^{\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{5}{2}}T^{-1}$	v^{-1}		
Energy*	$ML^{2}T^{-2}$	$ML^{2}T^{-2}$		Joule	10^{7}
Power*	$ML^{2}T^{-3}$	$ML^{2}T^{-3}$		Watt	10^{7}

* Units of energy and power are the same in both Electromagnetic and Electrostatic systems. They are *erg* and *erg per sec.*

†"Gauss" is the practical unit for magnetic induction. But it is also sometimes used as the practical unit for magnetic intensity.

In a similar way the ratio of units in the two systems for other physical quantities can be obtained. This ratio is given in column 4 in the Tables given on previous pages.

Art 189 Besides Electromagnetic and Electrostatic units there is another set of units known as practical units. These practical units are directly related to Electromagnetic units. Thus

One ohm (practical unit of resistance)

$= 10^9$ C. G. S. Electromagnetic units of resistance.

One Ampere (Practical unit of current)

$= 10^{-1}$ C. G. S. Electromagnetic unit of current; and so on.

The names of these practical units are given in column 5 of the table; their relations with Electromagnetic units are given in column 6 of the same table. The ratio of practical unit to E. S. unit may be readily obtained from columns 4 and 6. Thus in the case of E. M. F. $\frac{\text{Volt}}{\text{E.S. unit}} = 10^8 \times v^{-1} = \frac{10^8}{3 \times 10^{10}} = \frac{1}{300}$ *i.e.* one E. S. unit of E. M. F. is equal to 300 volts; and so on.

Exercise XIX

1. Express the dimensions of Electric charge, potential difference and magnetic pole in Electrostatic as well as in Electromagnetic systems.

The charge of an electron is $4{\cdot}80 \times 10^{-10}$ E. S. unit. What is its value in E. M. unit? Ans. $1{\cdot}60 \times 10^{-20}$ E. M. unit

2. How is it that the dimension of a physical quantity is different in E. M. and E. S. systems? What important conclusion can be arrived at by equating the dimensions of a quantity in the two systems? Illustrate your answer by expressing the dimensions of capacity in both E. M. and E. S. systems.

3. Describe how a capacity can be measured both in E. S. and in E. M. units. What is the ratio of these two measures?

4. Describe briefly what led Maxwell to develop the Electromagnetic Theory of light.

5. Assuming that the Earth is a negatively charged sphere of radius $6{\cdot}37 \times 10^8$ cms. placed in space, find the density of charge per square meter of its surface on a day when the fall of potential in the air around it is 300 volts per meter. C. U. 1936.

Ans. $7{\cdot}96$ E. S. unit

6. In a guard-ring air condenser, the plate is of diameter 50 cms and the distance between the plates $0{\cdot}2$ cm. Calculate the capacity in micro farads. Ans. $8{\cdot}68 \times 10^{-4}$ micro-farads.

7. If the work done in carrying a charge of 30 E. S. units between two points be 10 ergs, find the potential difference in volts between the two points. Ans. 100 volts.

8. If the surface density of a charged conductor be 25 coulombs per unit area, find the intensity at a point near the conductor. Ans. $94{\cdot}25 \times 10^{-10}$ dynes per E. S. unit of charge.

9. The plates of a parallel plate condenser are at a potential difference of 50 volts. If the distance between the plates be 1 m.m. find the intensity at points between the plates.

Ans. $1{\cdot}67$ dynes per E. S. unit of charge.

10. A potential of 100 volts is applied between the two plates of a parallel plate condenser. If the plates be of diameter 10 cms and are separated by a distance of 5 mm, calculate the force of attraction between the plates. Ans. $1{\cdot}39$ dynes.

CHAPTER XX

ELECTRONS AND ATOMIC STRUCTURE

Electrons

As early as the year 1879, Sir William Crookes carried out a remarkable series of experiments on the electric discharge through a rarefied gas. A cylindrical glass vessel containing air at atmospheric pressure is provided with two platinum electrodes at the two ends. The two terminals of an Induction Coil are connected to these electrodes and the following remarkable series of phenomena are observed as the air is gradually pumped out of the cylindrical vessel.

Art 190
Cathode Rays

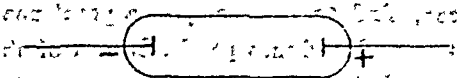
(1) 

Fig. 261

When the pressure is nearly atmospheric the resistance is enormous and sparking refuses to take place between the electrodes.

(2) At about 40 mm pressure irregular streamers begin to appear.

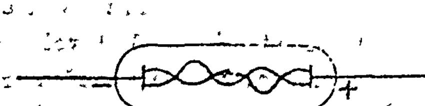


Fig. 262

These may be compared to lightning flashes seen across the sky during thunderstorms.

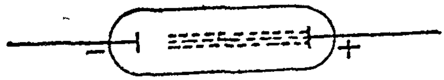
(2) 

Fig. 263

When the pressure is lowered to about 10 mm sparking becomes steady and regular. A column of beautiful pink colour—known as positive column—extends right up to the anode. But near the cathode there is a short dark region separating the positive column from the cathode. This dark

space was discovered by Faraday about the year 1838 and is now known as Faraday dark space. On the cathode itself there appears a small luminous blue spot.

(4) Gradual reduction of pressure causes the positive column to

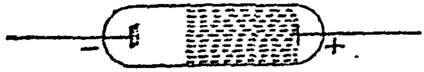


Fig. 264

thicken more and more, until at 2 to 4 mm pressure it fills the whole cross section of the tube. Faraday dark space becomes larger in length and the blue spot of light on the cathode increases in extent, producing a beautiful velvety glow. This is known as cathode glow.

(5)

Fig. 265

At about 1 mm. pressure the positive column breaks up into peculiar striae which are alternate bright and dark bands and are arranged at regular distances in the neighbourhood of the anode. At this stage the cathode glow just separates from the cathode and another dark space—now known as Crookes' dark space—appears between the cathode and the cathode glow.

(9) At about 0·5 mm pressure Crookes' dark space increases in size,

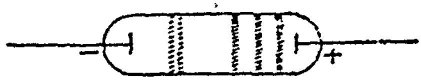


Fig. 266

cathode glow is pushed towards the anode, the positive column diminishes in size, striations are now fewer in number and are more widely separated.

(7)

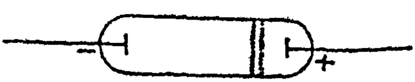


Fig. 267

At 0·1 mm pressure, Crookes' dark space increases very much in size, the cathode glow is pushed almost to the anode and the positive column vanishes almost completely.

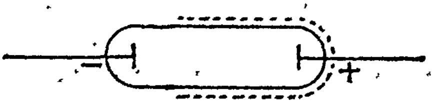


Fig. 268

(8) At pressures 0·02 mm and below the cathode glow also disappears and the tube is filled entirely by Crookes' dark space so that there is no trace of light within the tube. But the glass walls of the tube—especially, the portion opposite the cathode—are now luminous with a greenish glow.

Sir William Crookes was aware that this greenish glow was due to the impact of a kind of rays from the cathode on the walls of the vessel. Although he could not definitely establish the nature of these cathode rays he was fully conscious of the great importance of this phenomenon. He was of opinion that these cathode rays were a material radiation and he spoke of them as matter *in the fourth* state*

Art 191 Properties

Properties of cathode rays were thoroughly studied by numerous workers, Sir William Crookes, Sir J. J. Thomson, Lenard, Perrin and others. The properties are mainly as follows :—

(1) *Cathode rays travel in straight lines.*

If inside the discharge tube a mica cross be fixed in the path of the cathode rays a sharply defined shadow is produced

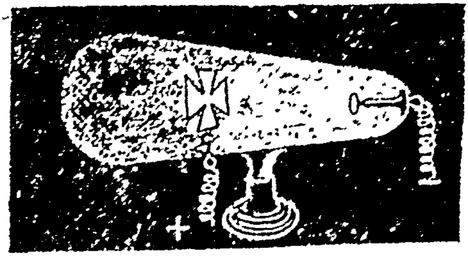


Fig. 269

on the walls of the tube. The sharpness of the shadow establishes the rectilinear propagation of the rays.

* Usually we have matter in three states,—solid, liquid and gaseous. Since cathode rays appeared to be rarer than a gas Crookes called them matter in the fourth state.

(2) *Cathode rays start out of the cathode in a direction perpendicular to the surface of the cathode.*

If the cathode be made concave in shape the rays follow the radial paths and are concentrated at the centre.

(3) *Cathode rays generate heat.*

A piece of platinum placed at the centre of curvature of a concave cathode is heated to redness when the cathode rays are incident on it.

(4) *Cathode rays possess inertia.*

If a light wheel with mica vanes be suitably placed with its axis on two horizontal rails the wheel is set into rotation,

Fig. 270

if only one half of the wheel is situated within the path of the cathode rays.

(5) *Cathode rays produce phosphorescent light.*

If the vanes of the mica wheel (in the previous experiment) be set with suitably chosen substances beautifully coloured phosphorescent light may be seen as the wheel rotates.

(6) *Cathode rays mutually repel one another.*

In Fig 271 C_1 and C_2 are both cathodes which can be jointly or separately connected to the induction coil. A is the anode. When C_1 alone is the cathode cathode rays follow the path C_1B_1. When C_2 alone is the cathode the rays move along C_2B_2. But when both C_1 and C_2 are simultaneously connected to the induction coil cathode rays

follow the paths C_1B_1 and C_2B_2 proving thereby that they mutually repel one another.

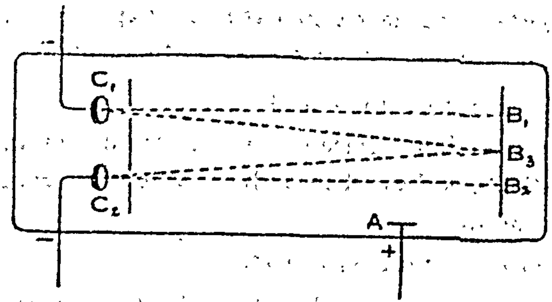


Fig. 271

(7) *Cathode rays are deflected by a magnetic field.*

If in experiment (1) one pole a magnet be brought near the discharge tube the shadow of the mica cross is appreciably displaced, proving thereby that the cathode rays are deviated by the magnet.

(8) *Cathode rays are deflected by an electric field.*

If the path of rays lies between two plates kept at a difference of potential the rays are found to be deflected from the rectilinear path. This was first observed by Srī J. J. Thomson in 1895.

(9) *Cathode rays can pass through thin metal foils.*

In a discharge tube a portion of the glass is replaced by a thin aluminium foil 0·001 *mm.* thick and free from holes. When the cathode rays are incident on this foil they pass right out of the apparatus. This was first observed by Lenard. For this reason the rays which come out of the apparatus are known as Lenard rays.

(10) *Cathode rays carry negative charge.*

This was first definitely established by Perrin in 1895 by allowing the rays to fall into a Faraday cage*. An electro-

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* A Faraday cage consists of two metallic vessels one within the other but insulated from each other. The outer vessel is connected to Earth and the inner to an electrometer. The outer vessel has a small window for the rays to enter.

meter connected to the inner vessel of the cage, indicated that negative charge was coming to the cage.

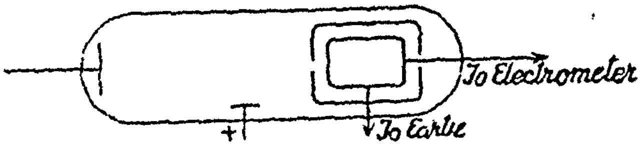


Fig. 272

Art 192
Nature of Cathode rays

Towards the end of the nineteenth century as the properties of cathode rays were being investigated, a heated controversy raged throughout the scientific world regarding the nature of these cathode rays. Hitherto, all known phenomena in connection with Electricity and Magnetism were fully explained by the wellknown Electrodynamic laws definitely established by this time. Interference, diffraction, polarisation and other properties of Light were also satisfactorily explained by the wave theory of light. The whole book of science seemed to be absolutely closed and nothing more—it was believed—could be got out of it. But these cathode rays—a small speck of cloud in an otherwise clear sky—were the starting point of a series of crucial experiments which completely upset all previous ideas about matter and energy. About this time Sir Oliver Lodge made the prophetic remark that the scientific horizon was practically clear excepting that a small patch of dark cloud had appeared in the sky. He prophesied that this cloud might grow bigger and bigger until it might fill up the whole of the sky.

We now know that this remark came to be very very true. The scientific horizon was completely overcast with clouds, so much so that until very recent times there was apparently no sign of the sky being cleared up. Even a simple question such as "What is Light" baffled solution for a long time and even now no very clear picture can be given to its answer.

During the latter part of the nineteenth century Light was believed to be definitely a wave in ether. It was therefore

natural for many of the physicists to explain these cathode rays also as a wave, although—they argued—it might be a new kind of wave. There was however another school of physicists who held that cathode rays were streams of particles—possibly charged particles. The properties of cathode rays—as they were discovered and studied—were applied to provide a crucial test between the two rival theories. The rectilinear propagation of cathode rays could be explained on either of the two theories and was therefore no such crucial property. The fact that cathode rays possess inertia, hardly requires an explanation if the rays were supposed to be streams of moving particles; but this property did not also disprove the wave theory; for by this time it was definitely known that Light—a wave in ether—exerts pressure and therefore possesses inertia. Deflection of cathode rays by electric and magnetic fields was advanced by the supporters of the particle theory as the decisive test. According to them the rays consisted of charged particles and could therefore be easily deflected by an electric field. The motion of these charged particles constituted an electric current* and hence they were also acted on by a magnetic field. The direction of deflection in either case showed that the charge on the cathode ray particles was negative. But by this time Kerr effect and Zeeman effect had been discovered. Intimate relation between Light and Electricity & Magnetism was long ago suspected by Faraday, Maxwell and others. Kerr effect and Zeeman effect proved the correctness of this point of view. Physicists who held that cathode rays were by nature waves, argued that just as ordinary Light was acted on by electric and magnetic fields as demonstrated by Kerr effect and Zeeman effect, cathode rays were also affected by these fields although the effect being different it might be that cathode rays were a new kind of wave.

* It should be remembered that an electric field acts on a charged particle; whereas a magnetic field produces its effect on a current but has no action on a charge at rest.

All speculations were however set at rest when Perrin definitely proved that cathode rays carried negative charge. A wave however peculiar it might be in nature can never be imagined to carry charge. The fact that negative charge was associated with cathode rays definitely established the nature of these rays. They must be *streams of negatively charged particles.* These particles were called *electrons.*

Art 193 : Determination of $\frac{e}{m}$ and v

Soon afterwards attempts were naturally made to determine the charge e, mass m and velocity v of these electrons. The electric and magnetic deflections provided methods for measuring the specific charge $\frac{e}{m}$ and the velocity v. We consider the electric deflection first.

Electric deflection

A beam of cathode rays made narrow by two parallel slits S_1 and S_2 is made to pass through two parallel plates between which an electric field can be established. At first when the electric field is not applied the rays are incident on a photographic plate at A; when the electric field is switched on the rays are deflected and are incident at B.

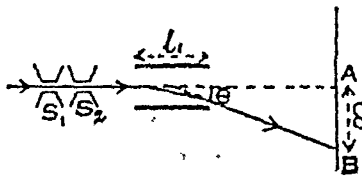


Fig. 278

If X be the strength of the electric field Xe is the force with which an electron—when it enters the electric field —is attracted towards the positive plate. $\frac{Xe}{m}$ is therefore the acceleration produced in a direction perpendicular to the original path of the electrons. If l_1 be the length of the electric field $\frac{l_1}{v}$ is the time which the electron takes to move across the field ; in this time $\frac{l_1}{v}$, the velocity generated at

right angles to its path is $\frac{Xe}{m} \frac{l_1}{v}$. Thus the electron—just when it leaves the electric field—has two velocities, v in the original direction and $\frac{Xe}{m} \cdot \frac{l_1}{v}$ at right angles to it. Hence if θ be the angle of deviation of the electrons,

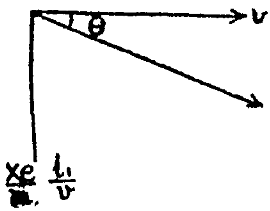


Fig. 274

$$\tan \theta = \frac{Xe}{m} \cdot \frac{l_1}{v} \Big/ v = \frac{Xel_1}{mv^2} \qquad \ldots \qquad (a)$$

Also, if δ_1 be the deflection AB on the photographic plate and D_1 the distance of the plate from the centre of the electric field we have

$$\tan \theta = \frac{\delta_1}{D_1} \qquad [\text{ Vide Fig. 273 }]$$

$$\text{Hence} \qquad \frac{Xel_1}{mv^2} = \frac{\delta_1}{D_1}$$

$$\therefore \quad \frac{e}{mv^2} = \frac{\delta_1}{Xl_1D_1} = \alpha \qquad \ldots \qquad \ldots \qquad (65)$$

where α is a measurable constant for the the given apparatus.

Magnetic deflection

We now come to the deflection by the magnetic field. As before let the beam of electrons narrowed by the parallel slits S_1 and S_2 enter the magnetic field of strength H. An electronic

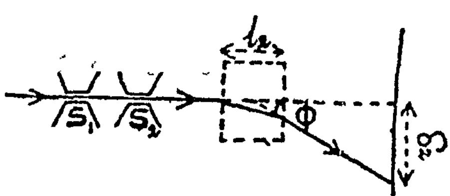


Fig. 275

charge e moving with a velocity v is equivalent to a current ev. The force on this current by the magnetic field is Hev and the acceleration produced is $\frac{Hev}{m}$. The deflection of a current is

perpendicular to both the current and the magnetic field. Hence if the magnetic field be at right angles to the plane of the paper, the deflection of the electron is in the paper. The electronic charge 'e' being negative the current ev is negative; it is therefore equivalent to a positive current in the opposite direction. If the magnetic field be directed from above towards the paper it may easily be seen, by applying Fleming's Left Hand Rule, that the force on the current is directed downwards, *i. e.* the deflection is downwards. [Vide Fig. 275].

If l_2 be the length of the magnetic field $\frac{l_2}{v}$ is the time in which the electron crosses the field. Hence when it passes out of the field the velocity generated at right angles to its path is

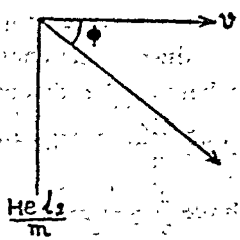


Fig. 276

$$\frac{Hev}{m} \cdot \frac{l_2}{v} = \frac{Hel_2}{m}.$$

Hence if ϕ be the deviation,

$$\tan\phi = \frac{Hel_2}{m} \Big/ v = \frac{Hel_2}{mv} \qquad (b)$$

Again if δ_2 be the deflection on the photographic plate and D_2 the distance of the plate from the centre of the magnetic field,

$$\tan\phi = \frac{\delta_2}{D_2} \text{ [Vide Fig 275]}$$

$$\text{Hence} \quad \frac{Hel_2}{mv} = \frac{\delta_2}{D_2}.$$

$$\therefore \quad \frac{e}{mv} = \frac{\delta_2}{Hl_2D_2} = \beta \qquad \dots \qquad (66)$$

where β is a constant for the given apparatus and can be measured.

Dividing (66) by (65). $v = \frac{\beta}{\alpha}$

Thus v is measured and knowing v, $\frac{e}{m}$ can be determined from any of the above two equations (65) or (66).

Art 194 It is to be noted that the path of the cathode rays is parabolic within the electric field but circular within the magnetic field. For, in the case of electric deflection if we suppose that the cathode rays travel with a velocity v along X axis the force Xe and hence the acceleration $\frac{Xe}{m}$ due to the electric field is along Y axis. The case is analogous to the case of a stone projected horizontally from the top of a tower. In time t distance traversed along X axis is given by $x=vt$; whereas the distance traversed along Y axis is given by $y=\frac{1}{2}\frac{Xe}{m}t^2$ Eliminating t between these two equations we have $\frac{x^2}{y}=\frac{2mv^2}{Xe}$. This evidently represents a parabola whose latus rectum is $\frac{2mv^2}{Xe}$.

In the case of the magnetic deflection howeve the direction of the force Hev due to magnetic field is *always* perpendicular to the path of the cathode rays. The case is analogous to a stone rotated with the help of a string; the force on the stone and hence the centripetal acceleration is along the string, *i. e.* perpendicular to the path of the stone. Exactly in a similar way as the cathode rays are deviated within the magnetic feld the force Hev also changes its direction and is always perpendicular to the path of the rays. The path of the cathode rays is therefore circular. The radius of the circular path is given by $Hev=\frac{mv^2}{r}$ or $r=\frac{mv}{He}$.

Art 195
Direct determination of v

The velocity v can also be measured directly. For this purpose both electric and magnetic fields are applied simultaneously in the same region but at right angles to each other—say, the

electric field in the plane of the paper and the magnetic field perpendicular to the plane of the paper. The deflection of the

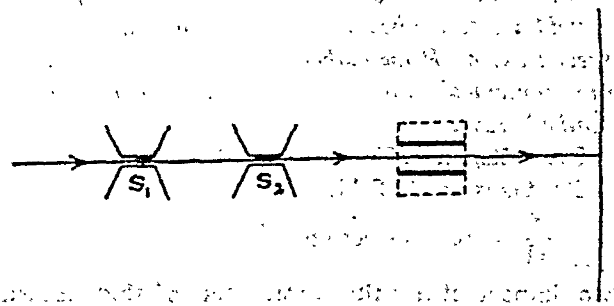


Fig. 277

cathode rays by both of them is therefore in the plane of the paper. The directions and strengths of the two fields are now so adjusted that the electric deviation θ is exactly equal and opposite to the magnetic deviation ϕ; cathode rays in that case pass undeviated. We therefore have $\theta=\phi$.

$\therefore$ from (a) and (b) $\dfrac{Xel_1}{mv^2}=\dfrac{Hel_2}{mv}$.

$$v=\frac{X}{H}\cdot\frac{l_1}{l_2}$$

If the lengths l_1 and l_2 of the two fields be also equal we have finally

$$v=\frac{X}{H} \tag{67}$$

Thus v can be determined.

Combining this experiment with *either* electric *or* magnetic deflection experiment both $\frac{e}{m}$ and v can be measured.

The charge of an electron is 4.8×10^{-10} E.S.U. while the specific charge (e/m) is 1.756×10^{7} E.M.U. Calculate the mass of the electron. *C.U. 1972*

Charge $=4.8\times10^{-10}$ E.S.U. $=1.6\times10^{-20}$ E.M.U.

$\therefore$ Mass $(m)=\dfrac{1.6\times10^{-20}}{1.756\times10^{7}}=9.11\times10^{-28}$ gm.

A beam of cathode rays is subjected to an electric field of 300 volts/cm at right angles to the direction of the incident beam. It is also subjected to a magnetic field of 300 Gauss normal to the electric field and to the beam direction. Both the fields have the same special extent. If the cathode ray beam remains undeflected due to the continued action of the two fields, calculate the velocity of the cathode rays. C.U. 1973.

$X = 300$ volts/cm$=300\times 10^8$ E.M.U. per cm.

$H = 300$ Gauss$=300$ E.M.U.

$\therefore\ v = \dfrac{X}{H} = 10^8$ cms per sec.

Art 196

Two important results came out of this measurement of $\dfrac{e}{m}$. First the value of $\dfrac{e}{m}$ for electrons was found to be approximately 1850 times that of the lightest atom, viz hydrogen atom [Vide Art 124, equation (44)]

i. e. $\dfrac{e}{m}$ for electron $= 1850 \times \dfrac{E}{M}$ for hydrogen

If it is assumed that the charge E on a hydrogen atom in electrolysis is the same as the electronic charge e, the mass m of the electron becomes 1850 times smaller than the lightest known elementary mass.

The second result was perhaps more conclusive. It was found that the value of $\dfrac{e}{m}$ was independent of the nature of the cathode, the anode or the gas within the discharge tube. That electrons were generated in the discharge tube was an undisputed fact. Clearly these electrons were produced from the cathode, the anode or the gas, presumably from the cathode. But if the same value of $\dfrac{e}{m}$ for the electrons be obtained in every case—whether the electrodes were made of copper, silver, aluminium or of any other metal and whether the tube contained air, hydrogen, nitrogen or any other gas the conclusion became irresistible that electrons were constituent parts of the atoms of all substances. Hitherto physicists

and chemists were of opinion that atoms were the least indivisible particles of all known elements. But here was a conclusive evidence which definitely established for the first time that atoms contained electrons. Clearly the view that the atoms were indivisible, could no longer be held to be tenable. Atoms could surely be broken up and smaller elementary mass could be obtained out of them.

Atoms as a whole are however neutral. If electrons which are negatively charged particles are present within atoms clearly there must also be equal amount of some positive charge to counteract the effect of the negative charge.

Art 197. The question is then how the charges are distributed within an atom. Obviously, the charges cannot remain at rest; for in that case mutual attraction of positive and negative charges would bring them together and they would be destroyed in no time. So far the conclusion rested on solid ground. In the next stage various speculations were made which could be tested and accepted or rejected in the light of subsequent experiments.

At first Sir J. J. Thomson put forward the hypothesis that positive charge was distributed uniformly over the surface of an atom (supposed to be hollow) and electrons rotated within the atom. This view was soon found to be untenable. For, according to this view two atoms cannot penetrate each other; the nearest approach of two atoms takes place when they touch each other, *i. e.* the shortest distance between the two centres is the diameter of an atom. But it was found that α rays which are positively charged particles emitted by radioactive substances (Vide Art 213) are sometimes deflected through large angles when they pass through air. We may visualise that this deflection of α rays is due to the repulsive action of the positive charges of the neighbouring atoms. Calculation however showed that this repulsive force—which becomes greater if the distance between the centres of repelling particles is smaller—is not

sufficiently large to deflect the fairly massive α particles through such large angles, unless it is assumed that α rays pentrated into the atoms.

The next theory is due to Rutherford according to whom positive charge is concentrated in a nucleus at the centre and electrons rotate round this nucleus. The analogy with the Solar system is now complete. The idea of a hard, elastic, indivisible atom is completely gone; in its stead we have now empty space in which a number of particles rotate round the nucleus. Into such atoms α particles can easily penetrate and be repelled by the positively charged nucleus. There being now practically no limit to the minimum distance of approach of the centres, the repulsive force may be as large as it is necessary to produce the deflection of α rays. As a matter of fact, later, on measuring this large angle scattering of α rays, Rutherford and Chadwick were succesful in determining the charge on the nucleus.

Without going through historical stages we may now sum up the final conclusions. The number of rotating electrons in an atom is the same as the position of the atom in the periodic table. This number is known as **atomic number**. Thus a hydrogen atom consists of a positively charged nucleus known as proton surrounded by only one rotating electron. The charge of the proton is numerically the same as that of the electron; but its mass is much larger. The mass of an electron being negligibly small—only $\frac{1}{1850}$ of that of a hydrogen atom—the mass of a proton is practically the same as that of a hydrogen atom. In 1932 another particle known as neutron was discovered; this is of the same mass as a proton but is neutral, *i. e.* it carries no charge. All atoms are believed to be built up of these neutrons, protons and electrons. Thus a He atom is of atomic number two. The number of rotating electrons in a He atom must therefore be two. Since the He atom as a whole is neutral we conclude that there are two protons in the

nucleus. And since the atomic weight of He is four, *i. e.* a He atom is four times as heavy as a H atom, the nucleus of a He atom must also contain two neutrons. Thus the nucleus of a He atom consists of 2 protons and 2 neutrons and this nucleus is surrounded by 2 rotating electrons.

Similarly for an oxygen atom,

$\left.\begin{array}{l}\text{At. Wt.}=16\\ \text{At. No.}=8\end{array}\right\}$ $\therefore$ No. of rotating electrons = 8 and the nucleus consists of 8 protons and 8 neutrons.

For a Na atom,

$\left.\begin{array}{l}\text{At. Wt.}=23\\ \text{At. No.}=11\end{array}\right\}$ $\therefore$ No. of rotating electrons = 11 and the nucleus consists of 11 protons and 12 neutrons.

And so on.

Thus the atomic number and the atomic weight are two characteristics of the atom of an element. It is customary to represent them as follows :—$_{11}Na^{23}$ denotes that the sodium atom has the atomic number 11 and the atomic weight 23, $_8O^{16}$ denotes that the oxygen atom has the atomic number 8 and atomic weight 16 and so on.

Art 198 All the rotating electrons however are not at the same distance from the nucleus. Groups of electrons lying approximately on one spherical shell* surrounding the nucleus, rotate approximately at the same distance from the nucleus and in all atoms (excepting hydrogen and helium) there are several such groups of electrons rotating in different shells Each shell is satisfied with a definite number of electrons, *i.e.* it can contain a definite number of electrons and no more. The first shell, *i. e.* the shell nearest to the nucleus is known as K shell ; it is satisfied by only 2 electrons. The next shell—L shell, can contain 8 electrons, the third shell—M shell, also 8 electrons, the fourth shell—N shell, 18 electrons and so on.

It has gradually been realised that physical and chemical properties of an atom depend upon the number of electrons in the *outermost* shell. Take for instance the inert gases—He,

* The word "shell" has been used here in the sense in which the word "orbit" is sometimes used,

Ne, A, Kr etc. respectively with atomic numbers 2, 10, 18, 36 etc. The number of rotating electrons in these substances is also respectively the same as these atomic numbers. The two electrons in a He atom are all in K shell; of the 10 electrons in a Ne atom 2 are in K shell and 8 in L shell. Similarly in an A atom 2 are in K shell, 8 in L shell and 8 in M shell and so on. The *outermost* shell is in all these cases satisfied, *i. e.* contains the requisite number of electrons and the result is that these substances have no tendency to combine with any other substance,—they are inert gases. The substances which follow the inert gases in the periodic table have one electron in excess and this extra electron is in the next higher shell. These substances have an electro-positive character and the valency is one. Li §, Na, K etc. are examples of this class. On the other hand those elements which precede the inert gases in the periodic table have one electron less than what is necessary to satisfy the outermost shell. In these cases also the valency is one but these substances are electro negative in character. F*, Cl, Br etc. are examples of this class. A stable compound is formed when one substance of the former class combines with one of the latter. The former readily loses the extra electron which the latter greedily absorbs. Similarly substances where there are *two electrons in excess* of those satisfying the different shells have a valency 2 and are electro-positive and substances where there is a *deficiency of two electrons* satisfying the outermost shell, have also a valency 2 and are electro-negative. Stable compounds are formed when

§ The atomic numbers of these substances are 3, 11, 19 etc. Of the 3 electrons in Li atom, 2 are in K shell and *one* in L shell; 11 electrons in Na atom are distributed—2 in K shell, 8 in L shell and *one* in M shell. In a K atom 2 electrons are in K shell, 8 in L shell, 8 in M shell and *one* in N shell; and so on. Thus in every case there is *one* electron in the outermost shell.

* The atomic numbers of these substances are respectively 9, 17, 35 etc. In a F atom 2 electrons are in K shell and 7 in L shell, in a Cl atom 2 electrons are in K shell, 8 in L shell and 7 in M shell; in the case of Br atom 2 electrons are in K shell 8 in L shell, 8 in M shell and 17 in N shell. Thus in every case there is *one* electron *short* of what would satisfy the outermost shell.

a substance of the first group combines with one of the second. And so on.

Art 199 One fundamental objection can however be raised against the above theory of the structure of atoms. It is well known from classical electrodynamics that a charge in rotatory motiop radiates energy. An electron rotating round the nucleus should therefore continuously loose energy and as a result its speed of rotation must continuously diminish and in no time it should coalesce with the nucleus. An atom with rotating electrons cannot therefore be stable.

Towards the close of the nineteenth century and in the beginning of the twentieth century a number of experiments was performed which shook the very foundation of the classical concept about Light and Electricity & Magnetism. Attacks on the classical theory were coming from different branches of Physics. In the year 1902 Planck a German physicist, in order to explain the nature of radiation emitted by a black body, put forward the revolutionary hypothesis that energy is emitted not continuously but in the form of quanta. These quanta are nothing but bundles of energy and according to Planck the energy of each quantum is $h\nu$ where ν is the frequency of radiated energy and h is a constant now known as Planck's constant. In the year 1912 Bohr a Danish scientist boldly asserted that classical theory is not applicable to the electron rotating within an atom, *i.e.*, an electron while rotating in any orbit inside the atom does not radiate energy. When however the atom is excited by electric sparking or by high temperature or by any other means, the electron in the outermost orbit is removed to still higher orbits; but these higher orbits being unstable the electron almost immediately jumps back to the lower orbits. And as the electron passes from a higher orbit to a lower one the difference of energy in the two orbits is radiated out in space as radiation. Bohr took up the idea of Planck and asserted that the frequency of this radiation is given by the relation—radiated energy $= h\nu$. The success with

which Bohr explained the spectra of hydrogen on this theory at once placed his theory on a sound basis. Although in later years many modifications have been introduced in this theory Bohr's theory still remains fundamentally sound even up to the present day.

Art 200 It will be seen from the foregoing articles
Atomic Number that the atomic number of a substance is more fundamental than the atomic weight. The latter gives us merely the weight of the substance relative to that of hydrogen ; it does not ordinarily give us any other information about the substance. The former, on the other hand, being the same as the number of rotating electrons, tells us how many electrons there are in the outermost orbit. And from this we know the valency of the substance, we also know whether the substance is electro-positive or electro-negative ; many other physical and chemical properties can also be deduced from these. There is no regularity or symmetry about atomic wts. of different substances. The atomic number on the other hand, increases regularly by unity from one element to the next in the periodic table. The atomic wt is more or less an accident whereas the atomic number is a fundamental property of the atom. In the earlier days when the periodic table was constructed on the basis of increasing atomic wts one serious discrepancy was noticed. The atomic wt. of Argon is greater than that of Potassium ; Argon was therefore placed after Potassium. But this placed Potassium in the group of inert gases and Argon in that of Alkaly metals. Clearly the properties of Argon and Potassium did not correspond to those of other substances in the same group. Later on when ideas about Atomic Number became clear it was realised that At. No. being more fundamental substances should be arranged in the periodic table not in the order of increasing At. wts. but in the order of increasing Atomic Numbers. The At. No. of Argon is less than that of Potassium. The positions of Argon and Potassium were therefore interchanged and the discrepancy was removed.

In modern chemistry the atomic weight of oxygen is taken to be 16 and those of all other substances are measured with reference to this atomic weight of oxygen. In this way, previously, except in the case of hydrogen, the atomic weights of almost all other substances came out to be integers. In modern times, however, due to very accurate measurements these atomic weights are no longer integers in almost all cases their values are somewhat different from integers. Such deviations from integral values are considered to be due to what is known as "Packing Effect". In modern times a new expression "Mass Number" has accordingly come into prominence. This mass number is the integer nearest to the atomic weight. It tells us precisely the number of protons and neutrons in the nucleus of an atom. The question naturally arises as to how these protons and neutrons hold themselves in equilibrium within the small compass of the nucleus. This has led to many researches as to the properties of protons and neutrons and attempts have been made for ascertaining the structure of the nucleus. It is in this way that an estimate may be made of the nuclear energy that may be obtained from the nucleus of an atom.

Art 201 As it often happens once the production of electrons by the electric discharge was definitely established, other methods were quickly discovered, whereby electrons could be produced in a much simpler way.

According to Bohr's theory an electron in the outermost orbit of an atom may be removed to higher orbits by exciting the atom, *i.e.* by imparting additional energy to the atom. If this imparted energy be sufficiently large the electron may ultimately be removed from the atom. The atom in that case is said to be ionised and this process of detaching the electron from the parent atom is known as *ionisation*. This additional energy may be imparted by simply heating the substance strongly. Thus mere heating to incandescence causes a metallic wire to emit electrons. Again, ultra-violet rays, X Rays, γ Rays etc. possess energy given by the equation $E = h\nu$ where h is Planck's constant and ν is the frequency of the rays. So when any of these rays is passed through a gas energy is

imparted to the atoms of the gas by these rays and electrons are detached from the atoms, *i.e.* the atoms are ionised.

The supply of energy to the atom may also conveniently be done by bombarding the atom by a stream of external electrons accelerated by a suitable potential gradient. If an electron falls through a potential drop V its velocity v generated thereby is given by $\frac{1}{2}mv^2 = eV$. If this velocity be sufficiently large the atom bombarded by the electron may be ionised. The value of V just sufficient to ionise an atom is known as the **Ionisation potential** of the atom. Sometimes after one electron has been removed from an atom a second electron may also be removed provided the energy of the bombarding electron is sufficiently high. The atom in this case is said to be doubly ionised. Thus there may be a second ionisation potential. The ionisation potential of Hydrogen is 13·6 volts. The two ionisation potentials for removing the two electrons of a He atom are 24·5 volts and 78·6 volts. A Mg atom has more than two electrons rotating round the nucleus. The ionisation potentials corresponding to removal of the outermost two electrons are 7·6 volts and 15 volts. And so on.

Sometimes the energy of the bombarding elecron is just sufficient to displace the electron of the bombarded atom to the next higher orbit. The potential drop V (through which the bombarding electron falls) is then known as resonance potential. The higher orbit being however unstable the electron (which is displaced to this higher orbit) immediately afterwards comes back to the original orbit and a wavelength λ of light corresponding to the difference of energies in the two orbits is emitted. The wavelength is then known as the resonance line. A ray of wavelength λ possesses an energy $h\nu$ where ν is the corresponding frequency. This energy $h\nu$ must be equal to the energy $\frac{1}{2}m v^2 = eV$ of the bombarding electron. Now $\lambda = \frac{C}{\nu}$ where C is the velocity of light. Hence

$$V = \frac{h\nu}{e} = \frac{hc}{e\lambda}$$

$h = 6{\cdot}62 \times 10^{-27}$ erg. sec $C = 3 \times 10^{10}$ cms/sec.

$e = 4{\cdot}80 \times 10^{-10}$ E. S. U. $1\mathring{A} = 10^{-8}$ cms.

$$\text{Hence } V \text{ (E. S. U)} = \frac{6{\cdot}62 \times 10^{-27} \times 3 \times 10^{10}}{4{\cdot}80 \times 10^{-10} \times \lambda}.$$

$$V \text{ (volts)} = \frac{6{\cdot}62 \times 10^{-27} \times 3 \times 10^{10} \times 300}{4{\cdot}80 \times 10^{-10} \times \lambda} = \frac{12{\cdot}4 \times 10^{-5}}{\lambda}$$

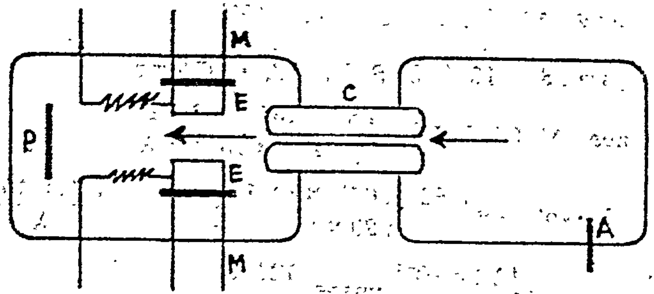
$$\therefore \quad \lambda = \frac{12{\cdot}4 \times 10^{-5}}{V} \text{cms} = \frac{12400}{V} \mathring{A}$$

where V is measured in volts.

For Hg the resonance lines corresponding to the first two higher orbits, are 2536 Å and 1862Å. Hence the corresponding resonance lines are given by $V_1 = \dfrac{12400}{2536} = 4{\cdot}85$ volts and $V_2 = \dfrac{12400}{1862} = 6{\cdot}66$ volts.

Art 202
Positive Rays

As the nature of cathode rays was gradually and firmly established another allied phenomenon was studied by Sir J. J. Thomson. We have seen in the previous articles that in a discharge tube cathode rays or electrons come out of the cathode and travel with fairly large velocities. The quantity of gas within the discharge tube, although extremely small is not however absolutely nil. Some of the electrons coming out of the cathode and travelling through the tube collide with the atoms of this residual gas. By this impact the atoms are ionised. These ionised atoms being positively charged (vide Art 201) are acted on by the electric field and move towards the cathode with considerable velocities. Sir Thomson used an aluminium cathode perforated along the length by a fine hole. The ionised atoms pass through the hole and come to the other side; they are then known as Positive rays. On this side also there is an evacuated tube—evacuated more completely than the main discharge tube, so that the com-



Main discharge tube

C ... Perforated Cathode.

A ... Anode

MM ... Horse shoe electromagnet.

Fig. 278

paratively massive* positive ray particles may move in straight lines without meeting with any collision.

To find $\frac{E}{M}$ for these positive rays, the usual electric and magnetic field were applied perpendicular to the rays. Sir J. J. Thomson applied these fields simultaneously and along the same direction. As has been explained in Art 195 the two fields being applied in the same direction, the deflections of the positive rays produced by these fields are at right angles to each other. The magnetic field is applied by the electromagnet MM. The end faces E, E of the poles of the electromagnet are insulated from the main body of the electromagnet. The electric field is applied between these end [illegible]ard E, E. A photographic plate P perpendicular to the path of the rays is placed at some distance and the rays are received by this plate. The electric deflection δ_1 and the magnetic deflection δ_2 are given by equations (65) and (66) [Art 193]. Substituting x and y for δ_1 and δ_2 in these equations we have $x = \frac{E}{Mv^2} X l_1 D_1$ and $y = \frac{E}{Mv} H l_2 D_2$ where E and M

* Positive ray particles are massive in comparison to cathode rays or electrons.

are the charge and mass of a positive ray particle and other quantities have their usual meanings.

$$\text{Thus } x = K_1 \frac{E}{Mv^2} \quad \text{and} \quad y = K_2 \frac{E}{Mv}$$

where K_1 and K_2 are constants depending on the apparatus.

Eliminating v between these two equations we have $\frac{y^2}{x} = \frac{K_2^2}{K_1} \cdot \frac{E}{M}$. Hence for particles which have the same value of $\frac{E}{M} \cdot \frac{y^2}{x} = \text{Const.}$ This obviously represents a parabola. Thus particles which have the same value of $\frac{E}{M}$ but different values of v, are distributed over a parabola on the photographic plate. If $\frac{E}{M}$ be different the constant is also different. Hence particles having different values of $\frac{E}{M}$ are distributed over different parabolas. From these parabolas $\frac{E}{M}$ and ultimately M of different particles can be determined.

Aston improved the arrangement by separating the electric field and the magnetic field. He arranged the fields in such a way that particles having the same value of $\frac{E}{M}$ were concentrated at one point, instead of being distributed over a parabola (as in the case of J. J. Thomson's apparatus). Thus the presence of even such particles as were small in number, could be detected. In this way M *i. e.* the atomic weight of different substances present even in small quantities in the discharge tube could be determined. The apparatus designed and used by Aston is now known as "Mass Spectrograph". The apparatus used by Sir J. J. Thomson is also sometimes called Thomson's Mass Spectrograph.

Art 203
Isotopes

In the early days of the development of chemistry Prout put forward the hypothesis that all atoms are built up of the lightest

atom, viz. that of hydrogen. According to this view the atomic weights of all substances should be integers. This view soon became untenable when by more accurate measurement the atomic weights of almost all substances were found to be fractional and not integers.

With the measurement of atomic weights of individual particles by J. J. Thomson's parabola method and later by Aston's Mass Spectrograph the same view has again come to the fore although in a modified form. It is now noticed that those substances whose atomic weights were hitherto known to be fractional are now found to be a mixture of two or more substances whose individual atomic weights are integers. Thus Chlorine whose atomic weight as determined by chemical methods is known to be 35'47, is now found to be consisting of two kinds of atoms whose individual weights are 35 and 37. These latter are always mixed in such a proportion that the average atomic weight comes out to be 35'47. These atoms although differing in weights have the same physical and chemical properties so that they can never be separated by physical or chemical methods. In Aston's mass Spectrograph however they are made to be concentrated at different points on the photographic plate and are thus differentiated. Such atoms which differ in weight but have the same physical and chemical properties are known as Isotopes. Thus Neon whose ordinary At. wt. is 20'2 has two isotopes of wts 20 and 22. Tin has six isotopes. Even substances whose atomic wts are found to be integers when measured by ordinary methods are also sometimes known to be consisting of isotopes. Thus oxygen in addition to the ordinary variety of At. wt. 16 has a rare isotope of At. wt. 17 and another of 18. The At. wt. of hydrogen is one. But two other varieties of At wts 2 and 3 have been discovered. They have the same physical and chemical properties as ordinary hydrogen. But they are heavier; and because of this heavi-

ness they are called Heavy Hydrogen.*

With the exception of a few almost all the elements are now found to be consisting of isotopes. It is also noticed that elements with odd atomic numbers almost never possess more than two stable isotopes. While those with even atomic numbers usually have a larger number of isotopes and these latter again occur in a fairly regular manner, e g. the isotopes of iron (At. No. 26) are 54, 56, 57 and 58 ; those of Zinc (At. No. 30) are 64, 66, 67, 68 and 70.

Art 204 Measurement of the charge *e*.

We have discussed how the specific charge $\frac{e}{m}$ of an electron was measured. Soon afterwards attempts were made to measure the charge *e*. One of the best methods for measuring the electronic charge is that due to Millikan.

The insulated metallic plates P_1 and P_2 separated by a small distance are placed within an air-tight chamber B. Oil (or mercury) is sprayed into the chamber B through the funnel F by the atomiser A. There being a small hole in the upper plate P_1, some of the drops of oil (or mercury) produced by the spraying, pass into the space between the two plates. X Ray radiation from the X Ray bulb X ionises the air within. so that the tiny droplets as they fall slowly, collide with either electrons or positively charged ions and become charged accordingly. An electric field strength of which can be varied as desired, is established between the

* Hydrogen of At. wt 2 was first discovered and was called Heavy Hydrogen. Hydrogen of At. wt 8, discovered later, is not usually so called. Hydrogen of At. wt 2 also k- as Deuterium was first discovered by Prof. Urey spectroscopically while he was studying the spectrum of Hydrogen. Clearly if ordinary hydrogen be replaced by Deuterium in a molecule of water the resulting water will be somewhat heavier. Such 'heavy water' was actually obtained by electrolysing large quantity of ordinary water by heavy current for a long time. The small quantity of water that ultimately remained was found to be 'heavy water'. Its properties are also different from those of ordinary water. Its freezing point is 3°·8C, boiling point 101°·4C and latent heat of vaporisation is 796 calories.

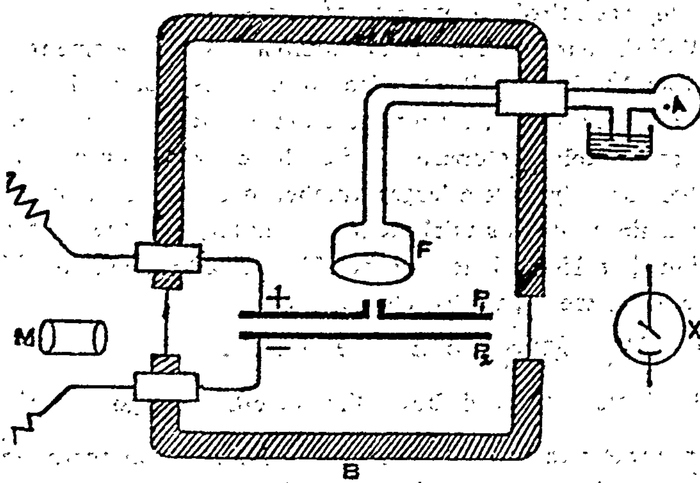


Fig. 279

plates. The upper plate being positive with respect to the lower, charged oil drops are attracted towards the upper or lower plate according as the charge is negative or positive. A powerful beam of light from an arc lamp, illuminates the drops so that they appear as specks of light in the field of view of the microscope M. Attention is concentrated on one of the drops pulled upwards by the electric field. If E be the charge on the drop and X the strength of the electric field, the upward force on the drop is XE; and if m be the mass of the oil drop, mg is the weight acting downwards. The strength X of the electric field is adjusted until

$$XE = mg \quad \ldots \quad \ldots \quad (a)$$

In that case the oil drop is not acted on by any force and can be observed for hours by the microscope as a stationary luminous point.

In equation (a) X and g are measurable quantities. To find 'm', the electric field is removed and the oil drop is allowed to fall downwards by the action of its own weight. As the size of the drop is extremely small it soon acquires

a *uniform* limiting velocity with which it goes down slowly in the field of view of the microscope. This limiting velocity is measured by means of a scale placed within the microscope and is given by Stokes' equation

$$v = \frac{2}{9} \frac{ga^2(\rho - \sigma)}{\eta} \quad \ldots \quad (b)$$

where a = radius of the drop, ρ = density of oil, σ = density of air and η = viscosity of air.

Thus in equation (*b*), all other quantities being known the radius 'a' of the drop can be determined. Hence the mass 'm' is obtained from the relation

$$\text{Mass '}m\text{'} = \text{volume} \times \text{density} = \frac{4}{3}\pi a^3 \rho$$

In equation (*a*), X, m, g being thus determined, the charge E on the drop can be measured. It is to be noted however that the oil drop—when it is observed by the microscope may have already captured one, two or more electrons. The charge E is therefore not necessarily the electronic charge itself but may be twice, thrice or even greater multiples of the same. The experiment was repeated a large number of times with hundreds of drops and at different intervals of time. It was noticed that different values of E obtained in this way, were all integral multiples of a single unit. This unit* is therefore the charge of the electron. Millikan obtained $4{\cdot}77 \times 10^{-10}$ E. S. unit as the electronic charge. The present day accepted value is $4{\cdot}80 \times 10^{-10}$ E. S. unit.

Exercise XX

1. Describe the properties of cathode rays and discuss briefly how they were applied to the determination of the nature of the cathode rays.

2. Explain how an electric field and a magnetic field may be simultaneously superposed on the path of cathode rays so that cathode rays are not deviated from their rectilinear path. Show how this leads to the direct determination of the velocity

* This was checked and corroborated in many other ways. For a fuller discussion, see 'Electrons' by Millikan.

of the cathode rays particles.

3. Explain why the deflections in the path of a moving charged particle by an electric field and a magnetic field applied in the same direction, are at right angles to each other.

4. A pair of plates, each of length 10 cms are placed at a distance of 5 mm from each other and an electronic particle of charge 4.80×10^{-10} E. S unit and mass 8.9×10^{-28} gms. is projected through them. If the plates are at a difference of potential of 6000 volts and if the particle is deflected through 30° find the velocity with which the particle is projected.

Ans. 1.93×10^{10} cms per sec.

5. Distinguish between atomic number and atomic weight. Which is more fundamental and why ?

6. Describe briefly the structure of an atom explaining how the valency of an atom is determined by the position of the atom in the periodic table.

C. U. Questions.

1965. Describe the construction and working of a Thomson's Mass Spectrograph Indicate the results obtained with it.

1966 Describe an experimental method of determining the specific charge of an electron. Discuss what led the scientists to believe that the electron is a common constituent of matter.

1967. Write short notes on (*a*) Cathode rays (*b*) Positive rays.

1968 (1) Describe and explain a method of determining the charge of an electron.

(2) Writes notes on "Structure of the nucleus".

1969. Give an account of a suitable method for the measurement of e/m of electrons.

Through what potential difference must an electron be accelerated in order to attain a velocity of 3×10^{8} cm/sec ? e/m for electrons $= 1.76 \times 10^{7}$ E.M.U. per gm.

1970 (1) Give an account of Millikan's method for the deter-

mination of the charge of an electron.

Calculate the electric field in volts/cm required in Millikan's experiment to balance an oil drop of radius 3.6×10^{-5} cm carrying an electronic charge from the following data :

Density of oil=0.8 gm cc ; g=981 cms/sec^2 ; $e = 4.80 \times 10^{-10}$ E.S. unit and 1 E.S. unit of E.M.F =300 volts.

(2) The atomic weight of sodium is 23 and its atomic number is 11. What is the structure of a sodium atom.

1971. What are cathode rays? Give an account of the method for the determination of e/m for cathode rays.

Calculate the ratio of the electric force to the gravitational force acting on an electron in an electric field of 300 volts/cm. e/m for electron = 1.778×10^7 e.m.u. Ans. 5.444×10^{14}

[Hints: Electric field is Xe and gravitational field is mg.

$$\text{Hence their ratio} = \frac{Xe}{mg} = \frac{X}{g} \cdot \frac{e}{m}\Big]$$

1972. (1) Describe Millikan's method for the determination of the charge of an electron.

(2) What are isotopes? How can their occurrence be explained? Uranium of atomic weight 92 has two isotopes of atomic weights 235 and 238 respectively. Explain the difference in the nuclear structures of the isotopes.

1973. What do you mean by the mass number and atomic number of an isotope? The mass number and the atomic number of an isotope are 27 and 12 respectively. Give the atomic and nuclear structure of the isotope. Can you identify the isotope?

1974. (1) The atomic number of an element is 20. What is its valency?

(2) In what way do two isotopes of the same element differ from each other, so far as the number of electrons, protons and neutrons are concerned.

1975. Describe with theory Millikan's method of determining the electronic charge.

An electron moves with a uniform velocity of 3×10^7 cm/sec under the action of mutually perpendicular electric and magnetic fields. If the electric field be 300 volts/cm; calculate the strength of the magnetic field. 1000 Oersted.

CHAPTER XXI

X RAYS AND RADIO-ACTIVITY

X Rays

Art 205
Discovery

Towards the end of the nineteenth century the properties of cathode rays as discovered and studied by Sir J. J. Thomson and others, attracted attention of numerous physicists all over the world. In the year 1895 W. K. Röntgen Professor of Physics in the university of Würzburg was working with such a highly evacuated cathode ray tube. Luckily for him and luckily for mankind a screen of barium platinocyanide was placed accidentally in the vicinity of his tube. He found to his surprise that as the cathode ray tube was being worked the screen began to glow with a faint greenish yellow fluorescent light. He thus discovered the generation of a new kind of radiation whose origin he ultimately traced to the sides of the glass vessel where the cathode rays impinged. Just as in algebra the letter 'X' stands for any unknown quantity so these rays, pending further enquiry as to their nature, were called X Rays; they are also nowadays sometimes called Röntgen rays after the name of the discoverer.

Properties

The most remarkable property that was soon found to be possessed by these rays was that the rays could pass more or less freely through many substances such as wood, paper, flesh etc. which are opaque to ordinary light. Bones, metals and other denser substances were however opaque to these rays also. Indeed opacity was found to be dependent on density. X-Rays were also found to ionise a gas and to affect a photographic plate.

Art 206

It was gradually established that X Rays are best produced by the impact of high speed cathode ray particles on a metallic target (now known

as anticathode) placed within the discharge tube. This fact ultimately led to the development of the common form of X Ray tubes usually seen in the market.

The property that cathode rays or electrons start out of the cathode in a direction perpendicular to the surface of the cathode (Vide Art 191), is utilised in the construction of an X Ray tube. The cathode C is made concave so that cathode Rays are all concentrated at the centre* of curvature of the cathode. At this centre the anti-cathode T generally in the form of a circular plate is placed inclined at an angle of 45° to the beam of cathode rays. Bombardment of the anti-cathode by the electrons being thus very much concentrated a powerful beam of X Rays is generated from the anti-cathode. The cathode is made of aluminium because aluminium sputters least. A large amount of heat is also generated at

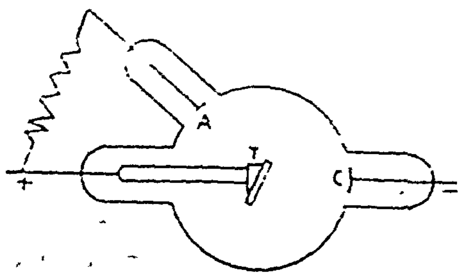


Fig. 280

the anti-cathode by the impact of cathode rays. To withstand this the anti-cathode is made of a heavy metal such as platinum or tungsten having a high melting point; it is sometimes attached to the end of a heavy copper tube which is hollow and cooled with water for carrying away heat. A separate anode A placed somewere in the tube is in metallic connection with the anti-cathode. The action of this separate anode is not definitely understood; but it is found in

* Actually cathode rays or electrons are concentrated a little beyond the centre of curvature; this is because of the mutual repulsion existing among the electrons.

practice to improve the working of the tube. The tube is generally run by an induction coil or a high tension step-up transformer, producing 30,000 to 50,000 volts. The pressure within the tube is of the order of 10^{-4} mm.

Art 207
Coolidge tube

Since 1913 an altogether new type of tube designed by W. D. Coolidge, has been placed on the market. Air is removed from within this tube as completely as possible so that ordinarily no discharge can be passed across the tube

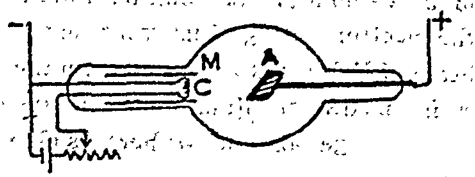


Fig. 281

owing to insufficient number of possible carriers left in the tube. The cathode C is however made of a spiral of tungsten wire which is heated to incandescence by means of an electric current. Large quantities of electrons (thermions) are thereby emitted and these serve as carriers in the discharge tube. These thermions have initially very small velocities but can be speeded up to any desired extent by applying corresponding potential difference across the tube. They are focussed into a beam by surrounding the cathode with a tube of molybdenum M. The anti-cathode A is made of a massive block of tungsten. No arrangement for cooling is usually made and no separate anode is provided for.

Art 208
Ionisation chamber

As has been stated earlier X Rays affect photographic plates. The presence of X Rays can therefore be detected by the use of these plates. An approximate estimate of the intensity of X Rays can also be obtained by the depth of *intensity* on the photographic plates produced by X Rays.

A more accurate estimate of the intensity can however be made by what is known as the Ionisation chamber. In this apparatus a metal rod R is placed on suitable insulating stands inside a metal chamber C. At one end the chamber has a window W made of aluminium through which X Rays can enter the chamber.

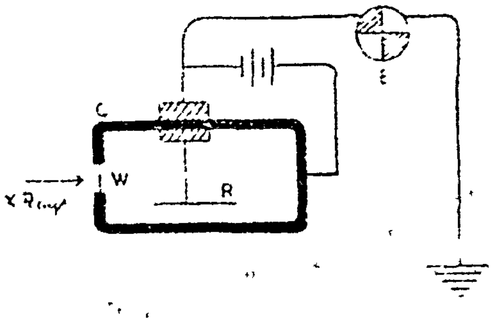


Fig. 282

A potential difference of several hundred volts is applied between the chamber and the rod by means of a battery. A quadrant electrometer E is also connected to the rod R so that it may detect any charge that may come to the rod. The chamber is filled with a gas, usually at atmospheric pressure. Hydrogen, air, carbon bisulphide, sulphur dioxide, methyl bromide are some of the gases used in an ionisation chamber.

When X Rays enter the chamber through the window they ionise the gas. Electrons and positive ions formed thereby are attracted towards the respective electrodes, viz the rod and the chamber and there is a current between the two. The rod being thus charged the electrometer shows deflection. This deflection is proportional to the rate at which the rod is being charged and hence to the intensity of of the X Rays. We have thus a measure of the intensity of X Rays.

The ionisation chamber has also been utilised in detecting the presence of and estimating the intensity of many other rays and particles.

Art 209
Nature of X Rays

Röntgen found that X Rays could not be deflected by a magnet. Unlike cathode rays they could not therefore be streams of charged particles. To all probability they were electromagnetic in nature like ordinary waves of light. But he did not

succeed in establishing their identity with light. For, all attempts to observe the usual properties associated with light, viz. reflection, refraction, interference, diffraction and polarisation, yielded only negative results.

It was strongly suspected that X Rays, if at all they are waves in ether, must be of very short wavelength at least 1000 times smaller than that of ordinary light. This would at least explain the apparent impossibility of demonstrating diffraction of X Rays by an ordinary grating. A beam of sodium light of wavelength 5890A is diffracted by about 19° in the first order by a grating with 5500 lines to the cm. To produce a similar deflection in X Rays would therefore require each of the spacings of the grating to be divided into 1000 spaces : this is obviously mechanically impossible.

In the year 1912 Prof. Laue conceived the brilliant idea that a crystal might provide a natural grating of suitable spacing for the diffraction of X Rays. There was however one important point of difference. An ordinary grating consists of parallel spacings all in one plane but the regularity of a crystal grating is in three dimensions instead of only two and may be roughly compared to a pile of gratings, one placed on the top of the other. The mathematics of the problem was indeed difficult but Prof Laue successfully tackled the problem and obtained a solution. According to him a narrow beam of X Rays passing symmetrically through a crystal would be diffracted in certain definite directions ; if a photographic plate be placed perpendicular to the beam, a symmetrical pattern of spots arranged according to definite laws would develop on the photographic plate.

Laue however was not an experimentalist ; the theory was put to a test by two of his students Friedrich and Knipping in 1913. A powerful beam of X Rays generated from the anti-cathode T and made very narrow by a series of slits in lead screens A, B and C was ultimately incident on the crystal X. A photographic plate P was placed perpendicular to the beam and an exposure was made lasting for several hours. When the plate was developed it was found that the central black

patch made by the undeflected beam of rays, was surrounded by a symmetrical pattern of spots exactly as indicated by the

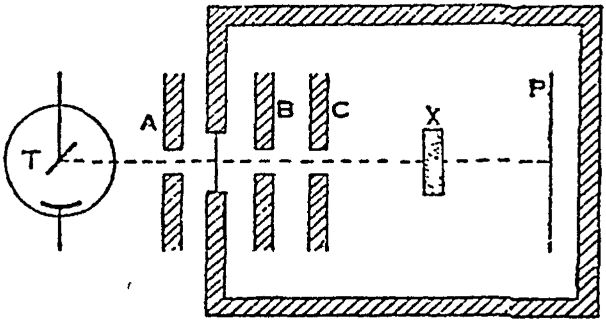


Fig. 283

theory. The theory was thus abundantly verified. This ultimately opened up a vast field of research probing into the structure of crystals.

Art 210
Bragg's Law

After the discovery by Prof. Laue that X Rays are waves in ether attempts were naturally made to measure the wavelength of X Rays. By this time Prof. W. L. Bragg developed a simple theory about the reflection of X Rays by crystal faces. Let a narrow beam of monochromatic X Rays of wavelength λ be incident on the face of a crystal at a small glancing angle θ. As X Rays can penetrate* into the crystal Bragg assumed that X Rays are partially reflected successively from the different layers. Let the incident beam AB be partially reflected by the

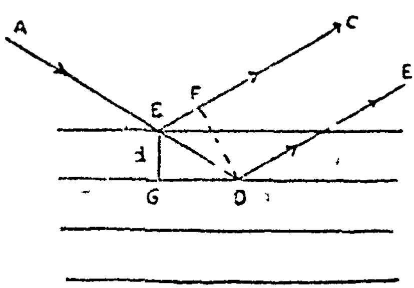


Fig. 284

* As X Rays penetrate into the crystal intensity of X Rays gradually diminishes, complete absorption taking place after the penetration of several layers of atoms.

first layer along BC. On entering the crystal along BD* let it again be partially reflected by the second layer in the direction DE. If DF be dropped perpendicular to BC the path difference between the two reflected rays is clearly equal to BD − BF. Let BG the distance between two successive layers be represented by d. Then $\sin\theta = \frac{d}{BD}$.

$$\text{Or}\quad BD = \frac{d}{\sin\theta}\quad\text{and}\quad BF = BD\cos 2\theta = \frac{d\cos 2\theta}{\sin\theta}.$$

Hence the path difference $= \frac{d}{\sin\theta} - \frac{d\cos 2\theta}{\sin\theta} = \frac{d(1-\cos 2\theta)}{\sin\theta}$ $= 2d\sin\theta$. If this path difference be equal to $n\lambda$ the two rays reflected from the first two layers reinforce each other. Clearly if this relation is satisfied rays reflected from all the layers also reinforce one another and we get a strong beam of reflected rays in the direction given by $2d\sin\theta = n\lambda$. This is known as Bragg's equation for reflection of X Rays by a crystal surface. It is to be noted that when the wavelength λ and the glancing angle θ satisfy this relation then and then only a strong beam of reflected rays is obtained ; for other glancing angles the beams reflected from different layers are out of phase with one another and as a result there is no effective reflected beam.

Art 211
X Ray Spectrometer

In order to measure the glancing angle θ Bragg designed an X Ray spectrometer. This is exactly analogous to an ordinary optical spectrometer. The collimator of the ordinary spectrometer is here replaced by two narrow slits S_1 and S_2 through which passes a narrow beam of X Rays generated by the X Ray bulb

* It is tacitly assumed here that the ray AB proceeds undeviated into the crystal along BD, *i.e.* the refractive index of the crystal for X Rays is equal to unity ; in that case the path BD in the crystal is equivalent to an equal path in air. Later by more accurate measurement it was established that the *refractive index of a crystal* for X Rays is slightly *less* than unity and X Rays are actually refracted into the crystal away from the normal.

This narrow beam is incident on the crystal face C placed

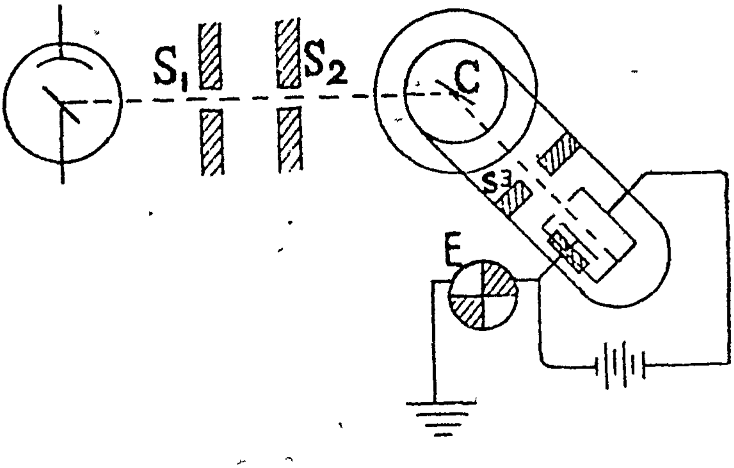


Fig. 285

the prism table. The telescope of the optical spectrometer replaced by an ionisation chamber. Just before the ıisation chamber there is a third slit S_3 which prevents any ttered radiation from entering the ionisation chamber. e ionisatioı chamber and the slit S_3 are carried by a handle ich rotates about the centre of the prism table. As the ıisation chamber is to receive X Rays reflected by the crystal e C it is obvious that as the crystal is rotated through a cern angle the ionisation chamber is also to be rotated through ıce the aı gle. When X Rays are reflected by the crystal face y enter the ionisation chamber. An ionisation current is ·reby produced and the quadrant elec.rometer E shows lection. These deflections are measured for different glancing gles According to Bragg's Law for a particular wavelength λ Rays should be reflected for glancing angles θ_1, θ_2 and θ_3 ven by $2d \sin \theta_1 = \lambda$, $2d \sin \theta_2 = 2\lambda$ and $2d \sin \theta_3 = 3\lambda$. Hence ı $\theta_1 : \sin \theta_2 : \sin \theta_3$ should be equal to 1 : 2 : 3. Actually ıen the angles θ_1, θ_2 and θ_3 were measured this relation s found to be true. Thus Bragg's Law was verified.

grams of radium. As a recognition of this they along with Becquerel had the honour of receiving the Nobel Prize in Physics in 1904*.

Later on a host of other radio-active substances was gradually discovered—Thorium, Ionium, Samarium (a rare earth discovered in 1933 by Hevesy), Actinium etc. etc.

Art 215
Constants of radio-active substances

It was gradually established that the emission of these rays is a spontaneous phenomenon. Strongest heat—enough to melt any element, intense cold—sufficient to freeze any substance, strong electric and magnetic fields, very high and very low pressures—all these were found to have no effect on the emission of these rays. As it was believed that these physical changes cannot produce any effect on the nucleii of atoms it was concluded that this emission must be due to the breaking up of the nucleii of atoms of radio-active substances.

Clearly the nucleii of all atoms present in a radio-active substance do not break up simultaneously or otherwise any radio-active substance would dis-integrate in no time, *i.e.* the rate of breaking up of the atoms cannot be infinitely large. It is reasonable to assume that at any instant this rate is proportional to the total number of atoms actually present at that instant. Thus if N be the number of atoms present at any instant the rate of breaking up, *i. e.* $\frac{dN}{dt}$ is proportional to N.

Thus $-\frac{dN}{dt} \propto N$ [The minus sign is added to indicate that N decreases with time t].

Or $\frac{dN}{dt} = -\lambda N$ where λ is a constant known as the disintegration or decay constant.

Integrating $\log_e N = -\lambda t + C$ where C is the constant of integration.

---

* In 1911 Madam Curie was awarded the Nobel Prize a second time—this time in Chemistry, the only instance hitherto of a second award. Pierre Curie had died prior to this by an accident.

If N_0 be the number of atoms initially present, *i. e.* if $N = N_0$ at $t = 0$ we have $C = \log_e N_0$.

Hence $$\log_e N = -\lambda t + \log_e N_0.$$

Or $$\log_e \frac{N}{N_0} = -\lambda t \quad \therefore N = N_0 e^{-\lambda t}.$$

This equation shows that N becomes *zero, i.e.* complete disintegration takes place only after infinite time. The time in which N decreases to half the original value is known as *Half period.* If T be this half period we have $\frac{N_0}{2} = N_0 e^{-\lambda T}$

or $$2 = e^{\lambda T} \quad \therefore \quad T = \frac{\log_e 2}{\lambda} = \frac{0{\cdot}693}{\lambda}.$$

Thus T is inversely proportional to λ; it is also therefore a constant for a given radio active substance. For radium T is 1590 years while for Radon (a radio-active gas) T is only 3·8 days

A third constant known as the mean life of a radio-active substance is defined to be the ratio of the total life time of all the radio-active atoms (of a substance) to the total number of all such atoms. It can be determined as follows :—

We know $\frac{dN}{dt} = -\lambda N$. Leaving out the minus sign which only signifies that N decreases with time t, we have $dN = \lambda N dt = \lambda N_0 e^{-\lambda t} dt$. These dN atoms disintegrate in time *dt after the lapse of time* t, *i.e.* they have had a life time t before they are disintegrated. Hence the total life time of these dN atoms is equal to $t dN$. Now the initial N_0 atoms disintegrate gradually until after infinite time all of them are disintegrated. Or, in other words, all these atoms have life times ranging from 0 to ∞. Hence total life time of all these atoms is $\int_0^{\infty} t dN$.

Hence mean life τ is given by

$$\tau=\frac{\int_0^\infty tdN}{N_0}=\frac{1}{N_0}\int_0^\infty t\,\lambda N_0 e^{-\lambda t}dt=\lambda\int_0^\infty t\,e^{-\lambda t}dt$$

$$=-\int_0^\infty td\left(e^{-\lambda t}\right)=-\left[t\,e^{-\lambda t}\right]_0^\infty+\int_0^\infty e^{-\lambda t}dt$$

The first term within the third bracket is zero for both the limits. Hence, finally

$$\tau=\int_0^\infty e^{-\lambda t}dt=-\frac{1}{\lambda}\left[e^{-\lambda t}\right]_0^\infty=\frac{1}{\lambda}$$

Thus the mean life τ turns out to be the reciprocal of λ. Since $T=\frac{0{\cdot}693}{\lambda}$ $\therefore$ $T=0{\cdot}693\,\tau$. The mean life of radium is therefore $\frac{1590}{0{\cdot}693}=2300$ years, and that of radon is $\frac{3{\cdot}8}{0{\cdot}693}$ $=5{\cdot}5$ days.

Art 216 Properties of these radio active rays were at first studied by the usual magnetic deflection. For this purpose different radio-active substances were taken in a small lead box provided with a narrow mica window at the

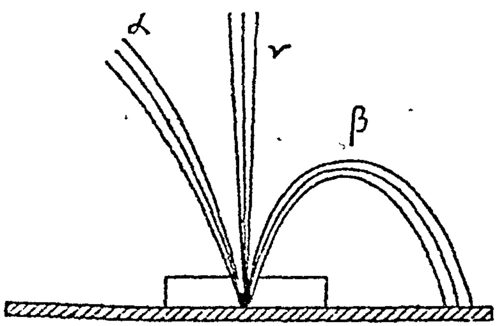


Fig. 286

top, so that the rays emitted by the substances in different angular directions were absorbed by thick lead walls on all sides and could pass out of the box only when they were

emilted vertically upwards through the mica window. When a strong magnetic field was applied perpendicular to the path of these rays, the rays were found to be split up into three portions. One portion was deflected to one side to a small extent, another portion was deviated to the other side to a much larger extent while a third portion passed out undeviated. These three were called α rays, β rays and γ rays.

Art 217
α rays

We shall first discuss the nature and property of α rays. From the direction of deflection by the magnetic field it was obvious that α rays consisted of positively charged particles They were also found to be deflected by an electric field. By measuring the deflections in the two cases the value of $\frac{E}{M}$ for the α particle was determined. It was noticed that $\frac{E}{M}$ for α particle was almost exactly half that of a hydrogen atom in electrolysis. [Vide Art 124 equation (44)]

i. e.
$$\left(\frac{E}{M}\right)_\alpha = \frac{1}{2}\left(\frac{E}{M}\right)_H$$

The charge on the particle was also later on measured* and was found to be equal to two electronic charges, *i. e.* E_α was equal to $2E_H$. It was therefore concluded $M_\alpha = 4M_H$.

Nature of α rays

Thus α rays consist of positively charged particles of mass equal to four times that of a hydrogen atom and of charge numerically equal to two electronic charges. What is therefore the nature of an α particle ?

We have seen that a helium atom consists of a positively charged nucleus surrounded by two electrons rotating round the nucleus. The nucleus itself consists of two protons and two neutrons. The mass of the nucleus is therefore equal to four times that of a hydrogen atom and the charge on the nucleus is twice that of an electron. Can it be

---

* Vide Art 219

therefore that an α particle is identical with the nucleus of a He atom ?

This point was satisfactorily settled by the simple but beautiful experiment of Rutherford. A glass tube A was fused into another glass tube B in which two electrodes were fitted. Both A and B were evacuated as completely as possible. A speck of radium was placed on the top of a wire placed in A. The portion of A *within* B was made of very thin glass through which α particles from radium could pass and enter the tube B. An electric discharge was passed

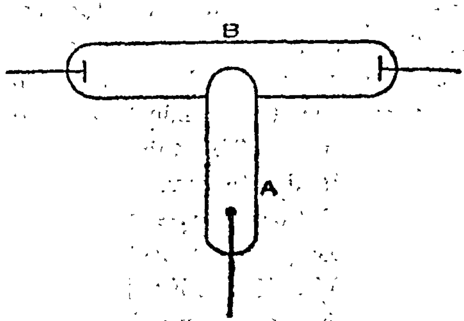


Fig. 287

between the electrodes in B. It was expected that if α particles were identical with He nucleus, they would, on coming to B, capture electrons produced by the electric discharge, and would form He gas. In that case the spectrum of He was expected to be produced by the discharge. Actually, after a few hours the brightest lines of He appeared in the spectrum and after a few days all the lines of He were produced. It was thus conclusively established that α particles were nothing but nuclei of He. The fact that in nature radio-active minerals always contain He gas in their cavities, also lead to the same conclusion, *viz*, He gas is a product of radio-activity.

Art 218
Geiger Counter

The Geiger Müller counter is a very efficient apparatus for *counting* individual particles that enter a chamber. It was first devised by Rutherford and Geiger in 1908 and later in 1928

it was brought to perfection by Geiger and Müller. It is now extensively used for counting α particles, β particles, neutrons, protons and for detecting the presence of various other rays. Essentially it consists of a metal tube T closed at two ends by ebonite plugs. Along the axis of this tube there is placed a very fine wire P usually made of tungsten. It passes out of the tube through one of the ebonite plugs. There is a circular opening in the other plug, the opening being however closed usually by an aluminium screen. This serves as the window W through which the rays enter the tube. The tube is first completely exhausted and is then filled with some suitable gas, *e. g.* air, hydrogen, argon or a mixture of these at some chosen pressure. A high tension battery is connected between the wire P and the tube T. The E. M. F is just short of sparking voltage so that ordinarily there is no discharge between the wire P and the tube T. A single α particle however, as it enters the tube through the aluminium window W, produces ionisation within the tube; as a result a discharge takes place between P and T. This may be detected by having a suitable instrument, such as a headphone or a galvanometer (G) placed in the circuit. It is essential that this discharge should not become permanent; it must be quickly and automatically extinguished. This is achieved by having a high leak resistance R in series with the battery. This being connected to the Earth electric charges generated by ionisation within the counter quickly pass on to the Earth and the counter is ready to detect the arrival of a fresh particle. The counter thus behaves as an automatic rifle which is fired by the trigger action of an in-coming particle and which quickly resets itself for further use.

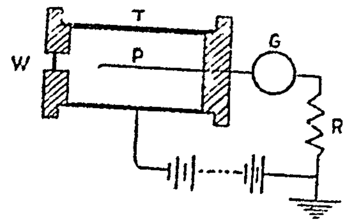


Fig. 288

For detecting heavy particles, such as protons, or α particles the gas within is at atmospheric pressure. If γ

rays are to be detected the window is done away with and the tube is closed on all sides. The tube itself having fairly thick walls (1 to 3 mm), with the arrival of γ rays secondary electrons emitted from the walls of the tube cause the discharge to take place. For the detection of neutrons the counter is filled with hydrogen. The protons liberated by the collision of neutrons, produce the necessary 'trigger' action.

Art 219
Properties of α rays

It was observed that α particles when incident on a zinc blende screen produce fluorescence. When examined by a low power microscope the fluorescence is found to be discontinuous ; individual scintillations can be seen. This provides a visual evidence that α rays consist of discrete particles. Counting the number of scintillations in a definite time over a small area AB placed at a known distance from a radio-active source S it is possible to have an estimate of the number of α particles that are emitted per sec per unit solid angle by the source. Afterwards the screen AB is replaced by a metallic plate connected to an electrometer and the total charge received by the plate in the same time is measured. From the previous experiment the number of α particles coming to the plate in a given time being known the charge of a single α particle is determined.

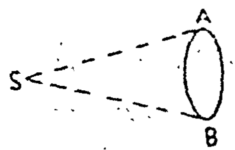


Fig. 289

α rays have also the power of affecting photographic plates. The presence of α rays at any place can therefore be readily detected by placing a photographic plate in the path of the rays.

When α particles pass through a substance they knock off electrons from the atoms lying along their paths, *i.e.* α particles have the power of ionising the atoms. When α particles pass through a gas the path is almost a straight line. The mass of α particles being very large in comparison to

that of an electron α particles are not deviated at all when they knock off electrons from the atoms. On rare occasions however α particles may collide with the comparatively massive nucleii of the atom. In that case the α particles may be deviated from their paths. Such deviation may in some cases be even greater than 90°. As a matter of fact this led Rutherford to the conception of the nuclear atom model [Vide Art 197].

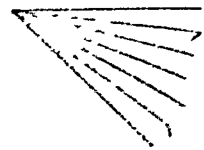


Fig. 290

A peculiar property of α rays is that when they pass through a gas they are stopped abruptly after a certain critical distance known as the *range of* α particles. Beyond this distance all the three properties of α rays, viz, ionising power, fluorescent effect and photographic effect disappear simultaneously. This discovery was made in 1904 by Bragg and Kleeman.

Art 220
β rays

By the usual method of electric and magnetic deflections, $\frac{e}{m}$ for β ray particle was determined. It was found to be the same as that of an electron. The direction of deflection of β rays in the magnetic or electric field, showed that they must be negatively charged particles. It was therefore established that β rays are nothing but cathode rays or electrons. The only characteristic that distinguishes them from an electron obtained otherwise, is the tremendous velocity which they usually possess. This velocity is comparable to the velocity of light, sometimes approaching the latter to within 98% of its value.

Variation of mass with velocity

It was known on theoretical grounds that the mass of a charged particle varies with velocity and one of the relations as stated by Lorentz, is

$$M_v = M_o (1-\beta^2)^{-\frac{1}{2}}$$

where M_0 = rest-mass, *i. e.*, mass of the particle when the

velocity is small or nil, M_v = mass of the particle with velocity v and $\beta = \frac{v}{C}$, C being the velocity of light.

Bucherer and other experimenters devised beautiful experiments with these β rays, which fully demonstrated the correctness of this relation.

β rays also produce scintillations on the zinc blende screen but to a much lesser extent than α rays. β rays affect the photographic plate and can ionise air. It may be noted that β rays being lighter particles, they are deviated by a magnetic field, much more than α rays which consist of much heavier* particles.

Art 221
γ Rays

Unlike α rays or β rays, γ rays are not deflected either by a magnetic field or by an electric field. They cannot therefore be streams of charged particles. It was gradually established that γ rays are electromagnetic waves, *i.e.* they are of the same nature as Light or X Rays from which they differ only in their much shorter wavelength. Light in the visible region extends roughly from about 4×10^{-5} cms. to about 8×10^{-5} cms. The wavelength of X Rays approximately lies in the region of 10^{-6} cm to 10^{-9} cm. The wavelength of γ rays, although overlapping somewhat with that of X Rays, is usually much smaller, it extends from 10^{-8} cm. to 10^{-11} cms.

β rays bear to γ rays exactly the same relation as the cathode rays bear to X rays. X rays are produced by the bombardment of cathode rays on the anticathode. Similarly β rays which are emitted deep within a solid radio-active substance—while tending to come out of the solid—strike against the substance itself; γ rays are thereby produced by the impact of β rays on the radio-active substance. As we have seen β rays possess greater velocity and hence greater

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* The mass of a β ray particle is $\frac{1}{1850}$ of that of a hydrogen atom where as an α ray particle is four times as heavy as a hydrogen atom.

kinetic energy than cathode rays. Their impact therefore produces radiations of greater energy, *i.e.* of greater frequency and hence of smaller wavelength. Thus γ rays have wavelengths much smaller than those of X Rays. This also explains the significant fact that while α rays alone are emitted by some substances, β rays and γ rays are always found to be associated with each other, *i.e.* substances which emit β rays also produce γ rays.

Art 221 (a)

Transformation by radioactive emission Radioactive rays from radioactive substances are emitted with such large velocities that they must be assumed to be coming out of the nuclei of the atoms. Naturally the question arises whether by such emission any change is produced in the structure of the nuclei and hence of the atoms. One difficulty however has to be faced immediately. It is well known that the nucleus of an atom contains only protons and neutrons. How is it then that β rays (which are nothing but electrons) come out of the nucleus? For a long time this was a pertinent question. Ultimately, however, it has been established that a neutron (which is a neutral particle) can break up into an electron and a proton. It is now believed that when a β particle comes out of the nucleus of an atom a neutron in the nucleus is transformed into a proton. Obviously by this process the number of protons in the nucleus of the atom increases by one. There being now an excess of protons, the atom becomes positively charged. If the atom captures one more electron in the outer orbits it becomes neutral; otherwise it behaves as an ionised atom. Anyway in this modified atom since in the neutral state the number of electrons in the outer orbits increases by one, the atomic number of the new substance so formed must increase by one, i.e. the substance moves one step forward in the Periodic Table. Since the total number of protons and neutrons in the nucleus remains the same the atomic weight of the substance remains unchanged. The substance is a new one with the same atomic weight as before but displaced one step forward in the Periodic Table.

An α particle has been established to be nothing but the nucleus of a Helium atom. As is well known the nucleus of a He

atom contains two protons and two neutrons. With the emission of an α particle from the nucleus of a radioactive substance the nucleus therefore loses two protons and two neutrons. The atomic weight of the new substance is therefore diminished by four. Again, as the nucleus loses two protons, two electrons in the outer orbits must also become detached from the atom. As a result the atomic number of the substance must decrease by two. The transformed substance is therefore a new one with its atomic weight less than that of the original one by four and displaced two places backward in the Periodic Table.

The original substance and the transformed substance are both elements. Thus we have here a case of transformation of elements by radioactive emission.

Gamma rays are, as is well known, nothing but electromagnetic waves of the same nature as that of visible light. These rays do not really come from the nucleus of any atom. But, as has been explained in the previous article they are generated by the impact of β particles (generated well within a radioactive substance) on the radioactive substance itself. As a matter of fact when β particles are stopped by the impact the enormous kinetic energy of these high speed particles is converted into electromagnetic waves known as Gamma rays. The question of transformation of any atom by the emission of Gamma rays does not therefore arise at all.

Thus uranium of atomic weight 92 and atomic number 238 emits α rays and is converted into Uranium X_1 of atomic weight 234 and atomic number 90; Uranium X_1 again emits β rays, the new element so formed being Uranium X_2 of atomic weight 234 and atomic number 91. Symbolically this can be stated as

$$_{238}U^{92} \xrightarrow[\alpha]{} {}_{234}UX_1^{90} \xrightarrow[\beta]{} {}_{234}UX_2^{91}$$

The process of transformation does not stop at UraniumX_2; it continues and the last product of this Uranium series is a stable substance Lead (or Radium G) of atomic weight 206 and atomic number 82. There are three more radioactive series—those of Actinium, Thorium and Neptunium. The end product of the first of the above three series is also Lead (or Actinium D) of atomic weight 207 and atomic number 82;

while that of the second series is also Lead (or Thorium D) of atomic weight 208 and atomic number 82. Thus there are three varieties of Lead, end products of the three series—Uranium, Actinium and Thorium. They are the isotopes of Lead. The end product of Neptunium series however is Bismuth of atomic weight 209 and atomic number 83.

Exercise XXI

1. Describe an X Ray tube pointing out the importance of its differents parts.

2. What is a Coolidge tube ? In what way is it different from an ordinary X Ray tube ?

3. Explain briefly how X Rays have been identified with waves in ether. In what way is it different from ordinary light ?

4. What are soft X Rays and hard X Rays ? How do they differ from each other ? On what does the softness or hardness of X Rays depend ?

5. What are radio-active substances ? Name some of them.

6. What are Becqueral rays and what is meant by activity of Becqueral rays.

7. Describe briefly how Radium was discovered.

8. How has it been established that usually three kinds of rays come out of a radio-active substance.

9. How has it been established that α rays are nothing but nucleii of He atoms ?

10. What are β rays ? In what way they are different from cathode rays ?

11. β rays bear to γ rays exactly the same relation as cathode rays bear to X Rays. Justify this statement.

C. U. Questions

1963. Write short notes on Properties of α, β and γ rays.

1966, 1974. what are the different characteristic properties of α particles ? How would you show that α particles are helium nucleii ?

1967. (1) Describe a modern X Ray tube with a neat diagram and explain how X Rays are. produced. What are the characteristic properties of X Rays ? Explain what is meant by hard X Rays.

(2) Give an account of the properties of radiations emitted by radio-active substances. Explain what is meant by the half period life of a radio-active substance.

1969. Write notes on (a) Coolidge tube for producing X-Rays and (b) Properties of β-rays and γ-rays.

1970. Explain how it has been experimentally demonstrated that α particles are nuclei of He atoms.

1971. Describe a modern X Ray tube for the production of X Rays. Discuss the important characteristics of X Rays.

1972. Discuss the properties of alpha and beta particles. How does the structure of nucleus change due to the emission of either of these particles?

1975. (1) Describe some modern X Ray tube. Give reasons to justify the conclusion that X Rays are essentially of the same nature as visible light.

Cite two uses of X Rays.

(2) Which particles are constituents of the nucleus of an atom? How do the atomic number and atomic weight change when a nucleus emits (*i*) α, (*ii*) β or (*iii*) γ rays ?

What do you mean by the half life of a radio-active element? What is its importance ?

CHAPTER XXII

ELECTRONICS AND WIRELESS TRANSMISSION

Electronics, a branch of physics, has come to play a very important part in modern times, not only in pure physics but also in engineering and industry. As is well known there are always free electrons within a metal. These electrons move within the metal like molecules of a gas with different velocities and hence with different energies; the maximum energy possessed by an electron depends on the temperature. At absolute zero the maximum energy W_F is known as Fermi level. As the temperature rises this maximum energy also increases. For every metal there is again an energy W_d (known as potential barrier) which must be exceeded before an electron can go out of the metal surface. Or, in other words, at any temperature if the maximum energy be greater than W_d some of the electrons having energy greater than W_d are emitted from the metal surface. The difference $W_d - W_F$

Art 222
Work function and thermionic emission

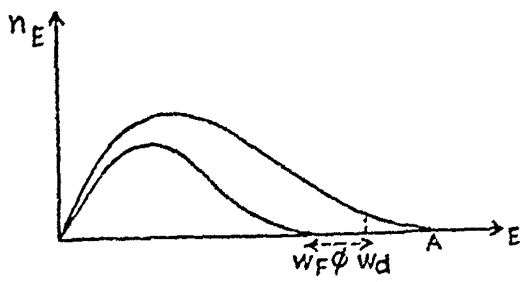


Fig. 291

is known as the Work Function ϕ. In Fig. 291 E represents the energy of an electron and n_E the number of electrons possessing the energy E. So long as the maximum energy is less than W_d no electron comes out of the surface of the metal. If the temperature be increased so that the maximum energy is represented by a point A beyond W_d the electrons

possessing energies represented by points between W_a and A have the chance to leave the metal surface. Thus if the temperature of a metallic wire be sufficiently high there is a copious supply of electrons from the wire. Electrons obtained in this way are known as **thermions.** Unlike cathode rays these thermions have initially very small velocities but they can be speeded up to any desired extent by a suitable potential gradient.

Art 223
Diode

In a Diode a metallic filament (sometimes called a cathode) is strongly heated by an electric current drawn from a small battery of low voltage E_1 (usually 2 to 8 volts) known as low tension (L. T.) battery. A metallic plate P known as anode is placed in the neighbourhood of the filament and is maintained at some high positive potential with respect to the filament by a battery of fairly high voltage E_2 (usually 80 to 200 volts) known as high tension (H. T.) battery. The electrons emitted by the heated filament are attracted by the anode and there is a flow of electrons from the filament to the anode, *i. e.* a negative current flows from the filament to the anode. This is equivalent to a positive current—known as plate current or anode current—from the anode to the filament. The filament and the anode are enclosed in a glass bulb which is ordinarily completely evacuated.

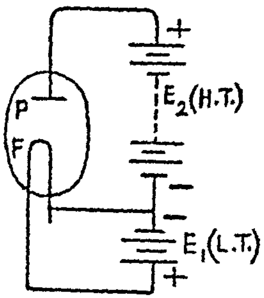


Fig. 292

Art 224
Cathode potential and space charge

Let the potential V at a point between the cathode and the anode be plotted against the distance x of the point from the cathode. When the cathode is cold there is negligible emission of electrons and the potential varies linearly from the zero at the cathode C to E_2 at the anode.

If the cathode temperature be raised there is some emission of electrons which move towards the anode under the

action of the electric field. Due to presence of electrons in space between the cathode and the anode the field strength near the cathode is reduced and the potential distribu-

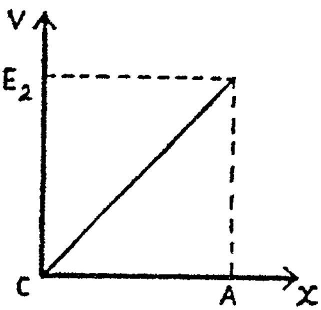


Fig. 293(a)

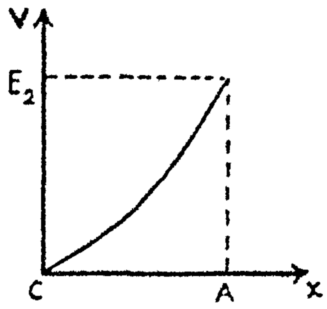


Fig. 293(b)

tion now takes the shape as in Fig 293(*b*). The cathode and the anode potentials, of course, retain their previous values.

As the temperature of the cathode is gradually raised, producing increased emission of electrons the potential near the cathode continues to fall. Ultimately this

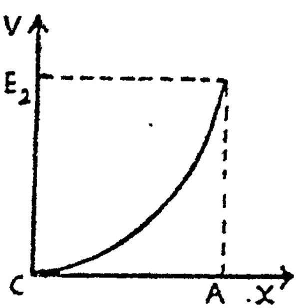
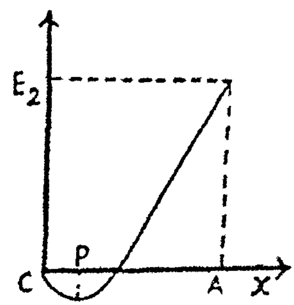


Fig. 293(c)



293(d)

may be zero [Fig. 293(*c*)] or even negative [Fig. 293(*d*)]. In the latter case the potential at some point P between the cathode and the anode is minimum and is negative. At this point there exists in space an accumulation of some electrons. The cloud of electrons accumulated there is neither attracted towards the anode nor repelled towards the cathode. This cloud is known as Space charge. On one

side of the point P the electrons are attracted towards the anode whereas on the other side the electrons are repelled towards the cathode. For an electron to reach the anode it must start from the cathode with a sufficiently high velocity so as to be able to pass through the region of the retarding field caused by the space charge. Electrons emitted with less velocities are brought to rest before P; they then return to the cathode. The strength of the anode current is therefore limited by the space charge and is called space charge limited current. The value of this current was first derived by Child and is given by $i_A = AV_A^{3/2}$ where A is a constant for the valve and V_A is the anode voltage. This relation is known as "Child's Law" or Three Halves Power equation" or "Langmuir's equation." When however the emission is small as in Fig 293 (*b*) the field strength near cathode, although reduced, is accelerating and all the electrons emitted from the cathode reach the anode. Under these conditions the anode current is determined by the cathode temperature only and is known as temperature limited current. In this case the current is given by Richardson's equation $i_A = AT^2 e^{-b/T}$ where T is the absolute temperature of the cathode, A is an absolute constant and b is equal to ϕ/K where ϕ is the work function of the metal and K is Boltzmann's constant. The limiting case between temperature limitation and space charge limitation is shown in Fig 293 (*c*).

Art 225
Gas Diodes

Sometimes the diode is filled with some gas which does not combine either with the anode or with the cathode. Fig 294 shows the anode current in a diode which has an oxide coated cathode and in which there is some mercury vapour. As the voltage V_A of the anode is gradually increased from zero the anode current at first rises slowly; when V_A reaches the value of about 10 volts the current is 1 m.A. If V_A be increased slightly beyond 10 V the current suddenly rises at

an enormons rate and a glow appears inside the diode. As the electrons are speeded up to a fairly large velocity, after successive collisions the atoms of the gas begin to be ionised. The glow is due to this ionisation of gas molecules. This takes place at some distance from the cathode. Beyond this there is neutralisation of space charge by the positive ions and the potential becomes practically constant. The potential variation between the cathode

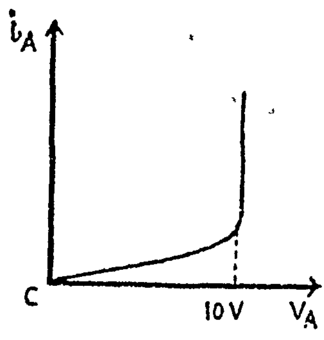


Fig. 294

and the anode is shown in Fig 295. Practically all the voltage drop across the diode occurs in a short region CP. The region CP round the cathode is known as the **positive ion sheath.** The region P to A where the voltage is nearly constant is known as **Plasma.**

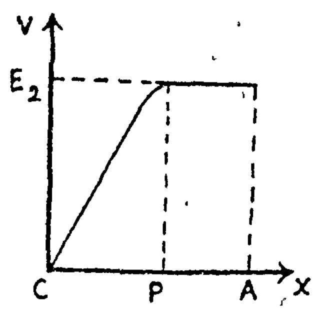


Fig. 295

Art 226
Diode as a rectifier

The diode is nowadays extensively used in rectifying alternating currents. In Fig 296 P and F are the anode and the filament in the diode valve. The filament is heated to incandescence by a low tension battery. AB and CD are two coils forming the primary and the secondary of a transformer. The A.C. current to be rectified is passed through the primary AB. Due to this an alternating potential difference is generated between the terminals of the secondary

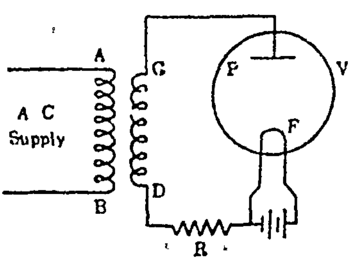


Fig. 296

CD. As C is connected to P and D to F the anode P becomes alternately positive and negative with respect to F. When P is positive there is a flow of electrons from the filament to the anode, *i. e.* there is a negative current from F to P and hence a positive current from P to F within the valve. Obviously this current flows from F to P outside the valve and passes through the external resistance R included in the circuit. When P is negative there is no flow of electrons from F to P and consequently there is no current through R. Thus an intermittent current—but always in the same direction—passes

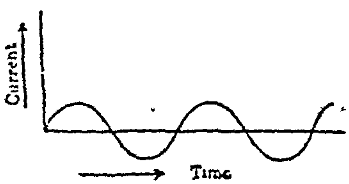


Fig. 297 (*a*)

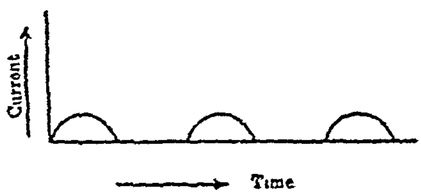


Fig. 297 (*b*)

through R. The alternating current through AB may be represented by the curve in Fig 297 (*a*); whereas the current through R is as shown in Fig 297 (*b*). Obviously the upper half of the A. C. supply produces the corresponding current in R but no current is produced in R by the lower half of the A. C. supply. Such rectification is known as *Half Wave Rectification.*

Full Wave Rectification may be produced by having two diodes. Thus in Fig 298 the terminals C and D of the secondary are respectively connected to the anodes P_1 and P_2 of the two valves V_1 and V_2. The mid point O of CD is joined to the filaments through the external resistance R. The filaments are as usual heated to incandescence by a low tension battery (not shown in the diagram). When the primary AB is connected to the A.C. supply the terminals C and D of the secondary become alternately positive and negative with respect to each other. When C is positive with respect to D the point O—the mid point of CD—becomes nega-

tive with respect to C but positive with respect to D. Thus in the valve V_1 P_1 is positive with respect to F_1 but in the valve V_2 P_2 is negative with respect to F_2. Hence there is a flow of electrons in V_1 but not in V_2. We say that the valve V_1 is conducting and the valve V_2 is non-conducting. After half a cycle when D becomes positive with respect to C the effect is reversed, *i. e.* the valve V_2 becomes conducting and the valve V_1 non-conducting. This goes on alternately. Now from Fig 298 it is obvious that whichever valve is conducting, due to flow of electrons there is always a positive current flowing through the resistance R in the direction indicated by the arrow. If the alternating current be represented by the

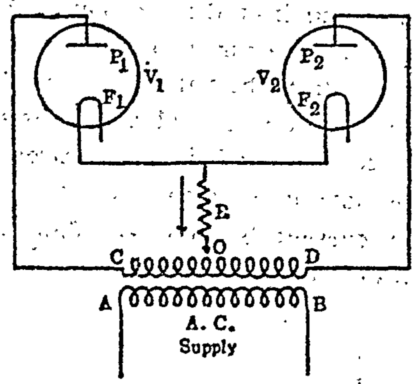


Fig. 298

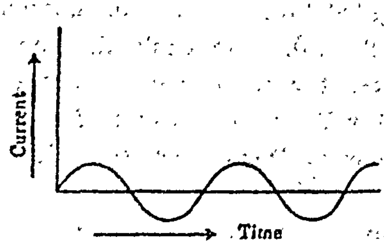


Fig. 299 (*a*)

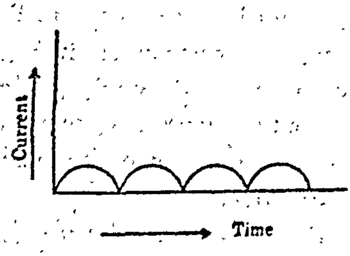


Fig. 299 (*b*)

curve in Fig 299 (*a*) the current through R is now of the form as shown in Fig 299 (*b*). Thus we get unidirectional current for both halves of the A. C. supply. This rectification is known as Full wave rectification.

Instead of having two separate diodes we may have a single diode with two anodes. Thus in the valve V [Vide Fig 300] the anodes P_1 and P_2 are respectively connected to the two terminals C and D of the secondary CD. The mid point

O of CD is connected to the filament F through the external resistance R. As the A. C. supply is applied to the primary AB the anodes P_1 and P_2 become alternately positive and negative with respect to the filament F. Hence flow of electrons takes place alternately to P_1 and P_2. In each case however a positive current flows through R in the direction indicated by the arrow. Such a diode with two anodes is known as Du-diode. Full wave rectification may thus be obtained with the help of a Du-diode.

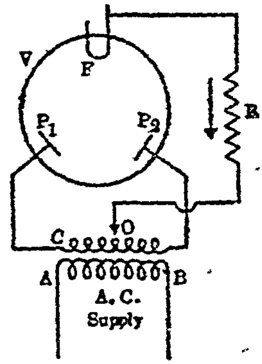


Fig. 300

It is seen from Fig 299 (b) that the current through R, although unidirectional, is not of uniform strength. The current may however be smoothed down by using a suitable choke in series and a condenser of required capacity in parallel with the resistance R.

Art 227
Triode and its constants

Let us now suppose that a wire gauze or a wire netting (technically know as a grid) is placed* between the plate and the filament. The valve is now called a triode, or a three electrode valve because it contains three electrodes, viz, the filament, the grid and the plate. Electrons from the filament F have got to pass through the meshes of the grid G before they can reach the plate P. The grid being nearer the filament it is evident that a small potential applied to the grid considerably modifies the motion

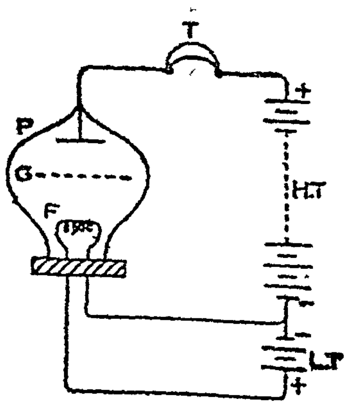


Fig 301

* The grid is usually a cylinder of wire netting surrounding the filament and the plate is another cylinder surrounding the grid.

of the electrons and therefore produces a corresponding change in the anode current. The anode current i_A is thus a function of two variables—the plate voltage V_A and the grid voltage V_G. Mathematically $i_A = f(V_A, V_G)$. Obviously if we keep the grid voltage V_G constant we can vary i_A by varying V_A. The family of curves so obtained corresponding to different constant values of V_G are known as Anode characteristic curves. These are shown in Fig. 302. Similarly if we keep the plate voltage V_A constant we can also vary i_A by changing V_G. The family of curves so obtained for different values of V_G are known as mutual characteristic curves. These are shown in Fig. 303. It will be seen that each of the mutual characteristic curves consists of a straight line in the middle together with two bends at the two ends. When the grid voltage is sufficiently high the curve becomes a straight line parallel to the V_G axis and the current is then maximum. The valve is usually worked at some particular point of a characteristic curve, the point being chosen either on the straight line portion or on the bending portion—the choice of the point depends on the purpose for which the valve is to be used. Obviously the point is determined by applying the corresponding voltages V_A and V_G.

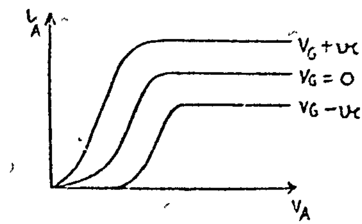


Fig. 802

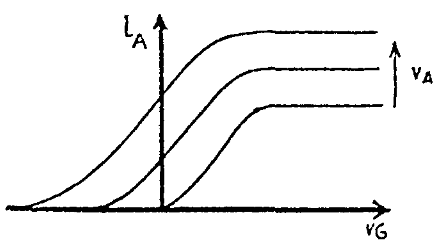


Fig. 308

When the valve is actually used we are concerned not so much with the actual values of V_A, V_G or i_A but with the changes of these three variables. Writing V_a for the small change δV_A, V_g for δV_G and i_a for δi_A we may now define the three constants of the valve.

If we keep V_G constant and if we obtain a change $i_a \left(= \delta i_A \right)$ in the anode current corresponding to the change $V_a \left(= \delta V_A \right)$ in the anode voltage we define the first constant as $\left(\frac{V_a}{i_a} \right)_{V_G = \text{const}} = R_a$ (internal resistance or anode slope resistance of the valve).

Similarly if the anode voltage V_A is kept constant and if a change i_a in the anode current be produced by a change $V_g \left(= \delta V_G \right)$ in the grid voltage we obtain the second constant as $\left(\frac{i_a}{V_g} \right)_{V_A = \text{const}} = g_m$ (Mutual conductance of the valve).

Lastly if i_A be kept constant the ratio V_a to V_g is known as voltage amplification factor. Hence the third constant is defined as $\left(\frac{V_a}{V_g} \right)_{i_A = \text{const}} = \mu$ (amplification factor).

Now since $i_A = f \left(V_A, V_G \right)$ we have

$$di_A = \left(\frac{\delta i_A}{\delta V_A}\right)_{V_G} dV_A + \left(\frac{\delta i_A}{\delta V_G}\right)_{V_A} dV_G$$

$$= \frac{1}{R_a} \cdot dV_A + g_m \cdot dV_G.$$

We may also arrange in such a way that the plate current remains unaltered, *i. e.* $di_A = 0$. Hence $\frac{1}{R_a} dV_A + g_m \, dV_G = 0$

$$\therefore \; g_m R_a = -\left(\frac{dV_A}{dV_G}\right)_{i_A} = -\mu.$$

$$i.\,e. \quad \mu = -g_m R_a \qquad \ldots \qquad \ldots \qquad (68)$$

Again since $\frac{V_a}{V_g} = \mu$ we have $V_a = \mu V_g$ *i. e.* the change V_g in the grid voltage is equivalent to the change μV_g in the anode voltage.

Art 228 Let us now consider two mutual characteristic curves for anode voltages V_A and $V_A + V_a$. Consider a point B for which the grid voltage is V_G and the anode current is i_A for the anode voltage V_A. Let the grid voltage change to $V_G + V_g$ so that the point moves to D. Hence $BC = V_g$. Since BD is small.

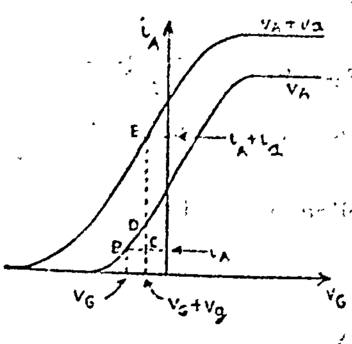


Fig. 304

$$\frac{CD}{BC}=\left[\frac{\text{Change in anode current}}{\text{Change in grid voltage}}\right]_{V_A}=g_m$$

$$\therefore\quad CD=g_m.\ V_g.$$

Also along DE grid voltage is constant. Hence

$$DE=\left(\frac{\delta i_A}{\delta V_A}\right)_{V_G}\quad\times\quad V_a=\frac{V_a}{R_a}$$

Since $\qquad CE=CD+DE$

$$\therefore\quad i_a=g_m\ V_g+\frac{V_a}{R_a}\qquad\qquad(69)$$

Art 229
Tetrode

In a triode the anode not only acts as the collector of electron current but also controls the strength of the current. It is possible to separate the two functions by introducing a second grid between the original grid and the anode. The original grid G_1 is now called the control grid and the new grid G_2 is known as the screen grid. This screen grid is maintained at some positive potential. For a given value of the control grid voltage the presence of the screen grid helps in overcoming the effect of the space charge; the flow of electrons leaving the cathode thus depends mainly on the voltage of the screen and very little on that of the anode.

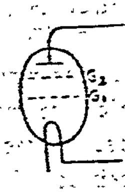


Fig. 805

Art 230
Pentode

The electrons from the filament as they reach the anode, strike the anode with fairly large velocities. This impact may cause secondary emission of electrons from the anode. In a triode the control grid being negative all these secondary electrons emitted from the anode are repelled by the grid and are pulled back to the anode. There is therefore no effect on the anode current. In a tetrode however the screen being at a positive potential

some of the secondary electrons from the anode may go to the screen thereby causing some reduction of the anode current. This effect can be eliminated by the insertion of a third grid G_3 between the screen and the anode. This additional grid known as the suppressor grid is usually maintained at zero potential, *i. e.* the potential of the cathode. When the screen and the anode are at positive potentials the space within the meshes of the new grid is at a potential somewhat above zero. With this arrangement the field outside the anode—even with low anode voltage—is such that secondary electrons return to the anode.

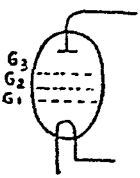


Fig. 306

We shall see in the next few article that wireless transmission and reception depend mainly on the presence of the original grid, though of course the screen grid and the suppressor grid are often used as practical improvements.

Art 231
Transmisson of electromagnetic waves

Wireless transmission, as the very name implies, is transmission without any wire. The message to be transmitted—music, speech or telegraphic code—is converted into electromagnetic* waves which propagate out into space. At the receiving station these waves are reconverted into sound and the message is raceived. There are thus two stages,—(1) transmission or generation of electromagnetic waves, and (2) reception or conversion of electromagnetic waves into sound.

As early as the year 1863 Maxwell predicted on theoretical gronnds that electromagnetic waves may be generated by rapid oscillations of electric potential difference between the two terminals of a sparking plug. Thus in Fig 307 when the key is pressed making contact with the stud S_1 the condenser C is charged by the battery B. When the key is released by the action of the spring S, contact is made with

* In such waves pulses of electric strain as well as pulses of magnetic strain follow one another. That is why these waves are called electromagnetic waves.

the stud S_2 and the condenser is discharged. In Arts 157 and 159 we have seen that both charging and discharging of a condenser are oscillatory when the capacity C, the inductance L and the resistance R of the circuit satisfy the relation $R < 2\sqrt{\frac{L}{C}}$. In wireless circuits resistance is kept extremely low so that this relation is satisfied and the frequency of oscillation is $\frac{1}{2\pi\sqrt{LC}}$ [Vide equation (59a), Art 157]. If a spark gap G is placed in parallel with the condenser [Fig 307] a high frequency alternating potential difference is produced at its two terminals ; electromagnetic waves are therefore generated at each charge and again at each discharge of the condenser. Suitably altering the values of L and C this frequency may be made as large as is desired.

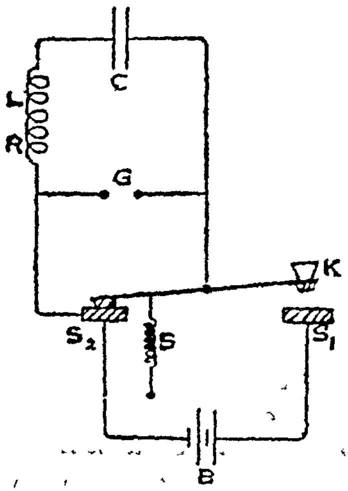


Fig. 307

Art 232 In 1888 Hertz first demonstrated experimentally the existence of such waves. About the year 1895 Marconi discovered that if the sparking plug be replaced by a vertical wire known as aerial (supported by suitable masts) connected to the Earth considerable energy may be sent out to space as electromagnetic waves. It must however be clearly understood that in order that considerable energy is transmitted as electromagnetic waves it is absolutely necessary that frequency of oscillations of electric potential difference must be very high :—of the order of 10^6 or more per sec. Such frequency is known as Radio frequency (R. F.)

Clearly a pulse of electromagnetic waves starts at the make and again another pulse at the break of the circuit. These pulses follow one another in space and they travel with the velocity of light. The interval between two successive pulses

obviously depends on the frequencey of making and breaking the circuit. This latter frequency is comparatively small and

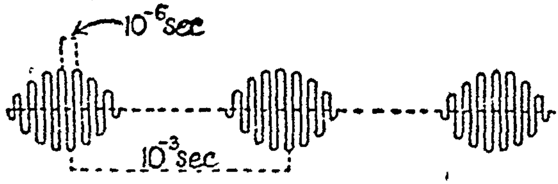


Fig. 308

is known as Audio frequency (A. F.). Obviously this can be adjusted.

Art 233
Reception of electromagnet c waves

We now come to the reception of electromagnetic waves. This reception is exactly analogous to the case of resonance in sound. When a tunning fork vibrates it sends out waves in air. If in the neighbourhood there be a sonometer wire tuned to the same frequency we know that the wire responds, *i. e.* the wire begins to vibrate. Thus the vibration of a tuning fork generates waves in air and these waves in their turn produce vibrations in a stretched wire tuned to the same frequency. Exactly in a similar way electric oscillations in a circuit generate electromagnetic waves in space. These *waves are of the same frequency as that of oscillations* in the transmitting circuit. When these waves are incident on another circuit (receiving circuit) tuned to the same frequency the second circuit responds, *i. e.* electric oscillations of the same frequency are generated in the receiving circuit. At the receiving station therefore we must have a circuit tuned to the same frequency as that of the transmitting circuit. Since the frequency of a circuit—when resistance is extremely small —is equal to $\frac{1}{2\pi\sqrt{LC}}$ [Vide (59*a*) Art 157] it is clear that in both transmitting and receiving circuits the product LC must be the same. By *altering* the values of L and C the same receiving apparatus may be made to receive different wave-

lengths.* This is what we call tuning of a radio set to different wavelengths.

Art 234
Detection and Amplification

Now electric oscillations generated in the circuit are of high frequency (10^6 or more)—the same as that in the transmitting station. This frequency is too high to work an ordinary instrument such as a galvanometer, a telephone or a loudspeaker. These high frequency oscillations must therefore be converted into low frequency before they can produce corresponding sound ; this conversion of high frequency into low frequency is what is technically known as "detection". Again as energy spreads out in different directions from the transmitting station a very small amount of energy comes to the receiving station and electric oscillations produced thereby are of small amplitude. These oscillations must also be amplified. Thus two things are necessary at the receiving station—detection and amplification. With the help of crystals detection is possible. Certain crystals have got the property that they allow currents to flow through them in one direction only ; currents in the reversed direction meet with an infinite resistance so that they are quenched or destroyed. If therefore one such crystal be included in the receiving circuit one half of electric oscillations is quenched and currents are made unidirectional. The way how a telephone also included in the circuit responds to these currents may be understood from an examination of Fig. 309. Fig (*a*) represents a series of incoming waves.

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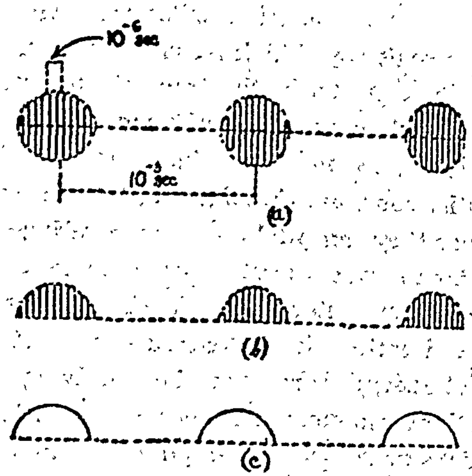
* As is well known wavelength and frequency are connected by the relation wavelength $= \frac{\text{Velocity}}{\text{Frequency}}$; in the case of wireless transmission the velolcity is the same as that of light, *viz.* 3×10^{10} cms per sec. Hence a wavelength of 300 metres corresponds to a frequency of $\frac{3\times10^{10}}{300\times100}=10^6$ oscillations per sec. But a kilocycle is 10^3 oscillations and a megacycle $=10^6$ oscillations. Hence a wavelength of 300 metres is equivalent to a frequency of 1000 kilocycles or one megacycle per sec. Similarly a wavelength of 30 metres corresponds to 10 megacycles per sec and so on. Frequencies also may similarly be converted into corresponding wavelengths.

Frequency of oscillations in each wave is very high, 10^6 or more (R. F.), whereas frequencies of waves following one



Fg. 309

another are comparatively lower—10^3 or less (A. F.) When these waves are incident on the receiving circuit they produce corresponding electric oscillations but due to the crystal one half of oscillations is cut off so that they are now as shown in Fig (*b*). The telephone is unable to respond to the separate high frequency pulses in each half wave; these separate pulses jointly produce the same effect as that of a unidirectional varying current [Fig. (*c*)]. This varying current of short duration, due to each wave, produces a short sound in the telephone, *i. e.* a signal is produced in the receiving station.

A receiving set using a crystal for detection is known as a crystal set. Crystals used for this purpose are generally divided into two classes :—(1) Catwhisker type, *i.e.* those which require a fine contact with a metal piece and (2) Perikon type, *i.e.* those which require another crystal in contact with them. Silicon, iron pyrites, carborundum, galena etc. belong to the former class and Zincite, bornite, copper pyrites etc. are examples of the latter. Galena may be used also as a perikon.

For crystals of the former class the nature of the metal piece is of importance for the achievement of good results. Thus carborundum

and silikon work best with steel, iron pyrites with gold, galena almost equally well with gold, copper or brass and so on, For crystals of the latter class also suitable combinations of crystals produce best results , thus Zincite works best with bornite, tellurium with zincite and so on.

Art 235 Wireless Telegraphy

The waves which we have discussed in the preceding articles are known as damped waves ; they are suitable for wireless telegraphy. Clearly intervals between successive waves are the same as those between successive make and break of the transmitting circuit. As we have seen in the last article each wave produces a signal (of very short duration) in the receiving station. The interval between two successive signals is therefore the same as that between make and break of the transmitting circuit ; clearly this can be easily adjusted. We have thus a method of transmitting telegraphic message. A short interval is technically known as a dot and a long interval—usually three times as large as a short interval—is what we call a dash. By combining dots with dashes various letters of the English alphabet can be signalised. Thus in Morse code 'A' is represented by .—, 'B' by — .., 'C' by —.—. and so on.

Thus a crystal set can detect high frequency oscillations but it can in no way amplify them. In the next few articles we shall see that both detection and amplification are possible by means of triodes.*

Art 236 Detection

At the receiving station electro-magnetic waves are incident on a tuned aerial circuit containing an inductance L and a variable condenser C producing thereby oscillating potential difference between the terminals of the inductance L. This inductance L being coupled with another coil N oscillating potential difference is produced between the terminals of N also. This is ultimately applied between the grid and the filament of a valve.

* Recently another instrument known as transistor is coming into use. It is gradually taking the place of the valve set.

Thus we may say that with the arrival of incoming waves an E.M.F. $E_0 \sin pt$ is applied to the grid, This necessarily affects the anode current. But as the frequency $\frac{p}{2\pi}$ is very high the resultant effect on the anode current due to one complete wave is practically nil. To get some resultant effect it is necessary that a symmetric variation of grid voltage should produce a non-symmetric variation of the anode current so that the mean value of the anode current when the E. M. F. applied to the grid is present, is different from that when the E.M.F. is absent. This is what is known as 'detection' by the valve.

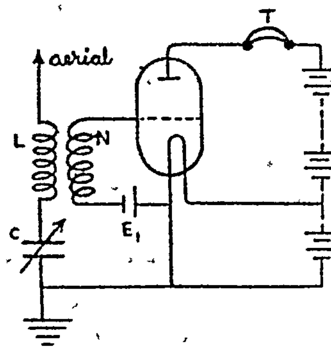


Fig. 310

Art 237
Triode as a detector

For the purpose of 'detection' a valve for which the bending portion of the mutual characteristic curve is prominent, is used A point $A^{\prime}$ on the bending portion is taken as the working point. Corresponding to this point a negative E.M.F. equal to OA [Fig 311] is applied to the grid by a battery E_1 [Fig 310] so that the anode current is reduced to $AA^{\prime}$. Let this be denoted by a_0. With the arrival of the incoming waves the grid voltage changes by V_g $(=E_0 \sin pt)$. The anode current may be represented by $i_A = a_0 + a_1 V_g + a_2 V_g^2 + \ldots$ For all practical purposes very good approximation is obtained if we retain only the first three terms. Thus we may take

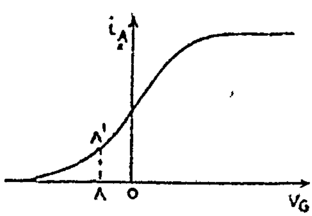


Fig. 311

$$i_A = a_0 + a_1 V_g + a_2 V_g^2$$

with the arrival of the incoming waves the anode current is given by

$$i_A = a_0 + a_1 E_0 \sin pt + a_2 E_0^2 \sin^2 pt$$
$$= a_0 + a_1 E_0 \sin pt + \tfrac{1}{2} a_2 E_0^2 (1 - \cos 2\,pt)$$

The current thus consists of two parts, firstly a high frequency component $a_1 E_0 \sin pt - \frac{1}{2} a_2 E_0^2 \cos 2\,pt$; this has no resultant residue. Secondly there is the steady portion $a_0 + \frac{1}{2} a_2 E_0^2$. Of this the part a_0 was present when the signal waves were absent. Thus the net effect of a signal wave is $\frac{1}{2} a_2 E_0^2$. Every time a pulse of high frequency wave is incident on the receiving aerial there is this resulting change in the anode current. If therefore a telephone T be included in the anode circuit [Fig 310] a short sound is thereby produced in the telephone. Since pulses of waves follow one another at a rather low frequency [Fig 309 (*a*)] short sounds are heard in the telephone at low frequency. As previously discussed in Art 235 intervals between successive sounds may be adjusted and thereby telegraphic messages may be sent.

It is to be noted that the resultant change in the anode current, viz., $\frac{1}{2} a_2 E_0^2$ varies as the square of the E.M.F and so the detector valve acts more efficiently if strong signals are received. We shall see in the next article that the strength of a signal may be amplified by a valve. This may be done either before or after detection. When very weak signals are received it is advantageous if they are first amplified in the high frequency stage, *i. e.* before detection. After detection they may be further amplified by successive valves; utlimately the signal becomes so strong that a loud-speaker may be used.

Art 238
Triode as an amplifier

We shall now discuss how a triode can amplify the incoming signals. For this purpose the valve for which the straight line portion of the mutual characteristic curve is prominent, is used. If B^1 be the centre of the straight line portion a negative E.M.F. OB corresponding to the point B^1 [Fig 312 (*a*)] is applied to the grid by the battery E_2 (Grid

bias battery) [Fig 312 (*b*)]. The anode current is therefore B′. With the arrival of signals the grid voltage fluctuates by an amount V_g. This change is equivalent to the change

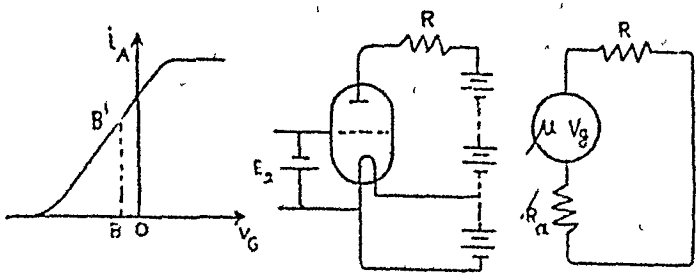


Fig. 312 (*a*) Fig. 312 (*b*) Fig. 312 (*c*)

μV_g in the anode voltage [Art 227] where μ is the voltage amplification factor. Thus so far as changes in the anode current are concerned we may suppose that the valve acts as a source of fluctuating potential μV_g. The valve has also an effective internal resistance R_a [Art 227]. If R be the external resistance the whole of the anode-filament circuit may be represented as in Fig 312 (*c*). As we are concerned only with *changes* in the voltage and in the current the battery is omitted in Fig 312 (*c*) The change in potential μV_g produces a change i_a in the anode current. This is given by $i_a = \dfrac{\mu V_g}{R+R_a}$. This in turn produces a change δV in the potential drop across the resistance R given by $\delta V = R i_a = \dfrac{\mu R V_g}{R+R_a} = G V_g$ where $G = \dfrac{\mu R}{R+R_a}$.

Thus the fluctuating potential V_g applied to the grid is amplified to the fluctuation GV_g between the terminals of R. This is available for application to the grid of the next valve. G is called the gain of the amplifier for this stage. Obviously G is less than the amplification factor μ of the valve. If R be sufficiently large G of course becomes very much the same as μ. In practice however R is made very nearly equal to R_a and in that case $G = \dfrac{\mu}{2}$.

to the terminals of R_4 responds to these oscillations. As will be seen the two valves are connected by the transformer R_1/R_2. This sort of coupling is known as transformer coupling. There are in reality three transformers in this circuit,—L/N, R_1/R_2 and R_3/R_4. Usually all of them are step-up transformers and oscillations are magnified by them also.

The terminals of R_4 instead of being connected to the telephone T may be connected through a suitable grid bias battery to the grid and the filament of another amplifier valve and further amplification may be produced. In this way by using a number of amplifier valves it is possible to amplify the oscillations to any desired extent.

In actual practice oscillations generated by the incident waves are at least once amplified in the high frequency stage. They are then converted to low frequency by means of the detector valve. In this low frequency stage they are again amplified successively by two or more valves until the oscillations are so strong that a loudspeaker may be satisfactorily worked.

Art 240
Wireless Telephony

The waves which we have so far discussed are damped waves. There is an interval between two successive waves and naturally there is a corresponding interval between two sounds in the telephone produced by these waves. Such waves are suitable for wireless telegraphy but not at all suitable for transmission of speech or music. For this latter purpose we require continuous waves. Speech or music produces waves in air; they can be converted into electric oscillations But sound waves being of low frequency electric oscillations produced thereby are also of low frequency. Very little energy can be transmitted into space as electromagnetic waves by such low frequency oscillations. Sound therefore cannot directly be converted into electromagnetic waves.

A high frequency continuous wave—known as a 'carrier wave'—is therefore first generated at the transmitting station.

We shall see in the next few articles that sound waves modify this carrier wave and the modified carrier wave is received at the receiving station and sound is reproduced there.

Art 241
Triode as an oscillator

This carrier wave, *i. e.* high frequency electromagnetic wave can be generated by a triode. A pendulum may be set swinging by giving a blow to the bob. But due to air resistance and friction the swing gradually dies down and the pendulum ultimately comes to rest. The swinging however may be maintained indefinitely if suitable blows are given to the bob at the end of each oscillation, *i. e.* if suitable energy be supplied to the pendulum from an external source to make up for the loss. In a similar way in an electric circuit electric oscillations may be generated in a variety of ways. Such oscillations may be represented by

$i = i_0 \, e^{-\frac{Rt}{L}} \sin \frac{t}{\sqrt{LC}}$ where R, L, C and t have their usual meanings. In an electric circuit R may be kept small but can never be made absolutely zero. The oscillation therefore dies down because of the factor $e^{-\frac{Rt}{L}}$. As will be seen below with the help of a triode valve it can be so arranged that the oscillation generates another E. M. F. by which the effect of the resistance R is annulled. L and N are two inductances coupled with each other with mutual inductance M. C is a variable condenser. Electric oscillations are somehow generated in the L – C circuit. Due to these oscillations the grid voltage fluctuates. Small changes in the grid potential cause large changes in the anode current. As the coil N is connected to the anode through a condenser, by means of

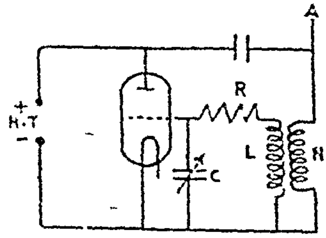


Fig. 314

the mutual inductance M a little of the energy of the anode circuit is fed back to the grid circuit so as to make up for the loss of the energy lost in the resistance R. This is equivalent to reducing the resistance by $\frac{\mu M}{CR_a}$ where μ is the voltage amplification factor and R_a is the anode slope resistance of the valve. The coupling between L and N is such, *i. e.* the value of M is so adjusted that the resistance R is exactly equal to $\frac{\mu M}{CR_a}$ and is therefore reduced to zero. In the L–C circuit the current i therefore becomes $i_0 \sin\frac{t}{\sqrt{LC}}$. The time decay factor having thus completely disappeared it follows that electric oscillations once set up in the circuit continue with a constant amplitude, *i. e.* the valve acts as a producer of electric oscillations of frequency $\frac{1}{2\pi\sqrt{LC}}$. By varying L and C any desired frequency—as high as is necessary—may be produced. If the inductance N be connected to the aerial as shown in Fig 314 high frequency continuous electromagnetic waves may be transmitted into space.

Art 242
Modulation

We have stated earlier that for the purpose of wireless telephony a continuous high frequency carrier wave is modified by a low frequency audio wave. The way how this is done is shown in Fig 315.

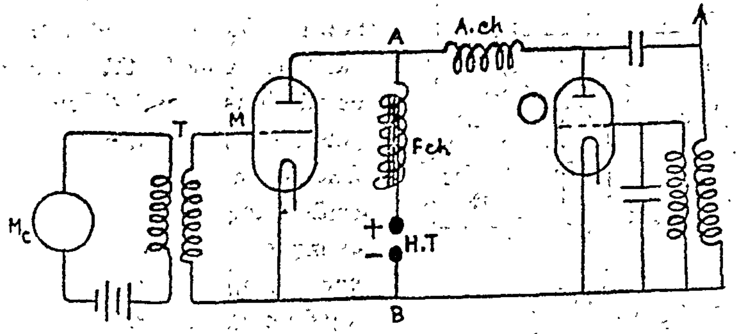


Fig. 315

The microphone M_c is connected through the microphone transmitter T to the grid of the modulating valve M. O is the main oscillating valve (oscillator) which produces high frequency continuous waves. The anodes of the two valves M

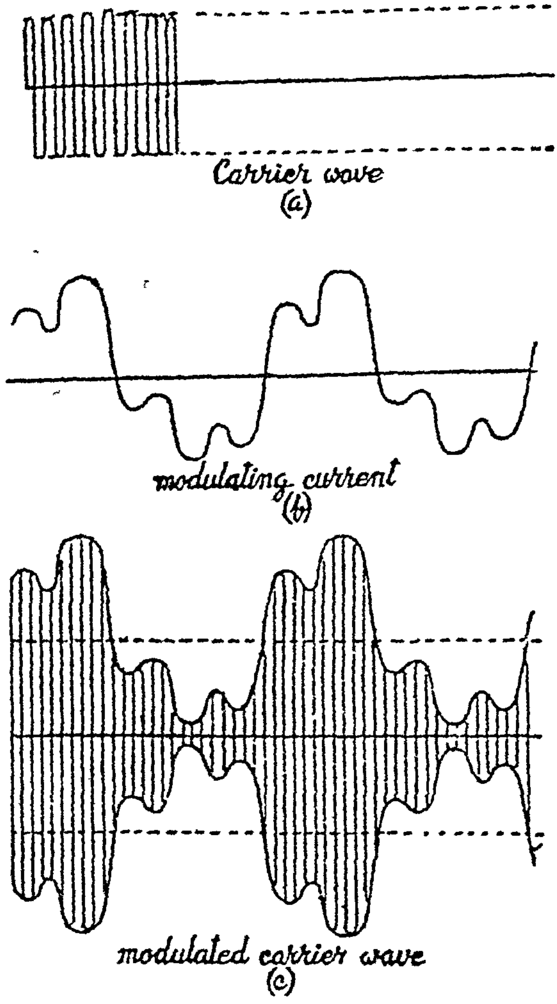


Fig. 816

and O are connected to a common high tension supply (H.T.) with which an iron cored choke coil F. *ch.* is placed in series

When sound waves of frequency w are incident on the microphone voltage variations are set up in the grid circuit of M and these cause the anode current of M to fluctuate at the same frequency. Now the presence of the choke coil F. *ch.* in series with H. T. supply makes the current in the high tension circuit (AB) constant. Since this current is equal to the sum of currents flowing to the anodes of the valves O and M it follows that the sum of these two latter currents remains constant. Hence when the anode current in the valve M is caused to fluctuate at the frequency w the supply of current to the valve O must also fluctuate at the same frequency. This fluctuation of the anode current in O causes variation of the amplitude of high frequency oscillations produced by the oscillating valve O and thus produces the desired modulation. In order that the high frequency oscillation produced in O is not transmitted to the valve M an air-cored choke (A. *ch*) is inserted as shown in the diagram. Since the impedance of a choke depends on the frequency of the current the air choke (A. *ch.*) effectively prevents the transmission of the high frequency oscillation to M but allows practically free passage to low frequency oscillations (generated in M) going to O. Thus the carrier wave is modified. If the carrier wave [Fig 316 (*a*)] is represented by $E_0 \sin pt$ the modified wave [*Fig* 316 (*c*)] is represented by $E_0 (1 + m \sin wt) \sin pt$. The quantity m represents the magnitude of the modulation effect and when expressed as percentage it is known as the percentage modulation.

Art 243
Reception of modulated wave

We have seen in Art 237 that if the characteristic of a detector valve be given by $i_A = a_0 + a_1 V_g + a_2 V_g^2$ and if a voltage $E_0 \sin pt$ be applied to the grid the resulting change in the anode current is equal to $\frac{1}{2} a_2 E_0^2$. When a modulated wave $E_0 (1 + m \sin wt) \sin pt$ is received by the aerial the constant amplitude E_0 is replaced by $E_0 (1 + m \sin wt)$. Hence the change in the anode current is $\frac{1}{2} a_2 E_0^2 (1 + m \sin wt)^2 = \frac{1}{2} a_2 E_0^2 (1 + 2m \sin wt + \frac{m^2}{2} - \frac{m^2}{2} \cos 2wt)$. The varying portion

of this change is evidently $\frac{1}{2} a_2 E_0^2 (2m \sin wt - \frac{m^2}{2} \cos 2wt)$.

The first term viz. $\frac{1}{2} a_2 E_0^2 . 2m \sin wt$ is of frequency w. The telephone (or louspeaker) placed in the anode circuit traversed by this current responds to the frequency w. This being of the same frequency as that originally impressed on the carrier wave at the transmitting station the original sound is reproduced. There is however the second term $\frac{1}{2} a_2 E_0^2 \frac{m^2}{2} \cos 2wt$ of frequency $2w$. This evidently produces an octave of the original sound. This is however not usually of much importance; for, in the ultimate analysis of a complex note there are usually octave components and the introduction of an extra double frequency component does not matter much. As this second term—distortion term, as it is called—contains square of the modulation factor m its effect can be minimised by making m rather small. Usually it is never greater than 60%.

Exercise XXII

1. What is an electromagnetic wave and why is it so called? Explain briefly how it can be produced.

2. Discuss briefly the principle of reception of wireless signals by an electric circuit.

3. Write short notes on Thermionic valves.

C. U. 1941, 1946

4. What is a diode valve and explain how it can rectify alternating current. What is meant by (a) Half wave rectifition. (b) Full wave rectification. Explain what modification is necessary in a simple diode valve to obtain Full wave rectification.

5. What is a triode valve and why is it so called? How can it be used for rectifying oscillations?

6. What is meant by the characteristic curve of a triode valve? Discuss its importance in (a) rectifying and (b) amplifying electric oscillations.

C. U. Question

1965. What do you understand by the term 'Detection of electromagnetic waves'? Explain the working of a diode as a detector.

1966. What do you mean by thermo-ionic emission? Describe a triode valve. How can you determine its amplification factor? Give details.

1966 Describe the construction of a triode valve. Define the constants of a triode valve and show how these are inter-related.

The anode slope resistance is 20,000 ohms and its amplification factor is 30. Find its mutual conductance.

1968. Write notes on (*a*) Triode valve as an amplifier and (*b*) Simple transmitting system.

1969. Describe the construction of a triode valve. What are its constants and what are the relations among them?

Explain how a triode valve can be used as an amplifier.

1970. Write notes on (*a*) Triode as an oscillator and (*b*) Principle of a simple radio receiving system.

1971. Describe the construction of a Diode and explain how it can be used as a rectifier.

1973. Write short notes on "Principle of radio reception."

CHAPTER XXIII

MODERN TOPICS

Art 244
Faraday effect

Faraday had long ago suspected that there is some intimate connection between Light and Magnetism. But having no very powerful electro-magnet at his disposal he could not proceed very far. One effect however he discovered and that is now known as Faraday effect.

A block G of dense glass is placed between the poles of an electromagnet and by boring holes through the pole pieces a ray of light is made to pass through the block along the lines of force. The ray is polarised by the polarising Nicol P and is analysed by the analysing Nicol A. Before switching on the electromagnet the analyser and the polariser are adjusted until they are crossed so that light is completely quenched by the analyser. It was noticed by Faraday that as soon as the electromagnet is switched on light re-appears in the analyser. By rotating the analyser light can again be extinguished. This shows that the plane of polarisation of the beam passing through the glass block is rotated by the magnetic field. It was gradually observed that this Faraday effect is not peculiar to glass alone; various substances such as quatz, methyl alcohol, water etc. exhibit this phenomenon.

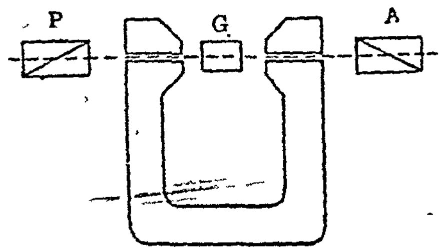


Fig. 817

The rotation of the plane of polarisation was studied extensively by Verdet for different substances and for different wavelengths of light. It was found by him that the angle of rotation is given by

$$\theta = \mathrm{KLH}\frac{\mu^2}{\lambda}\left(\mu-\lambda\frac{d\mu}{d\lambda}\right)$$

where L is the length of the substance (along the field), λ the wavelength, μ the refractive index of the substance and K a constant depending upon the nature of the substance. The ratio $\frac{\theta}{LH}$ *i. e.* the rotation produced by a unit field when a unit distance is traversed by the ray of light, is now known as Verdet's constant for the substance used.

It is wellknown that the plane of polarisation of a plane polarised beam is rotated when the beam is passed through various optically active substances. This rotation is however different from that by a magnetic field in one important respect. In the case of optically active substances the rotation depends in some way upon the arrangement of molecules in the substance. Hence if on emergence the beam is reflected back on its own path so that the beam traverses the substance twice, once in each direction, the rotation produced in the two cases are equal but *opposite in direction*: the resultant rotation is therefore nil. On the other hand in the case of the magnetic field the direction of rotation depends upon that of the magnetic field. Thus if the beam of light passing through the substance along the magnetic field be reflected back on its own path the rotations produced in the two cases are both *in the same direction*; the resultant rotation is therefore doubled.

Art 245
Zeeman effect

As early as the year 1870 Faraday had the prophetic vision that spectral lines emitted by a substance can be modified by a powerful magnetic field. He could not however discover this effect. We now know that he was unsuccessful only because he had not a powerful electromagnet at his disposal. In the year 1896 Zeeman discovered the effect long sought for by Faraday. A Bunsen burner fed with sodium salt is placed between the pole pieces of a powerful electromagnet. Light emitted by the sodium vapour is analysed by a spectroscope of high resolving power. Obviously light may be received by the spectroscope quite

easily in a direction *perpendicular to the magnetic field.* In order that light from the Bunsen burner may also be received by the spectroscope in the direction of the magnetic field, holes are bored in the polepieces along the magnetic field. Thus light is analysed both along the magnetic field and also in a direction perpendicular to the magnetic field. As is well known sodium light consists

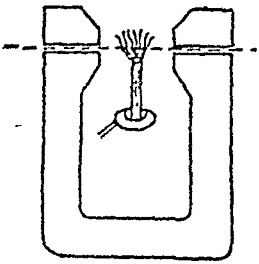


Fig. 318

of two close lines D_1 and D_2 of wavelengths 5890 and 5896 Angstrom units. When the electromagnet is not switched on, in both cases the usual D_1 and D_2 lines appear in their proper places. When however the electromagnet is switched on a curious phenomenon is observed When light is analysed in the direction perpendicular to the magnetic field, each of D_1 and D_2 lines is found to be split up into three lines; the middle one is in the same position as the original D_1 or D_2 line and two other lines appear at short but equal distances on two sides of the central line. And what is more curious— all the three lines are found to be plane polarised. But they are not polarised in the same plane; the direction of vibration of the central line is parallel to the magnetic field but that of the outer lines is perpendicular to this.

When however light is analysed along the magnetic field each of D_1 and D_2 lines is observed to be split up into two lines (and *not three*, as in the former case)—these two lines being in the same positions as the outer lines in the former case. The lines however are now found to be both circularly polarised—one right handed and the other left handed.

This phenomenon now known as Zeeman effect is apparently very surprising. In 1900 Lorentz offered an explanation of this on the classical theory. But gradually more complex phenomena were discovered in the case of many other substances and the classical theory failed to

explain them. The correct explanation of all these phenomena is based on Quantum theory.

Art 246
Photo-electricity

As early as the year 1887 Hertz found that in a vacuum tube even if the potential difference between the electrodes be quite small, a discharge passes if the cathode be exposed to ultra-violet light. In 1888 Hallwachs discovered that when ultraviolet light is incident on an insulated zinc sheet the sheet gradually acquires positive charge. If the sheet be initially negatively charged, with the incidence of ultraviolet light the sheet gradually loses its negative charge and ultimately becomes positively charged. On the other hand if the sheet be originally positively charged no change is observed.

These phenomena are not peculiar to zinc alone. Almost all metals exhibit these phenomena when ultraviolet light is incident on them. In the case of alkaly metals even visible light produces the same effect. All these phenomena have been correctly explained on the hypothesis that negatively charged particles of electrons come out of the metallic surface when light is incident on it. For a good many years the correct relation between the velocity & number of electrons on one hand and the wavelength & intensity of the incident light on the other, could not be correctly established. It was of course obvious that electrons in the atoms of the metallic surface somehow absorb energy from the incident beam and come out with a certain velocity. On this view the velocity should depend on the energy of the incident beam. Now according to the wave theory the energy of a beam of light depends on the intensity. It was accordingly expected that with the increase of the intensity the emissive velocity of electrons would also increase. But such was not found to be the case. Rather the velocity seemed to depend not on the intensity but on the frequency of the incident beam Experiments conducted during these years by different scientists often contradicted one another and nothing was decisive; everything seemed to be in the melting pot.

At last in the year 1905 Einstein published what is now regarded as the famous photo-electric equation. It has been stated in Art 199 that in order to explain black body radiation Planck had put forward the hypothesis that energy is emitted not continuously but in bundles of energy or quanta. Einstein took up this idea and went one step further; he boldly asserted that energy is also absorbed not continuously but in quanta, each quantum being nothing but a bundle of energy $h\nu$, where ν is the frequency and h is Planck's constant. Planck's hypothesis shook the very foundation of the wave theory of Light and Einstein's equation practically gave a death blow* to it. If at the start energy is emitted discontinuously in bundles of energy or quanta as stated by Planck and at the end also if it is absorbed discontinuously as asserted by Einstein, are there any valid reasons to suppose that in the intermediate stage, *i. e.* the stage between emission and absorption energy is propagated continuously as waves? Obviously the answer is in the negative. The only conclusion possible is therefore that throughout its career light consists of bundles of energy or quanta. Each quantum is also called a photon. Thus according to this theory—Quantum theory—light is nothing but a stream§ of photons.

Since one quantum of energy (or photon) is equal to $h\nu$ Einstein supposed that either the whole of $h\nu$ is absorbed or none at all. According to Einstein when this energy $h\nu$ is absorbed, a portion P is spent in detaching an electron from its parent atom and the remaining portion, viz. $h\nu-\mathrm{P}$ is

* The wave theory of light still held its ground because it successfully explained all phenomena in connection with Interference, Diffraction and Polarisation. Although the quantum theory could explain quite satisfactorily Photo-electric effect and other such phenomena, it could not explain Interference, Diffraction and Polarisation. Thus apparently there were two theories, each theory explaining one group of phenomena but not the other. From the year 1926 however attempts have been made fairly satisfactorily to unify these two rival theories into one theory. Vide Optics by D. P. Acharya.

§ It will be seen that the old corpuscular theory of Newton is thus revived in a modified form in the quantum theory.

utilised in creating the kinetic energy of the electron. P is obviously a constant depending on the nature of the metal. Thus if m be the mass of the electron and v the velocity with which the electron comes out of the metal surface, its kinetic energy is $\frac{1}{2}mv^2$. Hence $h\nu - P = \frac{1}{2}mv^2$.

This is the famous photo-electric equation of Einstein. It should be remembered that at the time when this equation was published there was not an iota of evidence in its support. It was only the prophetic vision of the great scientist Albert Einstein — the greatest scientist of modern age—that could predict this famous relation. Even after this equation was published for a number of years it could neither be proved nor disproved by experiments—experimental difficulties were so great. At last seven years later in the year 1912 the famous experimental physicist Millikan of America—the king of experimenters—overcame all difficulties and definitely established the correctness of this equation. From his experiments he found out the value of Planck's constant h and this also agreed with the value already found out from experiments on black body radiation.

If the constant P be replaced by $h\nu_0$ where ν_0 is a new constant depending on the nature of the metal the equation takes the shape $h(\nu - \nu_0) = \frac{1}{2}mv^2$. Obviously ν cannot be less than ν_0 as otherwise $\nu - \nu_0$ becomes negative and v becomes imaginary, *i.e.* electrons do not come out of the metal when ν is less than ν_0. ν_0 is called the *threshold frequency* for the metal. Thus we have the following laws for Photo-electric emission :—

(1) The *velocity* of the electrons which come out of the metal surface is independent of the intensity of the incident beam. The *number* of electrons is however proportional to the intensity.

(2) The velocity depends on the *frequency* of the incident beam. The relation between the square of the velocity and the frequency is linear.

(3) For every metal there is a threshold frequency and

frequency of the incident beam must be greater than the threshold frequency; otherwise electrons are not emitted.

For alkaly metals the threshold frequency lies in the visible region. This is why electrons are emitted from alkaly metals even when visible light is incident on them.

Art 247 Photo-electric cell The phenomenon of photo-electric emission has been utilised in many ways the most important of which is perhaps the construction of photo-electric cells or Photo-cells. A thin film of an alkaly compound—usually caesium oxide—is deposited on the inner side of a non conducting surface bent in the form of one half of a hollow cylinder. A metal rod R (known as collector) is placed along the axis of the cylinder and a fairly high potential difference is applied between the collector R and the inside coating C of the alkaly compound deposited on the cylindrical surface, the collector being positive with respect to the alkaly coating. The whole thing is enclosed in a glass envelope. The entire thing is known as a photo-electric cell or a photo cell. When a beam of light is incident on the metallic coating, electrons—these are sometimes called photo-electrons—come out and due to the electric field they are drawn towards the collector R. If a circuit be completed as shown in the diagram a negative current flows in the circuit in the direction from C to R; this is equivalent to a positive current in the opposite direction. If a galvanometer G be included in the circuit a deflection is produced therein. If the incident light be of varying intensity the strength of the current also becomes correspondingly variable.

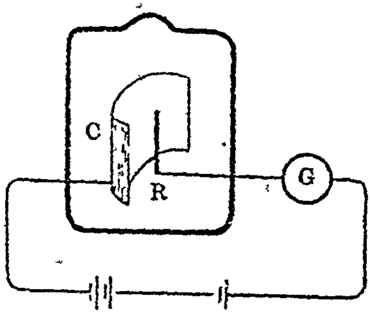


Fig. 319

Photo-cells are generally completely exhausted. But in some cases they are filled with some inert gas, such as argon.

The photo electrons while travelling towards the collector collide with argon atoms and dislodge electrons from them. These electrons dislodged from argon atoms are also drawn towards the collector. The positively charged ionised argon atoms also move towards the alkaly coating thus producing a positive current from R to C. Thus total current generated in the photo cell is increased 10 to 100 times the initial photo current.

In talkies a picture film and the corresponding sound film are placed side by side. When a beam of light passes through both portions light passing through the picture film produces the picture on the screen in the usual way by a magic lantern. The other portion which passes through the sound film, is made to be incident on a photo-electric cell. The transparency of the sound film in different portions depends upon the sound originally produced during the production of the talky film. Light which passes through the sound film becomes therefore of varying intensity, the variation corresponding to the original sound. A current of correspondingly varying strength is thus produced in the photo-cell circuit. If, instead of a galvanometer, a loudspeaker be placed in the circuit sound is exactly reproduced.

Photo cells are often used in the construction of what is known as "Burglar's Alarm." A beam of light incident on a photo cell is so arranged that a burglar approaching a door way intercepts the beam. The sudden decrease in photo current may be used to start a current (in a separate circuit) which rings an alarm.

In the manufacture or processing of materials photo cells are sometimes used to detect imperfect articles and to actuate mechanisms which reject them. The speed and accuracy with which this may be done are far greater than what is possible by human agency.

Exercise XXIII

1. What is Faraday effect with regard to a plane polarised beam of light passing through the pole pieces of an electro-

magnet along the lines of force? In what way is the rotation of the plane of polarisation produced in the beam of light different from the rotation produced by an optically active substance?

2. Describe fully the phenomena known as Zeeman effect when a source of Na light is placed between the pole pieces of an electromagnet.

3. What is photo-electricity? Explain how Einstein explained this with the help of his now famous photo-electric equation.

4. Explain how photo-electricity has been utilised to reproduce sound in cinema films.

C. U. Questions

1963. Write short notes on Faraday effect.

1958, 1960. 1961, 1967. Write short notes on Photoelectric cell and its applications.

1968. What is photo-electric effect? How has it been explained on the basis of the quantum nature of light?

1969, 1970, 1973, 1975. Write notes on "Photo-electric effect."

APPENDIX A

Tan A Position

$$\frac{M}{H}=\frac{(r_1^2-l^2)^2}{2r_1}\tan\theta_1=\tfrac{1}{2}r_1^3\left(1-\frac{l^2}{r_1^2}\right)^2\tan\theta_1$$

$$\therefore\ \frac{M}{H}\left(1-\frac{l^2}{r_1^2}\right)^{-2}=\tfrac{1}{2}r_1^3\tan\theta_1 \text{ or } \frac{M}{H}\left(1+\frac{2l^2}{r_1^2}\right)=\tfrac{1}{2}r_1^3\tan\theta_1$$

$$\therefore\ \frac{M}{H}(r_1^2+2l^2)=\tfrac{1}{2}r_1^5\tan\theta_1$$

Similarly $\quad \frac{M}{H}(r_2^2+2l^2)=\frac{1}{2}r_2^5\tan\theta_2$

$\therefore$ by subtraction $\frac{M}{H}(r_1^2-r_2^2)=\frac{1}{2}\{r_1^5\tan\theta_1-r_2^5\tan\theta_2\}$

$$\therefore\ \frac{M}{H}=\tfrac{1}{2}.\ \frac{r_1^5\tan\theta_1-r_2^5\tan\theta_2}{r_1^2-r_2^2}$$

Tan B Position

$$\frac{M}{H}=(r_1^2+l^2)^{3/2}\tan\theta_1=r_1^3\left(1+\frac{l^2}{r_1^2}\right)^{3/2}\tan\theta_1$$

$$\therefore\ \frac{M}{H}\left(1+\frac{l^2}{r_1^2}\right)^{-\frac{3}{2}}=r_1^3\tan\theta_1 \text{ or } \frac{M}{H}\left(1-\frac{3l^2}{2r_1^2}\right)=r_1^3\tan\theta_1$$

$$\therefore\ \frac{M}{H}(r_1^2-\tfrac{3}{2}l^2)=r_1^5\tan\theta_1.$$

Similarly $\quad \frac{M}{H}(r_2^2-\frac{3}{2}l^2)=r_2^5\tan\theta_2$

$\therefore$ by subtraction $\frac{M}{H}(r_1^2-r_2^2)=r_1^5\tan\theta_1-r_2^5\tan\theta_2$

$$\therefore\ \frac{M}{H}=\frac{r_1^5\tan\theta_1-r_2^5\tan\theta_2}{r_1^2-r_2^2}$$

APPENDIX B

In modern engineering the units of length, mass and time are taken to be a Metre, Kilogram and a Second. And the fundamental unit in Electricity & Magnetism is taken to be that of current, viz. Ampere. This system is therefore known as MKSA system.

In this system the unit of force is one Newton. It is defined to be that force which produces an acceleration of 1 metre per sec^2 in a mass of 1 kilogram. Thus 1 Newton $=$ 1 kg $\times$ 1 metre/sec$^2 = 10^5$ gm cm/sec$^2 = 10^5$ dynes. Unit of energy is one Joule which is equal to 1 Newton $\times$ 1 metre $= 10^7$ dyne cm $= 10^7$ ergs.

The ampere its defined to be that current which when present in two parallel straight wires of infinite length and of negligible cross-section, placed at the distance of 1 metre apart, produces a force of 2×10^{-7} newton (per unit length *i.e*, per metre). Starting with this definition of unit current we may define units of all other electric and magnetic quantities. It will be seen that all these units are the same as practical units defined in the ordinary way.

The two fundamental equations in Electrostatics and Magnetism are now (1) $F = \frac{Q_1 Q_2}{4\pi k r^2}$ and (2) $F = \frac{m_1 m_2}{4\pi \mu r^2}$ where k and μ are respectively called electric and magnetic permittivity of the medium.

As a result of this modification Gauss's theorem in Electrostatics is modified as follows :—

The total normal induction over a closed surface is equal to the total charge inside the surface.

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